

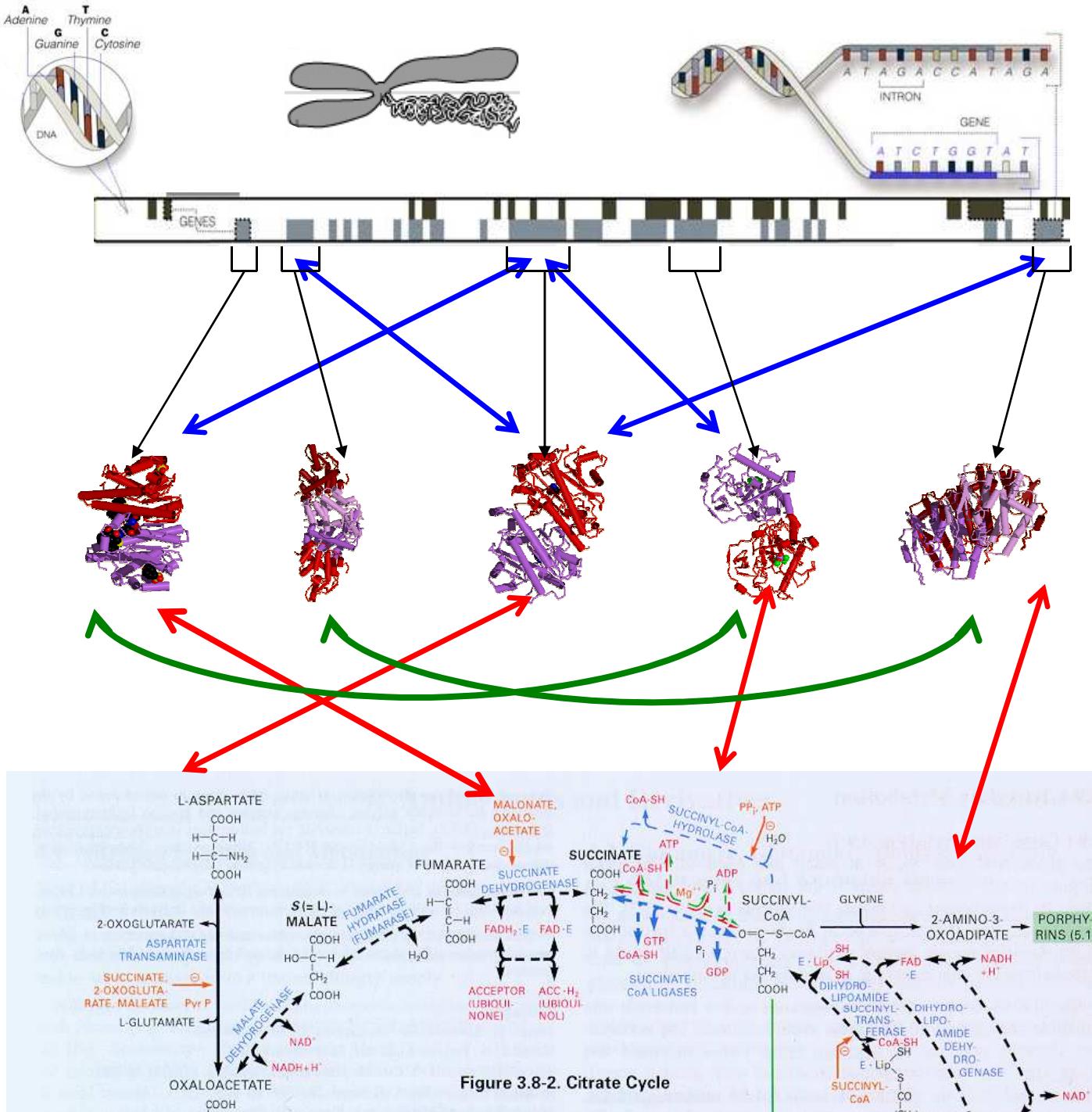
Winter School in Network Theory and Applications

