Long range correlations in non-equilibrium systems

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Joint works with

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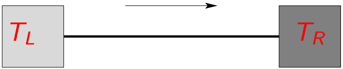
Outline

- Stochastic dynamics & Invariant measure
- Equivalence of ensembles for non-equilibrium systems
- Correlations and non-equilibrium phase transitions

Open systems with reservoirs



Heat reservoirs $T_L \neq T_R$

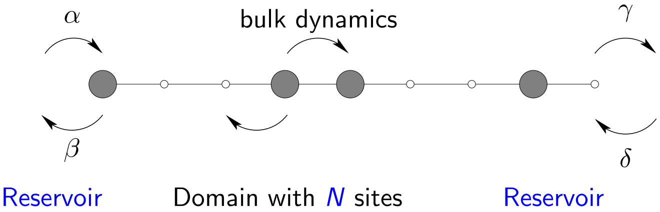
Current flowing through the system : $\begin{cases} \textit{Heat} \\ \textit{Electrons} \\ \textit{Particles} \end{cases}$

Questions.

- Structure of the steady state
- Long range correlations

Stochastic particle systems

Particles: $\eta(t) = \{\eta_i(t)\}_{i \leq N} \in \{0,1\}^N$



Left reservoir acting at site 0:

- ➪ particle creation at rate α
- ➪ particle annihilation at rate β

Right reservoir acting at site N

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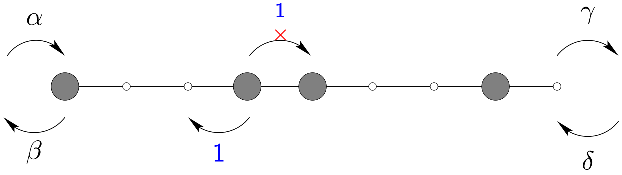

- Symmetric Simple Exclusion Process (SSEP)

$(\alpha, \beta) = (\gamma, \delta)$ Reversible markov chain.
Steady state = product Bernoulli measure

$(\alpha, \beta) \neq (\gamma, \delta)$ Non reversible markov chain.

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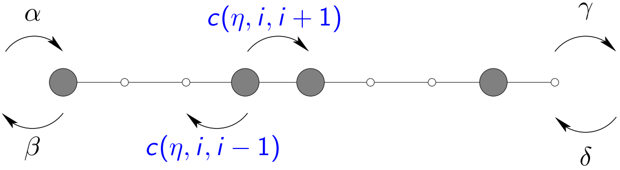

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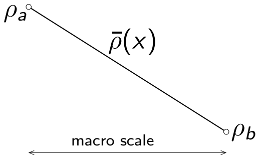
- More general dynamics (Kawasaki dynamics)

SSEP invariant measure



Linear density profile $\bar{\rho}(x)$

$$\rho_a = \frac{\alpha}{\alpha+\beta}, \qquad \rho_b = \frac{\delta}{\delta+\gamma}$$

Truncated two-point correlation function

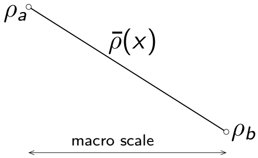
$$i < j, \qquad \langle \eta_i ; \eta_j \rangle = \langle \eta_i \eta_j \rangle - \langle \eta_i \rangle \langle \eta_j \rangle = \frac{1}{N} C^{\text{open}}\left(\frac{i}{N}, \frac{j}{N}\right)$$

with

$$0 \leq x < y \leq 1, \qquad C^{\text{open}}(x,y) = -(\rho_a - \rho_b)^2 \, x(1-y) \, .$$

- Current $= \rho_a - \rho_b$
- Local equilibrium : Correlations are of order $1/N$
- Global contribution : $\sum_{i \neq j} \langle \eta_i ; \eta_j \rangle = -\frac{N}{12}$

SSEP invariant measure



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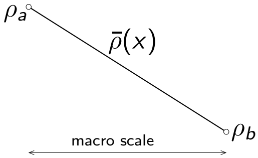

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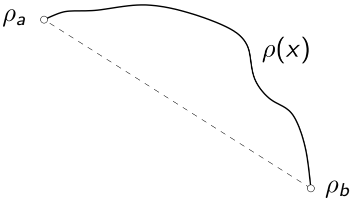
Derivation of the correlations

