Analysis and approximation of coupled Navier–Stokes–Fokker–Planck systems in kinetic models of dilute polymers

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We review recent analytical and computational results for macroscopic-microscopic bead-spring models that arise from the kinetic theory of dilute solutions of incompressible polymeric fluids with noninteracting polymer chains, involving the coupling of the unsteady Navier–Stokes system in a bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2$ or 3, with an elastic extra-stress tensor as right-hand side in the momentum equation, and a (possibly degenerate) Fokker–Planck equation over the $(2d+1)$-dimensional region $\Omega \times D \times [0, T]$, where $D \subset \mathbb{R}^d$ is the configuration domain and $[0, T]$ is the temporal domain. The Fokker–Planck equation arises from a system of (Itô) stochastic differential equations, which models the evolution of a $2d$-component vectorial stochastic process comprised by the d-component centre-of-mass vector and the d-component orientation (or configuration) vector of the polymer chain. We show the existence of global-in-time weak solutions to the coupled Navier–Stokes–Fokker–Planck system for a general class of spring potentials including, in particular, the widely used finitely extensible nonlinear elastic (FENE) potential. The numerical approximation of this high-dimensional coupled system is a formidable computational challenge, complicated by the fact that for practically relevant spring potentials, such as the FENE potential, the drift term in the Fokker–Planck equation is unbounded on ∂D.