Warwick, Jan 5-9 2011

Evolution of biological networks

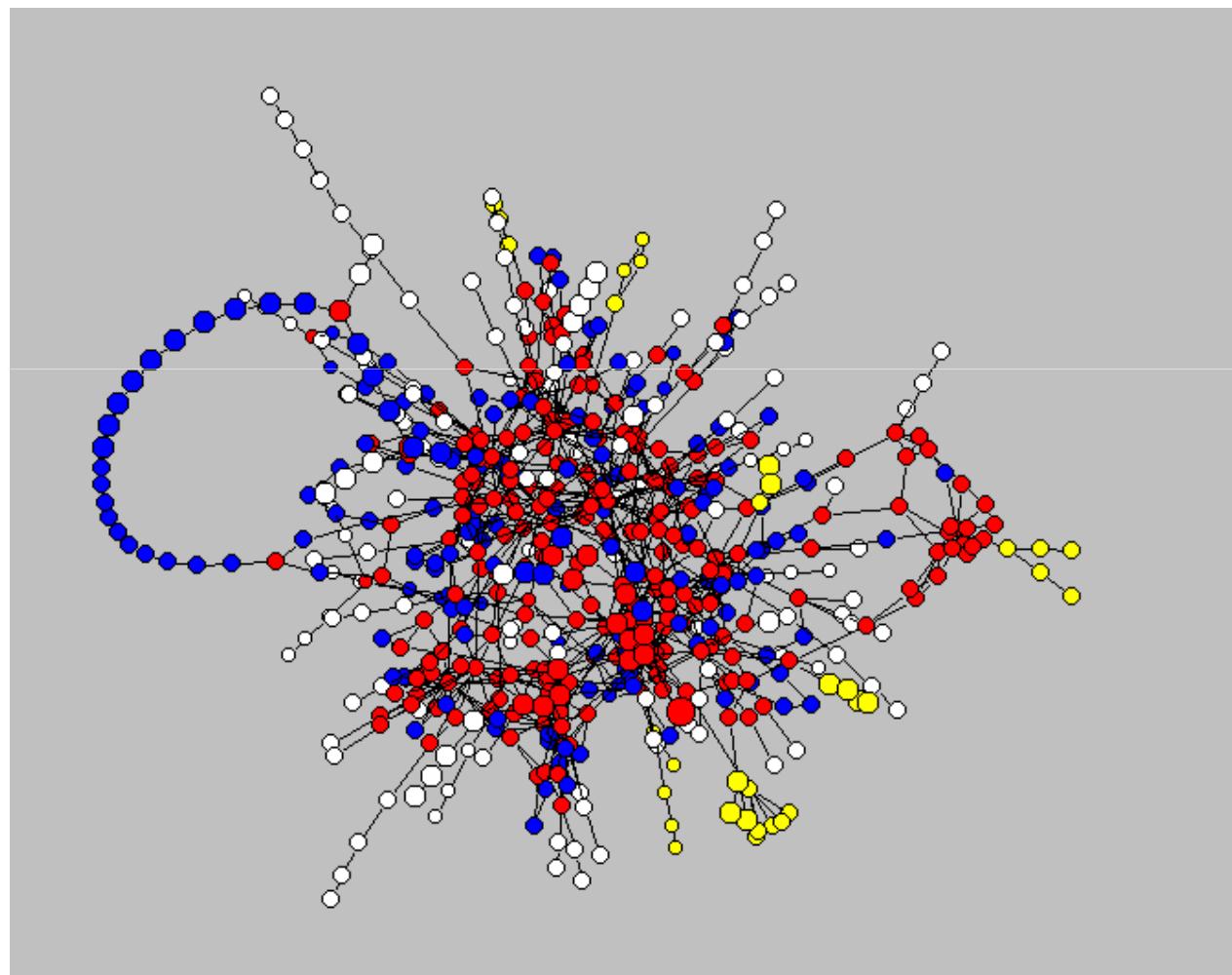
Ginestra Bianconi

Physics Department, Northeastern University, Boston, USA



Metabolic Network

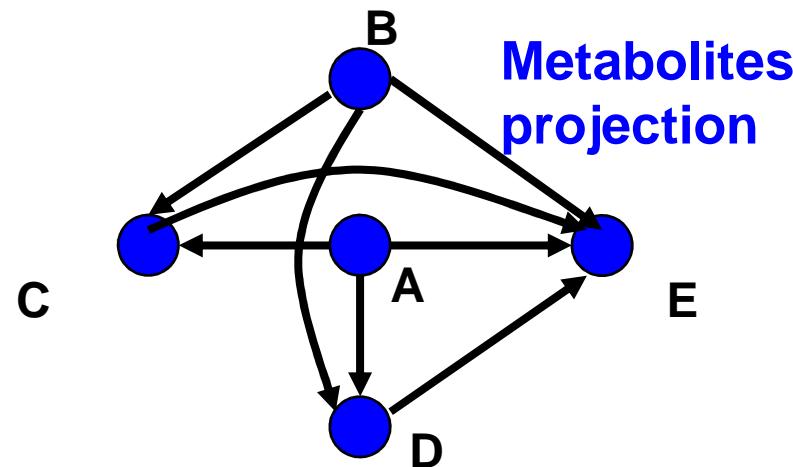
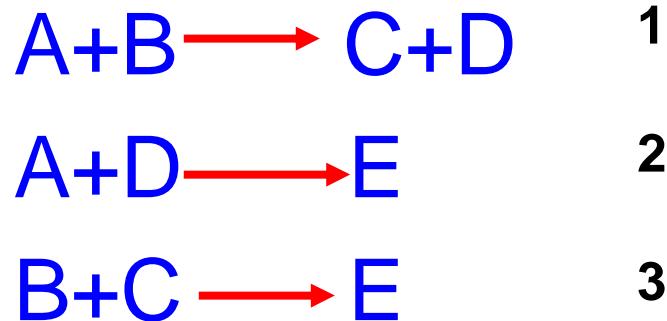
Nodes: chemicals (substrates)
Links: bio-chemical reactions



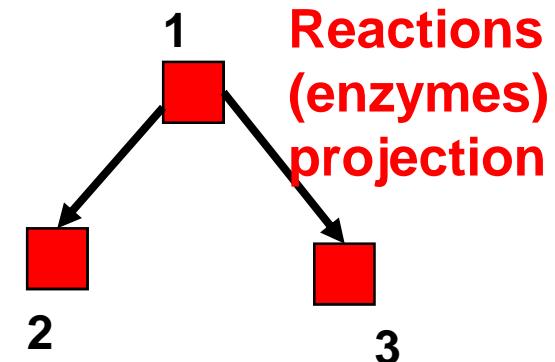
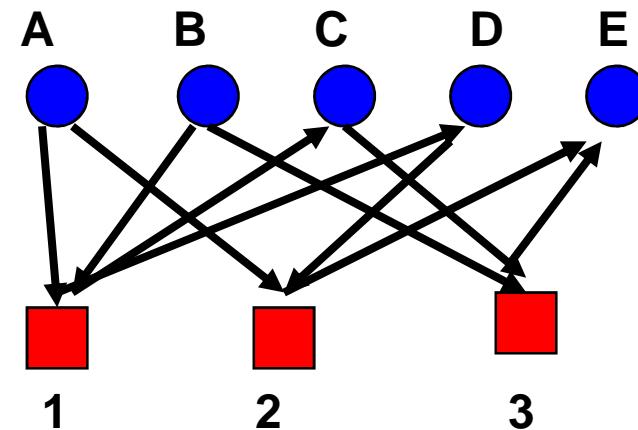
S.cerevisiae

Metabolic network

Reaction pathway



Bipartite Graph



Stoichiometric matrix

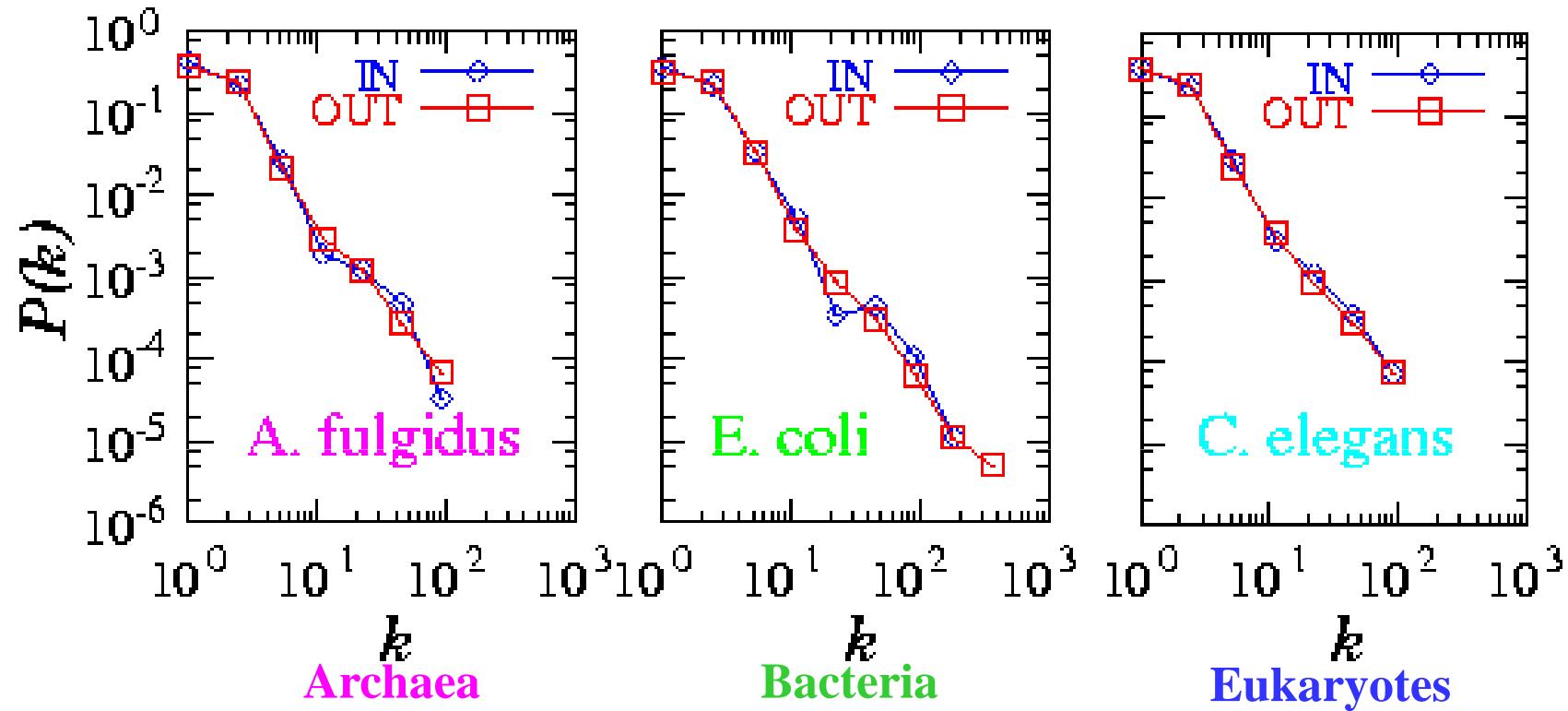


Metabolic Reaction

Substrates	Reactions				
	v_1	...	v_j	...	
A	-a
B	0
C	-c
D	d
E	0

- Each **gene** can contribute to the flux of multiple reactions.
- **Every reaction** can be catalyzed by the complex of more than one gene product.