- Exact microscopic computations [Spohn]
- Density large deviations [Derrida, Lebowitz, Speer]

$$\mathcal{F}(\rho) = \lim_{N\to\infty} -\frac{1}{N} \log \langle \text{observing } \rho \rangle$$



Macroscopic density profile:

$0 \leq x \leq 1, \qquad \rho(x)$

Large deviations : $\langle \text{observing } \rho \rangle \simeq \exp\left(- N\mathcal{F}(\rho)\right)$

Let λ be a smooth function in $[0,1]$

$$\frac{1}{N} \log \Big\langle \exp\Big(\sum_{i=1}^{N} \lambda\left(\frac{i}{N}\right) \eta_i \Big) \Big\rangle \to \mathcal{G}(\lambda)$$

with $\mathcal{G}(\lambda) = \sup\{\int_0^1 dx\, \rho(x)\lambda(x) - \mathcal{F}(\rho)\}$.

For λ small

$$\mathcal{G}(\lambda) = \int_0^1 dx\, \bar\rho(x)\,\lambda(x) + \frac{1}{2} \int_0^1 dx\, \bar\rho(x)(1-\bar\rho(x))\lambda(x)^2 + \frac{1}{2} \int_0^1 \int_0^1 dxdy\, C^{\text{open}}(x,y)\lambda(x)\lambda(y)$$

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Hydrodynamic limit

Diffusive hydrodynamic scaling $(x = i/N, t = \tau/N^2) \Rightarrow \begin{cases} \{0, N\} \hookrightarrow [0,1] \\ \eta_i(\tau) \hookrightarrow \rho_{(x,t)} \end{cases}$

Typical density: $\partial_t \bar{\rho}_{(x,t)} = \partial_x \big(D(\bar{\rho}_{(x,t)}) \partial_x \bar{\rho}_{(x,t)} \big)$

Typical current: $\bar{q}_{(x,t)} = -D(\bar{\rho}_{(x,t)}) \partial_x \bar{\rho}_{(x,t)}$

An arbitrary macrosocopic evolution $(\rho_{(x,t)}, q_{(x,t)})$ with $t \in [0, T]$

$$\big\langle \text{observing } (\rho_{(x,t)}, q_{(x,t)}) \big\rangle \approx \exp\big(- N \mathcal{I}_{[0,T]}(\rho, q) \big)$$

with $\partial_t \rho = -\partial_x q$ then

$$\mathcal{I}_{[0,T]}(\rho, q) = \int_0^T dt \int_0^1 dx \, \frac{\big(q_{(x,t)} + D(\rho_{(x,t)}) \partial_x \rho_{(x,t)}\big)^2}{2\sigma(\rho_{(x,t)})}$$

[B. , Derrida]

[Bertini, De Sole, Gabrielli, Jona-lasinio, Landim]

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A dynamical approach to compute the steady state

$$\Big\langle \text{observing the density } \gamma(x) \Big\rangle = \Big\langle \text{observing } \gamma(x) \text{ at time } T \Big\rangle$$

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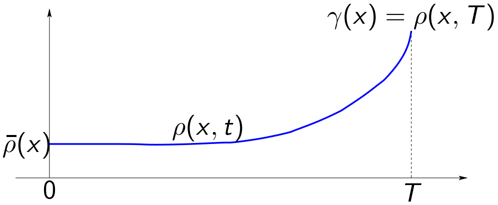
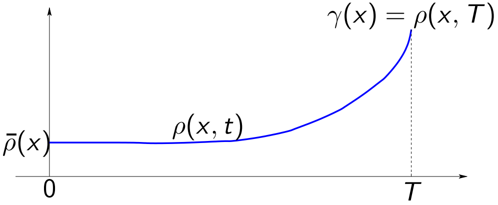


A dynamical approach to compute the steady state

$$\big\langle \text{observing the density } \gamma(x) \big\rangle = \big\langle \text{observing } \gamma(x) \text{ at time } T \big\rangle$$
$$\simeq \exp\Big(-N \inf_{\rho,q} \mathcal{I}_{[0,T]}(\rho, q) \Big)$$



$$\mathcal{F}(\gamma) = \lim_T \inf\{\mathcal{I}_{[0,T]}(\rho, q); \qquad \rho(x,0) = \bar\rho(x),\ \rho(x,T) = \gamma(x)\}$$

