International workshop on

Mathematics in the Science of Complex Systems

Background Paper

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Over the past decade the Science of Complex Systems has developed considerably. It is now widely accepted that the emerging science of complex systems is ICT enabled while most of the practical and theoretical problems in information and communications technologies arise because they are complex multilevel socio-technical systems. ICT is revolutionising the concept of data in the sciences. Traditional data collection instruments are giving way to massive real time data flows from telephony, electronic transactions and ubiquitous sensors. It is now commonplace to see systems modelled as discrete agent interactions with emergent micro- and meso-level dynamics aggregating into continuous upper meso- and macro level dynamics, with higher order emergence as top-down dynamics changing micro-level relationships and substructures.

Despite these huge steps forward, there remain problems that will not be solved by computer power, mass storage and ever-improving communications alone. For example, from biology to economies there is no formalism able to integrate the representation and dynamics of multilevel systems. The scientific problems of complex systems science are conceptual. Complex systems have new behaviours yet we try to model these behaviours using mathematics fine-tuned over hundreds of years to represent the relatively simple dynamics of physical systems.

Our workshop challenges the status quo and suggests that there is an exciting universe of new mathematical structures waiting to be constructed - new kinds of spaces with new kinds of algebraic, topological, and analytic properties that require new methods of investigation to make them tractable and comprehensible. Or is mathematics done – with all possible structures already discovered, making further exploration unnecessary?

The situation can be likened to the great voyages of the fifteenth century that discovered exotic new lands and continents. We have a vision that there are mathematical continents yet to be discovered, but to find them we must leave the familiar topography of smooth if occasionally craggy landscapes with continuous topologies and well manicured algebras. There is uncertainty and possible shipwreck on voyage, but also the possibility of discovering wonderful new things.

This workshop is intended to ask whether such new mathematical universes exist? To do so we will reflect on the mathematical world we know, trying to map it out and understand where there are gaps and opportunities. We will ask what this new universe might look like. Might the classical harmony and beauty of mathematics give way to discord and new generations of ugly objects that transform in ungainly and irregular ways; where closure is hard to achieve, operators have no simple inverses, and symbolic forms grow combinatorially making succinct identities rare and requiring knowledge of many unique objects with many unique properties interacting in ways with many exceptions. Or is it possible that by visiting such dystopian landscapes there may be completely new organising patterns to be discovered that make accessible the mysterious dynamics of complex systems - completely new mathematical systems to able to stimulate and sustain mathematical research for the next century?

The questions do not begin and end with mathematics. The science of complex systems is ICT-enabled with mathematical theories and models entangled with computation. Mathematical structures translate into data

structures, and data structures have mathematical properties. Computation has its own dynamics, and this fact impacts on ICT-based science. Thus the mathematics of complex systems science may have to advance in conjunction with new theories of computation.

The questions do not begin and end with mathematics and computation. Complex systems science is data-driven with ICT providing completely new kinds of data in volumes unthinkable just a decade ago. Many of the notions of prediction that underlie much conventional science are not applicable in the science of complex systems. In global systems scientists and policy makers are in the loop, making some predictions self-fulfilling prophesies and others inspired but often incorrect guesswork. Laboratory experiments are often not possible leaving policy and its implementation as the only practical means of experiment. But policy is conducted in vernacular language, and when policy is part of the system it too has to be modelled. Thus theories of complex systems may need to be meta-modelled in appropriate new and possibly computer-implemented logical frameworks. The dynamics of complex systems science present new kinds of empirical questions which require mathematics for modelling dynamics, computation for processing data and dynamics, and statistical theory for experimental design, data modelling and interpretation. Thus statistical theory and practice become entangled with mathematics and computation in the science of complex systems.

In this spirit of questioning the status quo and urging ourselves to think beyond current limits the workshop will address the questions:

- which areas of mathematics are used in complex systems science?
- what is the historical context? Have any outstanding problems been solved?
- have any new fields of mathematics or problems been generated ?
- how can statistical research contribute to complex systems science?
- how does mathematics interface to computation in CS science?
- what are the implications for applications in industry and for policy makers?
- are new logical frameworks necessary for the science of complex systems?
- what are the implications for education and training in CS science?
- are there completely new areas of mathematics waiting to be discovered?
- what are the ‘grand challenges’ for mathematics and mathematicians?

This workshop is the second in a series that began at ECLT in March 2008. In the meantime the science of complex systems has made great advances, and we can reflect on what has changed in the mathematical landscape since then. Have the changes been incremental, or are there some completely new areas emerging?

The form of the workshop centres on discussion and speculation. All participants are requested to fill no more than one page giving their answers to one or more of the questions above, or to provide their answer to what they think are better questions. These answers will be bound into a document to be circulated and discussed at the workshop. All participants will have fifteen minutes or more to present their ideas and/or answers to the questions, but the emphasis will be on open-ended discussion and the search for new perspectives.

The series of workshops is intended to result in action. We need an answer to the question “is existing mathematics sufficient for a science of complex systems”. If it is not we need new initiatives in mathematics research, including making funders aware of the challenges at national and European level. The workshop will end with a session to discuss these practical issues. The open-ended and divergent approach of this workshop will lay the foundations for more tangible outputs from the planned workshops in Paris in 2011and at ECLT in 2012. These will include an edited volume giving the state of the art of mathematics in the science of complex systems. This session will also include making plans to work with mathematical societies in Europe and elsewhere to drive this forward. If appropriate we will lobby for major new initiatives to support new and unusual lines of research in mathematics and its applications.