Metabolic network



Organisms from all three domains of life are
scale-free networks

Some example of hub
substrates are ATP, ADP

H. Jeong et al. Nature, 407 651 (2000)

Enzymatic reactions

Michael-Menten model



Enzymes speed up reactions of a factor $10^3\text{-}10^{17}$

$$V = [ES]k_{cat}$$

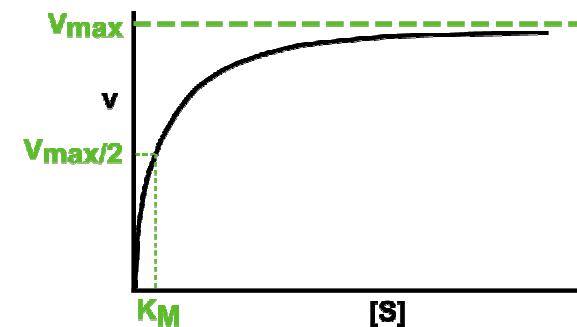
$$[ES](k_- + k_{cat}) = k_+[E][S]$$

$$[ES] + [E] = E_0$$

$$V = V_{max} S / (S + K_M)$$

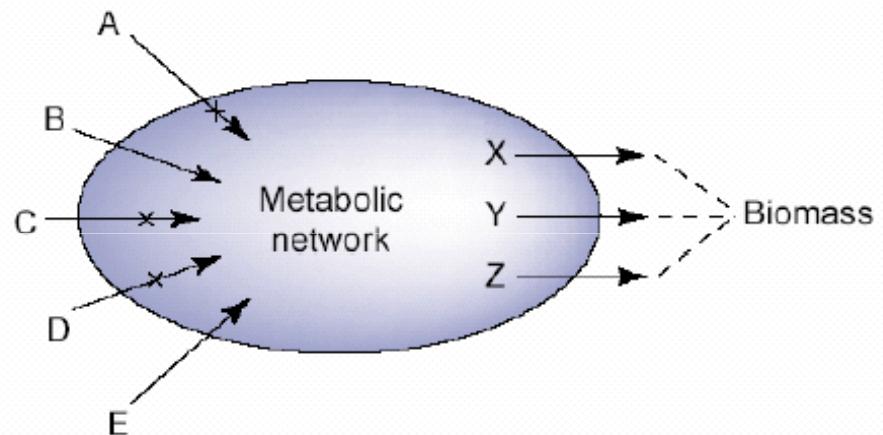
$$V_{max} = k_{cat} E_0$$

$$K_M = (k_{cat} + k_-) / k_+$$



Flux balance analysis

- Reconstruct the network from genomic and biochemical studies
 - Transport processes
 - Direction of reactions
 - Stoichiometry of reactions
- Identify major metabolic components of the cell:
biomass (X,Y,Z)
- Specify the **nutrient** present in the environment



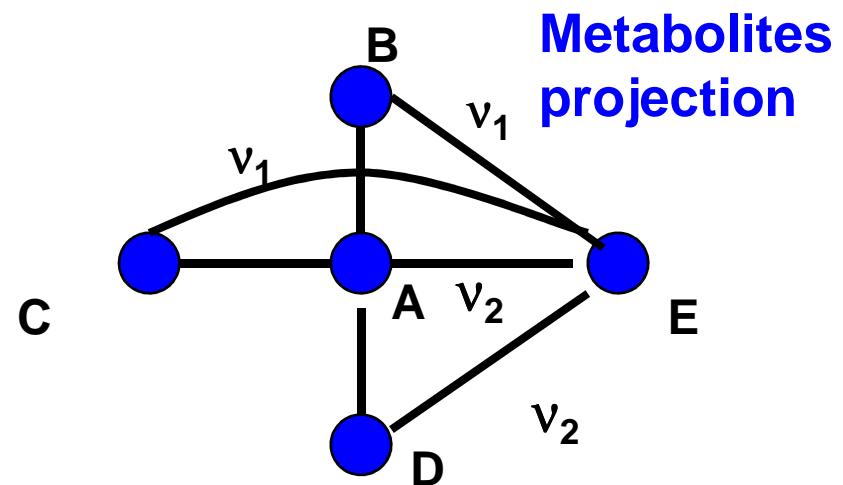
Flux-Balance-Analysis

Flux Balance-Analysis assume that the metabolic network is in the steady state that maximize biomass production and satisfy the physical limitation to the fluxes.

$$\frac{d[x_i]}{dt} = \sum_{j=1,N} S_{i,j} v_j = 0$$

$$\alpha_j < v_j < \beta_j$$

$$Z = \sum_{\substack{i \in \text{Biomass} \\ j}} S_{ij} v_j$$



$i=1, \dots, M$ with typically $M < N$
Null space

Limitations

Maximize Z i.e. biomass production

What is flux-balance analysis good for?

1. It finds flux distribution which **maximize** **biomass** production (at steady state for given network and nutrients)
2. **Predicts enzymatic flux** distribution for wild type and gene knockout experiments
3. Predicts relative growth rate (**fitness**) estimates for gene knockout strains to the wild type

Epistatic network

Interaction between couple of mutations
in the network of 890 metabolic
genes in *S. Cervisiae*.

Fitness of mutations:

$$W^X = V_{\text{growth}}^{\Delta X} / V_{\text{growth}}^{\text{WT}},$$

V rate of biomass

production.

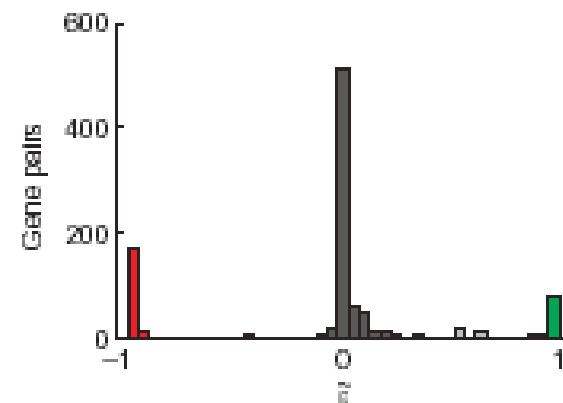
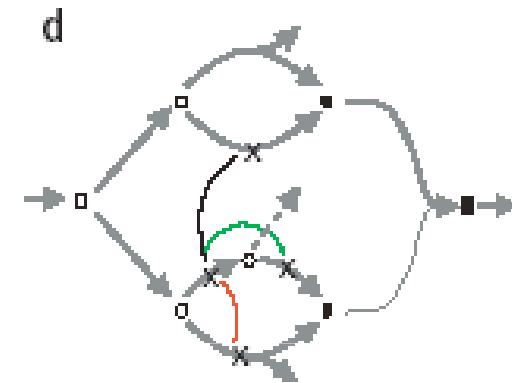
Types of interactions between mutations:

