

Characterization and Construction of the Nearest Defective Matrix via Coalescence of Pseudospectra; Michael Overton, NYU

Let $w(A)$ be the distance from a matrix A to the set of defective matrices, using either the 2-norm or the Frobenius norm, and let $c(A)$ be the supremum of all ϵ with the property that the open ϵ -pseudospectrum of A has n distinct components in the complex plane. Demmel and Wilkinson independently observed in the early 1980s that $w(A) \geq c(A)$, and equality was established for the 2-norm by Alam and Bora in 2005. We give new results on the geometry of the pseudospectrum near points where coalescence of the components occurs, characterizing such points as the lowest generalized saddle point of the smallest singular value of $A - zI$ over $z \in C$. One consequence is that $w(A) = c(A)$ for the Frobenius norm too, and another is the perhaps surprising result that the minimal distance is attained by a defective matrix in all cases. Our results suggest a new computational approach to approximating the nearest defective matrix; numerical experiments that support the usefulness of this idea are given. In addition we present several open questions and conjectures.

Michael L. Overton Coauthors: R. Alam, S. Bora, R. Byers