The Implicit Closest Point Method for the Numerical Solution of Partial Differential Equations on Surfaces

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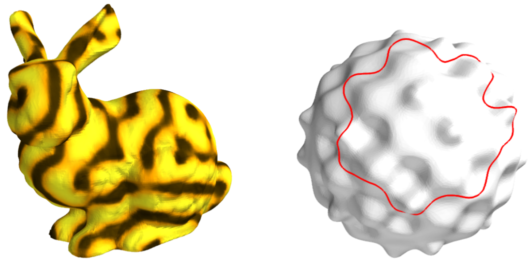
The Closest Point Method is a recent numerical technique for solving partial differential equations (PDEs) on general surfaces. For example, it can be used to solve a pattern-formation PDE on the surface of a rabbit or evolve a curve on a bumpy surface as shown below.



The Closest Point Method is simple to understand and implement. It combines standard finite difference schemes with an interpolation scheme in a narrow band enveloping the surface. However, the original formulation [1, 2] is designed to use explicit time stepping which may lead to a strict time-step restriction for some important PDEs.

In this presentation, we describe an implicit Closest Point Method [3] which allows large, stable time steps for high-order PDEs while retaining the principal benefits of the original method. Example computations (including the in-surface heat equation, reaction-diffusion on surfaces, Laplace–Beltrami eigenmodes, and fourth-order interface motion) on a variety of surfaces demonstrate the effectiveness of the method.

References

[1] S. J. Ruuth and B. Merriman, *A simple embedding method for solving partial differential equations on surfaces*, J. Comput. Phys., 227 (2008), pp. 1943–1961.

[2] C. B. Macdonald and S. J. Ruuth, *Level set equations on surfaces via the Closest Point Method*, J. Sci. Comput., 35 (2008), pp. 219–240.

[3] C. B. Macdonald and S. J. Ruuth, *The implicit Closest Point Method for the numerical solution of partial differential equations on surfaces*, (2008). Submitted to SIAM J. Sci. Comp.