$$\varepsilon = W^{XY} - W^X W^Y$$

No epistasis $\varepsilon=0$

Aggravating $\varepsilon>0$

Buffering $\varepsilon<0$

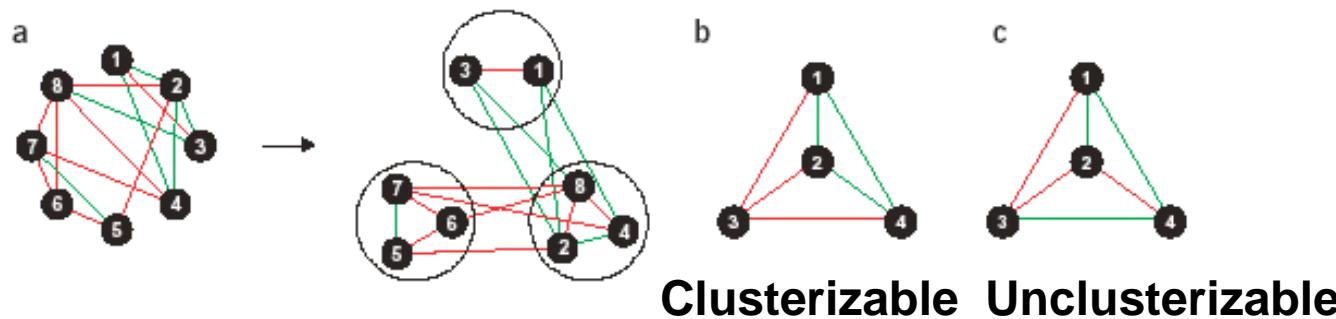


Monochromatic network

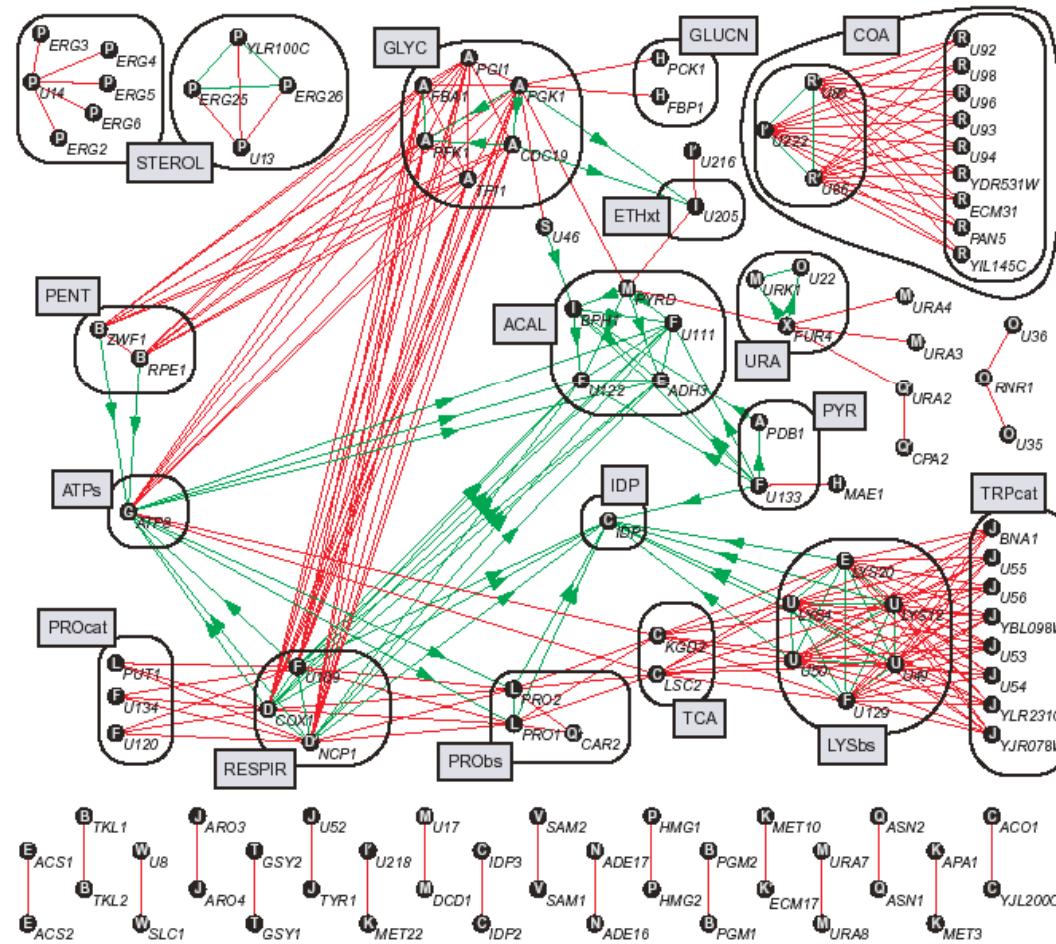
Epistatic interaction between genes assigned to function tend to be monochromatic (of the same sign).

One can then tend to do un unsupervised clustering analysis decomposing the network in monochromatically interacting modules for which the color of intra modules interactions is maximally monochromatic