[Freidlin,Wentzell]
[Bertini, De Sole, Gabrielli, Jona-Iasinio, Landim]

Two-point correlation function

$$\mathcal{F}(\gamma) = \lim_{T\to\infty} \inf \left\{ \mathcal{I}_{[0,T]}(\rho, q); \qquad \rho(x,0) = \bar{\rho}(x), \ \rho(x,T) = \gamma(x) \right\}$$

Perturbation around the steady state: $\gamma(x) = \bar{\rho}(x) + \varepsilon\varphi(x)$
Optimal trajectory at the second order in ε

➪ Expansion of $\mathcal{F}$ wrt ε

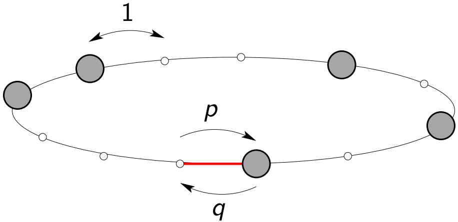
Alternative approaches.

- Fluctuating hydrodynamics
- Martin-Siggia-Rose

SSEP driven by a Battery

Periodic system: $\eta(t) = \{\eta_i(t)\}_{i \leq N} \in \{0,1\}^N$

Total number of particles conserved



At the Battery: $p \left\langle \eta_N(1-\eta_1) \right\rangle = q \left\langle \eta_1(1-\eta_N) \right\rangle$

Local Equilibrium :

$\Rightarrow$ Product measure with different densities across the battery

$$p\langle\eta_N\rangle(1-\langle\eta_1\rangle) = q\langle\eta_1\rangle(1-\langle\eta_N\rangle)$$

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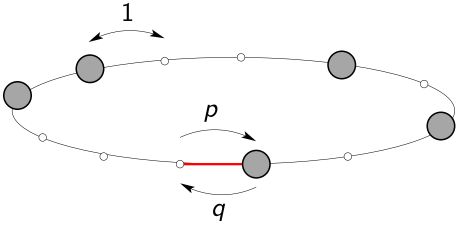

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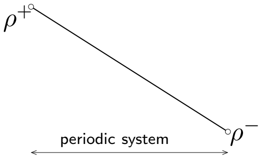
Steady State: Mean density $\bar{\rho}$



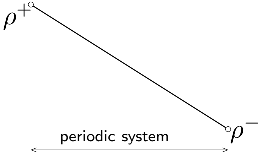
$$\rho^+ + \rho^- = 2\bar{\rho}$$

$$p\ \rho^-(1-\rho^+) = q\ \rho^+(1-\rho^-)$$

- Same density profile as in the open system
- At the leading order the stationary measure is locally a product measure.

SSEP driven by a Battery

Steady State: Mean density $\bar{\rho}$



$$\rho^+ + \rho^- = 2\bar{\rho}$$

$$p\ \rho^-(1-\rho^+) = q\ \rho^+(1-\rho^-)$$

Questions.

1. Equivalence of ensembles
2. Long range correlations ?
   ➪ Non-equilibrium + Micro-canonical constraint

Hydrodynamic limit

Heat equation: $\partial_t \rho(x,t) = \Delta \rho(x,t)$
Non linear boundary conditions:
$p\,\rho(1,t)(1-\rho(0,t)\,) = q\,\rho(0,t)(1-\rho(1,t)\,), \qquad \partial_x \rho(0,t) = \partial_x \rho(1,t)$

Fluctuation around the steady state : $\rho(x,t) = \bar{\rho}(x) + \varepsilon f(x,t)$

$$\partial_t \rho(x,t) = \Delta \rho(x,t), \qquad f(1,t) = a f(0,t), \quad \partial_x \rho(0,t) = \partial_x \rho(1,t)$$

with $a = \frac{\rho^+(1-\rho^+)}{\rho^-(1-\rho^-)}$.