– Prism algorithm

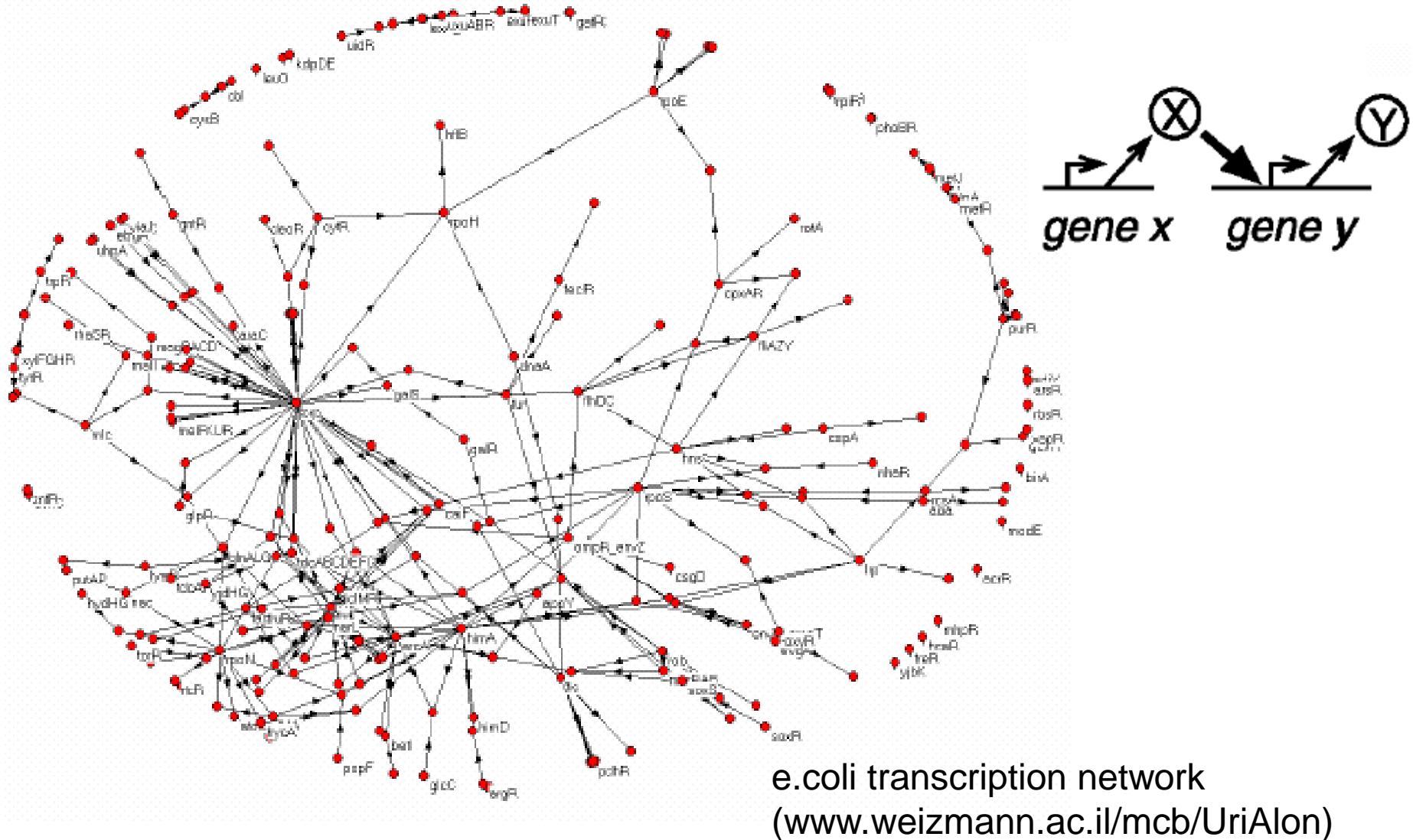


S. Cervisiae epistatic network

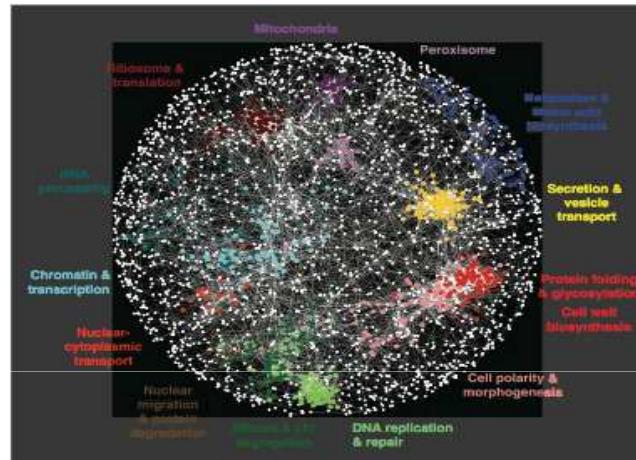


Segrè et al. Nature Gen.(2005)

Transcription network

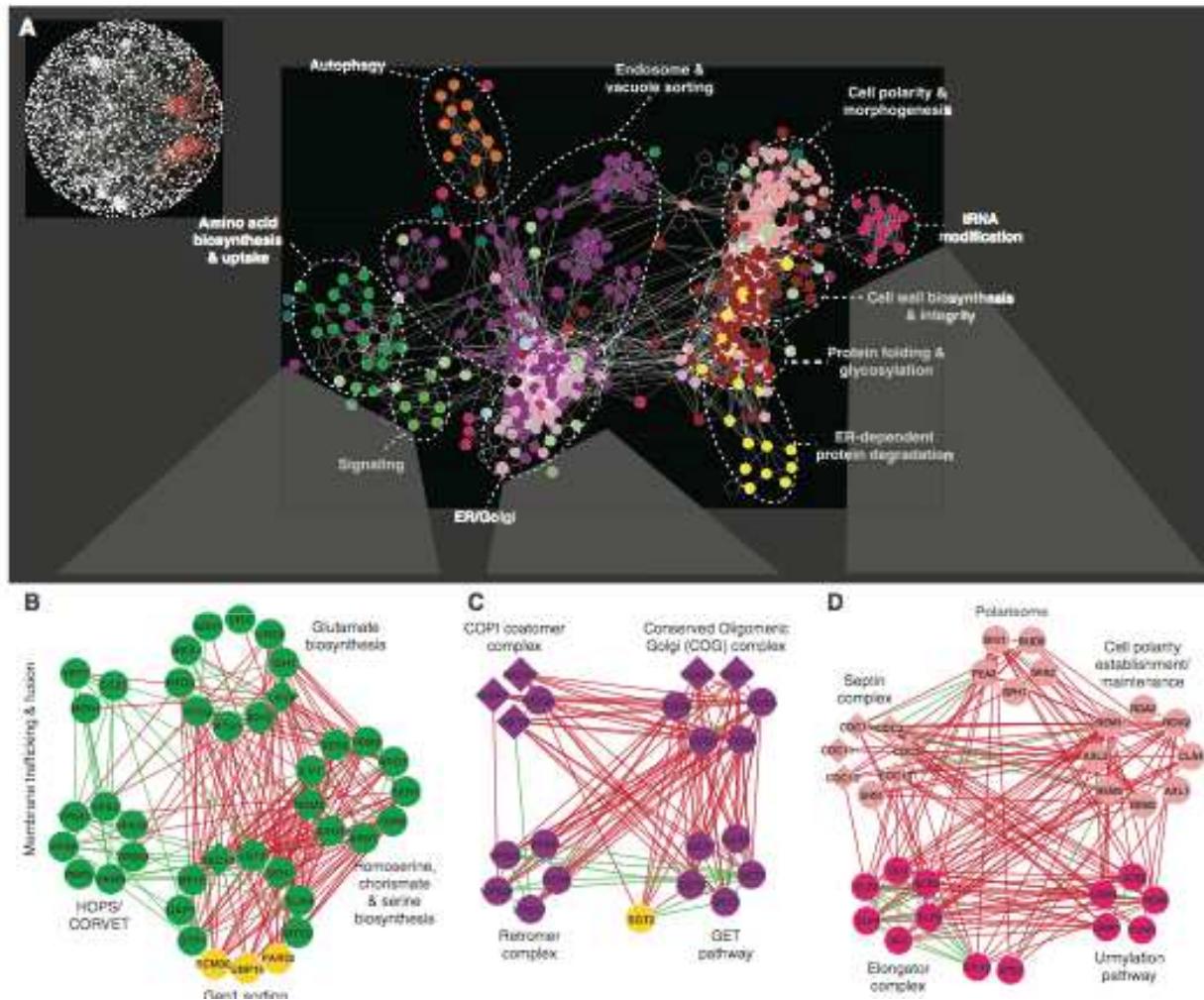


Epistatic network of *S. cerevisiae*



~5.4 million pairs of double mutations
in 1712 genes out of ~6000 genes
20% genes are essential

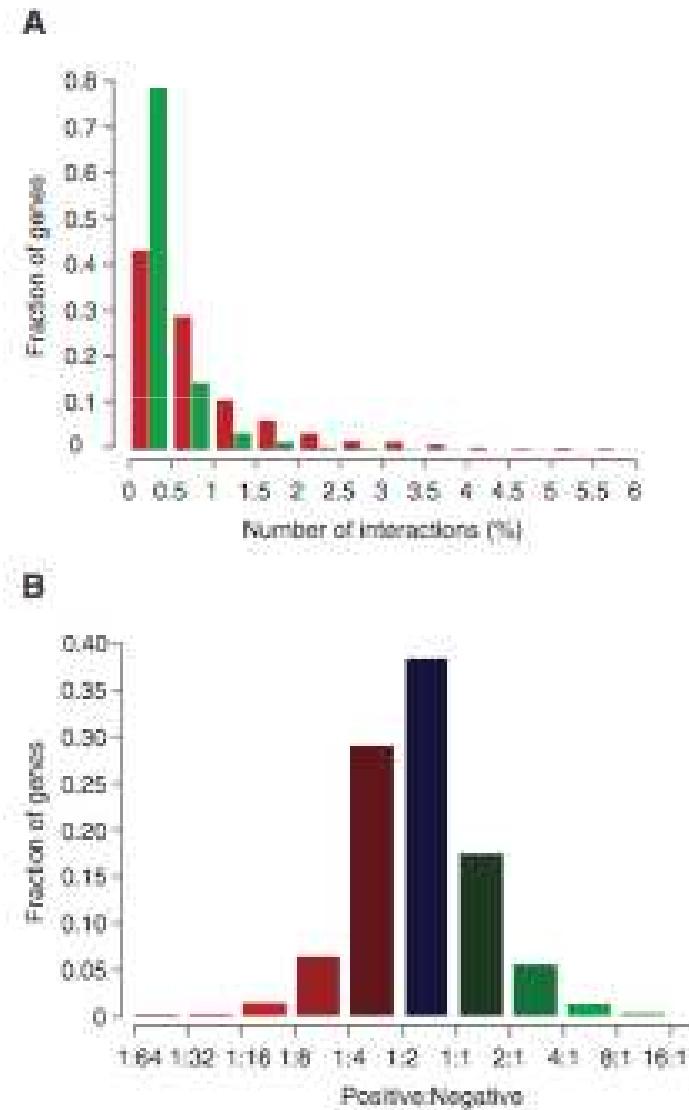
More aggravating interactions than buffering interactions
Costanzo et al. Science 2010

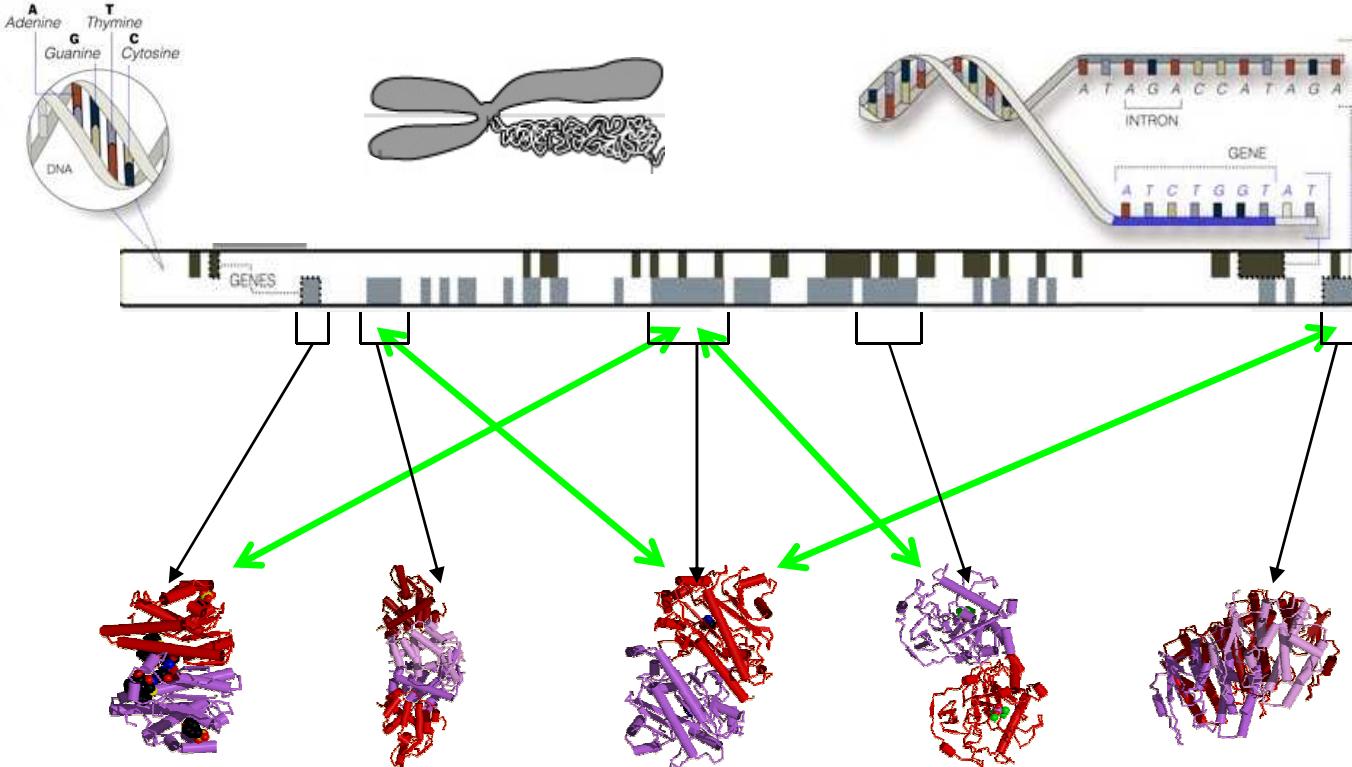


- Genes belonging to the same pathway or biological process tend share similar profile of genetic interactions
- Functional modules tend to interact monochromatically

Degree distribution of the epistatic network

- Degree distribution of the epistatic network
- The ratio between positive and negative interactions