Green's function

$$G_t(x,y) = \exp(-t\Delta)$$

Two-point correlation function

Current : $\mathcal{J} = \rho^+ - \bar{\rho}^-$

$0 \leq x \leq y \leq 1$

$$C^{\mathrm{bat}}(x,y) = -\frac{2}{(a+1)^2}\left(\frac{\sigma(\bar{\rho}^+)+\sigma(\bar{\rho}^-)}{2} + \frac{\mathcal{J}^2}{3}\right)(ax+1-x)(ay+1-y)$$
$$-2\mathcal{J}^2 \int_0^\infty dt \int_0^1 dz\, G_t(x,z)G_t(y,z) - G_t(x,0)G_t(y,0)$$

$$C^{\mathrm{open}}(x,y) = -\mathcal{J}^2\, \Delta^{-1}_{\mathrm{Dirichlet}}(x,y)$$

➪ Similar structure, Singularity at the battery.

➪ Small current $\mathcal{J}$:

$$C^{\mathrm{bat}}(x,y) = -\frac{\sigma(\bar{\rho})}{2}\left(1 + \mathcal{J}\frac{\sigma'(\bar{\rho})}{\sigma(\bar{\rho})}(-1+x+y)\right)$$

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Half-filling: $\bar{\rho} = 1/2$

$0 \leq x \leq y \leq 1$

$$C^{\mathrm{bat}}(x,y) = -\frac{1}{2}\sigma(\bar{\rho}^{+}) + \frac{1}{12}\mathcal{J}^2 - \mathcal{J}^2\left(\frac{1}{2}(x+y)(1-(x+y)) + x\right)$$

$$+8\mathcal{J}^2 \sum_{k,n\geq 1}^{\infty} \frac{\cos(2\pi nx)\cos(2\pi ky)}{(2\pi)^2(k^2+n^2)}$$

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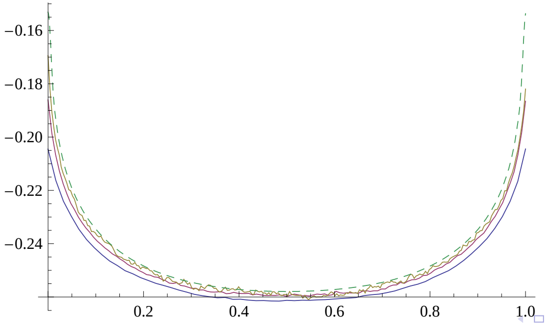

$i \to N\langle \eta_i ; \eta_{i+1} \rangle$

System sizes:
$N = 64, 128, 256$

Zero range with battery

Zero range with jump rates $g(n)$.

Driven by reservoirs. [De Masi, Ferrari]
Product measure with local density $\bar{\rho}(i)$

Driven by a battery.
Product measure with local density $\bar{\rho}(i)$ conditioned to a fixed density

ABC

q	1
$AB \rightarrow BA$	$BA \rightarrow AB$
$BC \rightarrow CB$	$CB \rightarrow BC$
$CA \rightarrow AC$	$AC \rightarrow AC$

Densities: r_A, r_B, r_C

System on the ring $\{1, N\}$

$$q = \exp(-\frac{\beta}{N})$$

[Evans, Kafri, Koduvely, Mukamel]
[Kafri, Biron, Evans, Mukamel]
[Clincy, Derrida, Evans]
[Ayyer, Carlen, Lebowitz, Mohanty, Mukamel, Speer]

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Phase transition.

$\beta < \beta_c$: Density profiles are flat $= r_A, r_B, r_C$

$\beta > \beta_c$: Densities are space dependent: phase segregation

ABC: Two-point correlation function

$r_A = r_B = r_C = 1/3.$ [Evans, Kafri, Koduvely, Mukamel]

- ➪ Explicit expression for the steady state (reversibility)
- ➪ Large deviation function

General case.

When the phase transition is 2nd order: $\beta_c = \frac{2\pi}{\sqrt{1-2(r_A^2+r_B^2+r_B^2)}}$

$$\langle A_i ; A_j \rangle = \langle A_i A_j \rangle - \langle A_i \rangle \langle A_j \rangle = \frac{1}{N} C_{AA}\left(\frac{i}{N} - \frac{j}{N}\right)$$

$$C_{AA}(x) = -r_A(1-r_A) - \frac{\alpha}{\sin(\frac{\pi\beta}{\beta_c})} \left(\beta \cos\left(\frac{\pi\beta}{\beta_c}(2x-1)\right) - \frac{\beta_c}{\pi} \sin(\frac{\pi\beta}{\beta_c}) \right)$$

$$\lim_{\beta \to \beta_c} C_{AA}(x) = \infty$$

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Phase transition WASEP

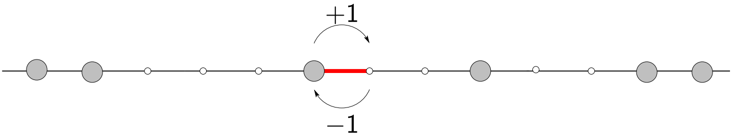
WASEP on a ring $\{1, N\}$

$$\text{jump rates} = \begin{cases} 1 + \frac{\nu}{N} & \bullet \curvearrowright \circ \\ 1 - \frac{\nu}{N} & \circ \curvearrowleft \bullet \end{cases}$$