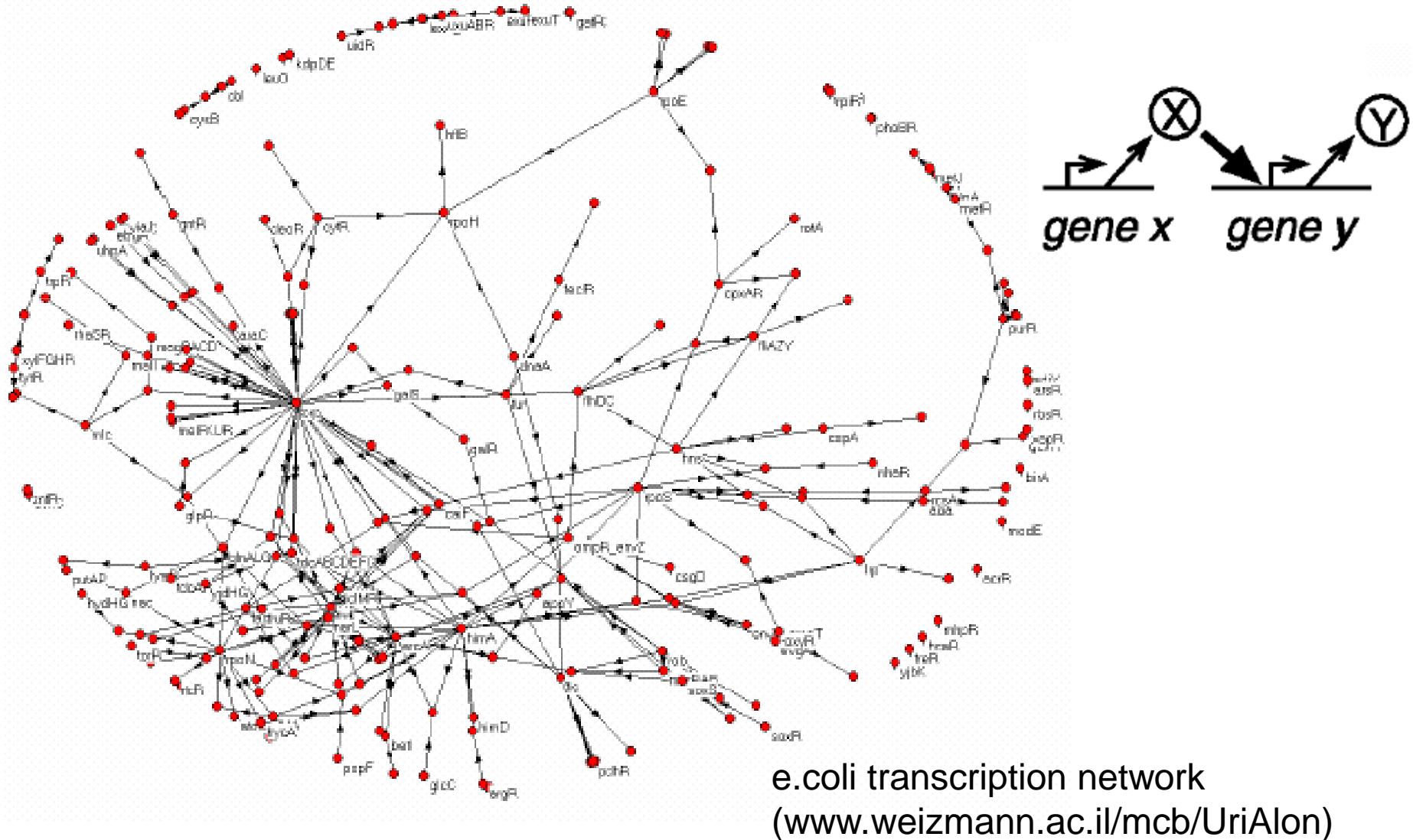
GENOME

transcription
networks

PROTEOME

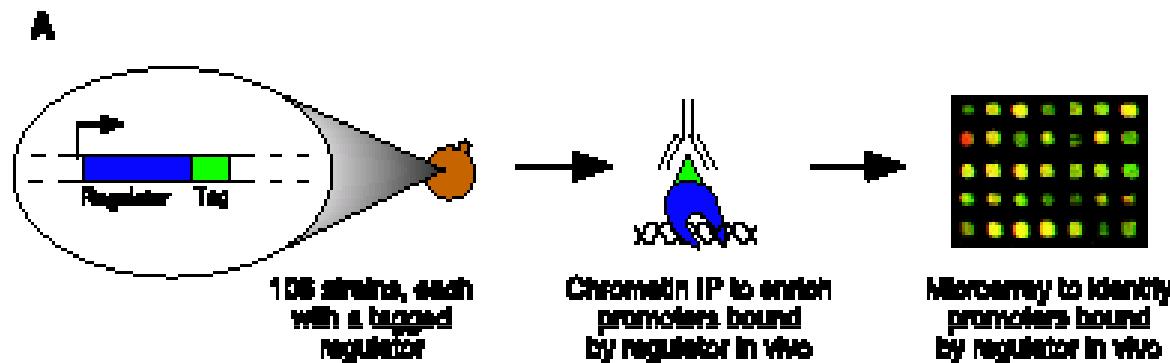
Transcription networks

Transcription network



High-throughput experiments

RESEARCH ARTICLES



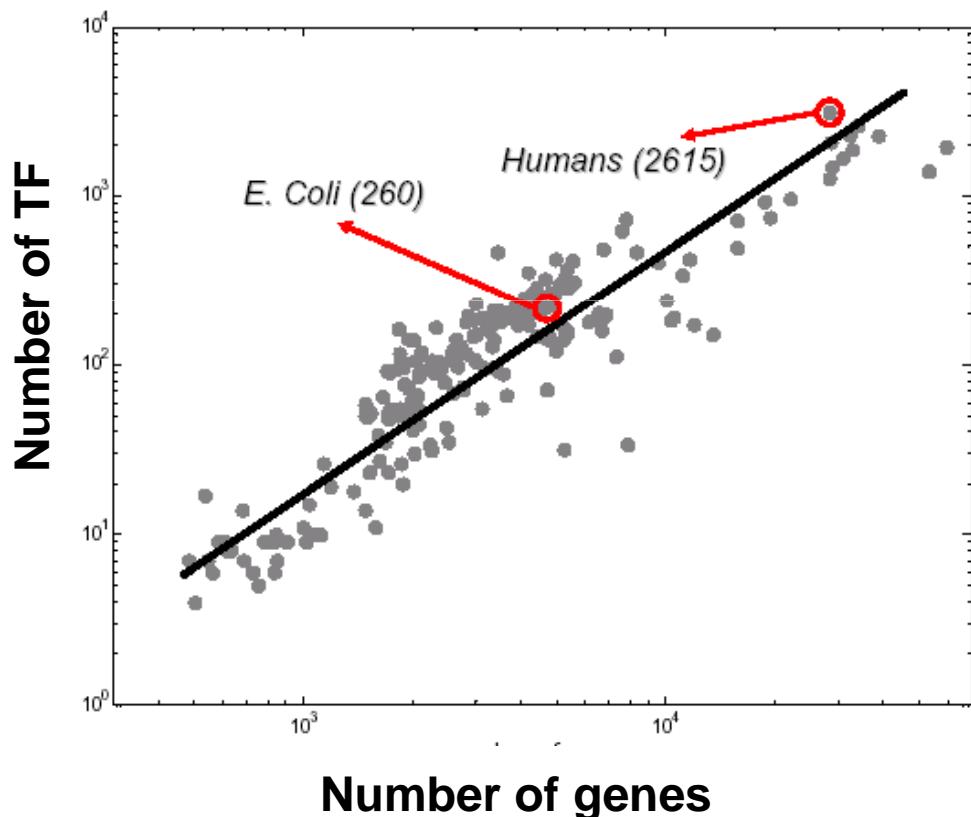
Yeast
4000 interactions
2343 regulators and promoter regions
106 transcription factors
Lee et al. Nature 2002

Alternatively or
additionally one
can consider

Collection or known regulations
from the literature

www.weizmann.ac.il/mcb/UriAlon

Number of Transcription Factors in different organisms

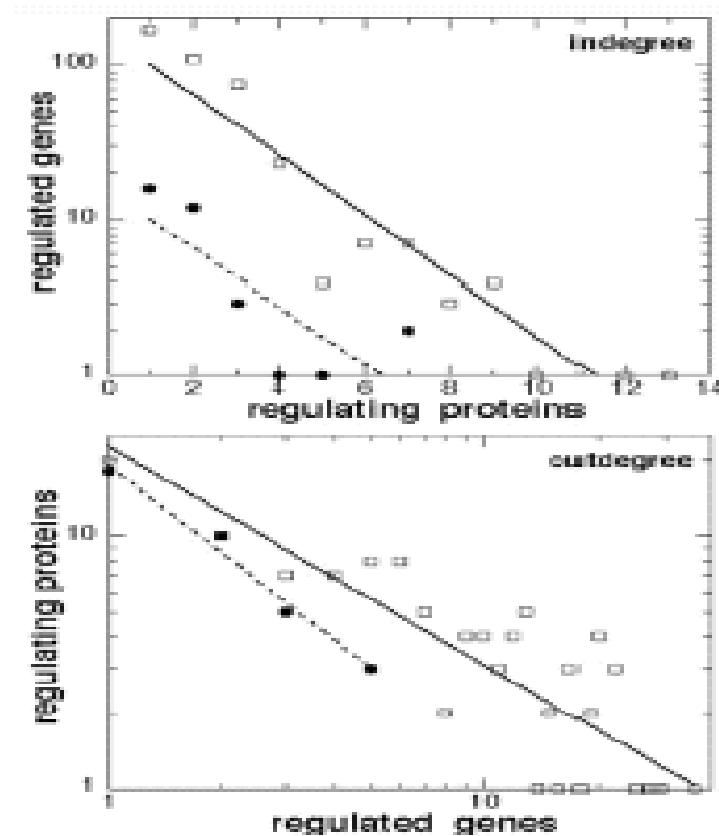


The growth in the picture is faster than linear:
organism with larger number of genes have a larger number of transcription factors per gene.