μ_N stationary measure:
Bernoulli with density $\bar{\rho}$

Integrated current through the edge $(i, i+1)$:

$$Q_{[0,\tau]} = \text{Number of jumps from } i \text{ to } i+1 \text{ during } [0,\tau]$$
$$- \text{Number of jumps from } i+1 \text{ to } i \text{ during } [0,\tau]$$



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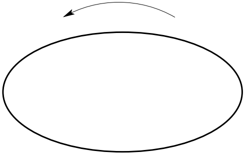

Mean current:

$$\mathbb{E}^{\nu}_{[0,\tau]}\left(\frac{Q_{[0,\tau]}}{\tau}\right) = \frac{\nu}{N} 2\bar{\rho}(1-\bar{\rho}) = \frac{\nu}{N}\sigma(\bar{\rho})$$

Definition

For a current $\mathcal{J}$, the constrained mesure is given by

$$\mu_{\mathcal{J},N}(F(\eta)) = \lim_{T} \mathbb{E}\Big(F(\eta(t=0)) \Big| \quad \frac{1}{2T} Q_{[-T,T]} = \mathcal{J} \Big)$$

The two-point correlation function is

$$\mu_{\mathcal{J},N}(\eta_i;\eta_j) = \mu_{\mathcal{J},N}(\eta_i;\eta_j) - \mu_{\mathcal{J},N}(\eta_i)\mu_{\mathcal{J},N}(\eta_j)$$

For $\mathcal{J} > q_c^* = \nu\sigma(\bar{\rho})\sqrt{1 - \frac{\pi^2}{2\nu^2\sigma(\bar{\rho})}}$

The correlation scales for large N as

$$\mu_{\mathcal{J},N}(\eta_i;\eta_j) = \frac{1}{N}\frac{\sigma(\bar{\rho})}{2}\left(-1 + \sum_{k\geq 1} 2C_k \cos(2\pi k \frac{i-j}{N})\right)$$

with $\sigma(\rho) = \rho(1-\rho)$

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Consequence. When $\mathcal{J} \to q_c^*$ the correlations blow ($C_1 \to \infty$). Precursor of the macroscopic clustering which occurs after the transition.

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Phase transition on a ring

Large deviations on the ring $\{1, N\}$ for the WASEP

$$\lim_T \lim_N \frac{1}{TN^2} \log \mathbb{P}^{\nu}_{[0,TN^2]} \left(\frac{Q_{TN^2}}{TN^2} \sim \frac{\mathcal{J}}{N} \right) = G^{\nu}(\mathcal{J})$$

with $G^{\nu}(\mathcal{J}) = \lim_{T\to\infty} \inf_{\rho} \left\{ \frac{1}{T} \mathcal{I}^{\nu}_{[0,T]}(\rho) \right\}$

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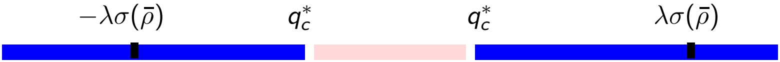

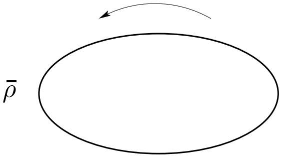

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Optimal density profile $= \bar\rho$

Stability of the flat profile $\bar\rho$

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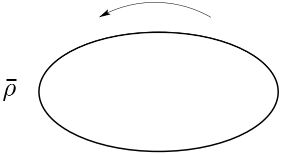

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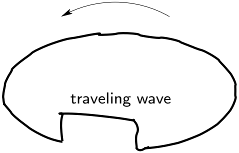

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Conlusion

- Long range correlations in non-equilibrium models
- Equivalence of ensembles
- Long range correlations and non-equilibrium phase transitions