Degree distribution of transcription network

The in-degree distribution is exponential

the out-degree distribution has fat tails

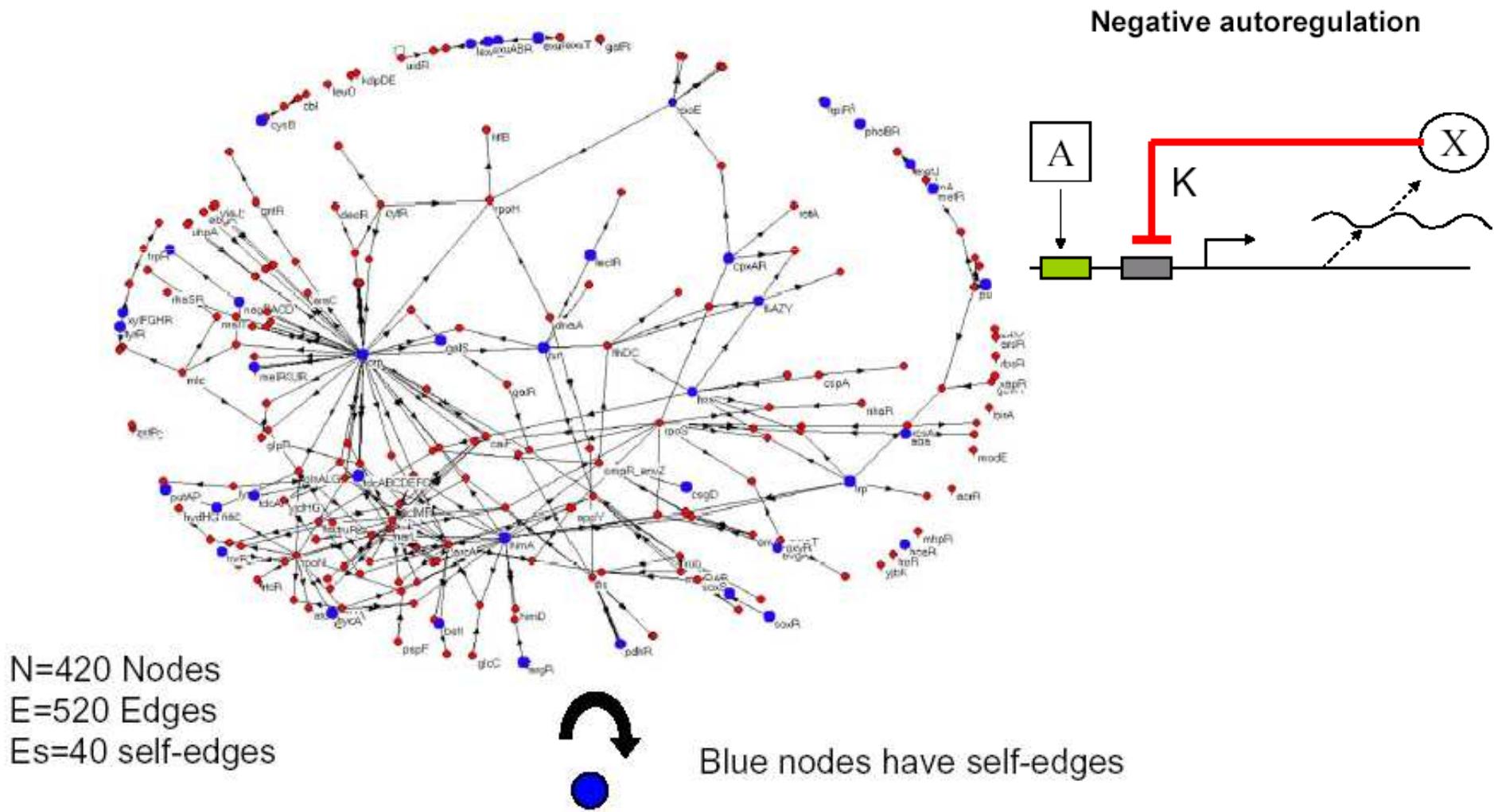


Guelzim et al. *Nature Genetics* 31, 60 (2002)

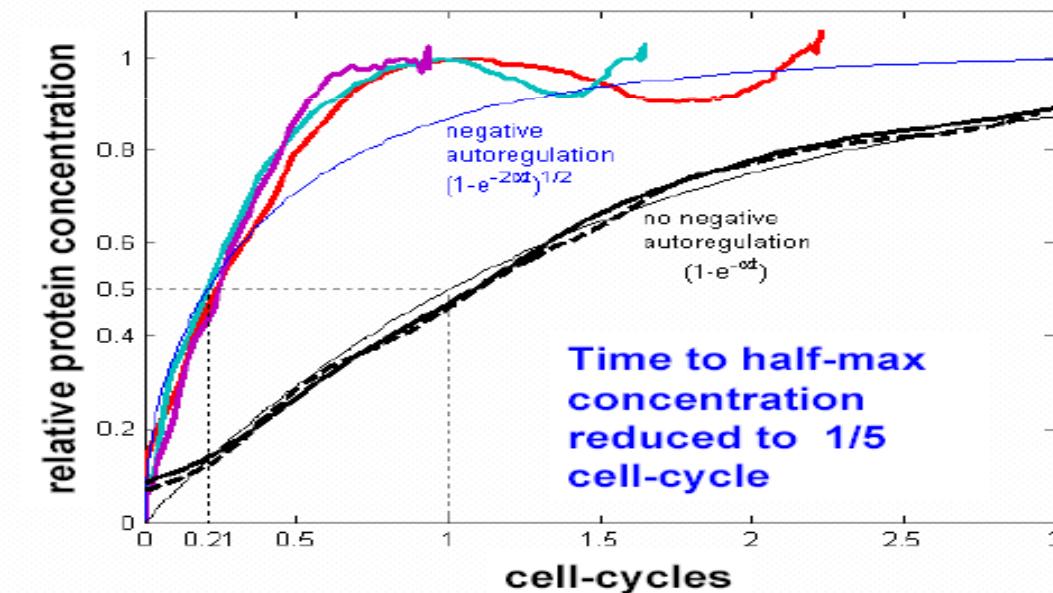
Lee et al. *Science* 298 799 (2002)

S. cerevisiae

Negative autoregulation is over-expressed in yeast transcription network



Negative autoregulation speeds up the response time of transcriptional networks



Rosenfeld et al. JMB (2002) 323,785

Motifs in transcription networks (*e.coli* and *s.cervisiae*)

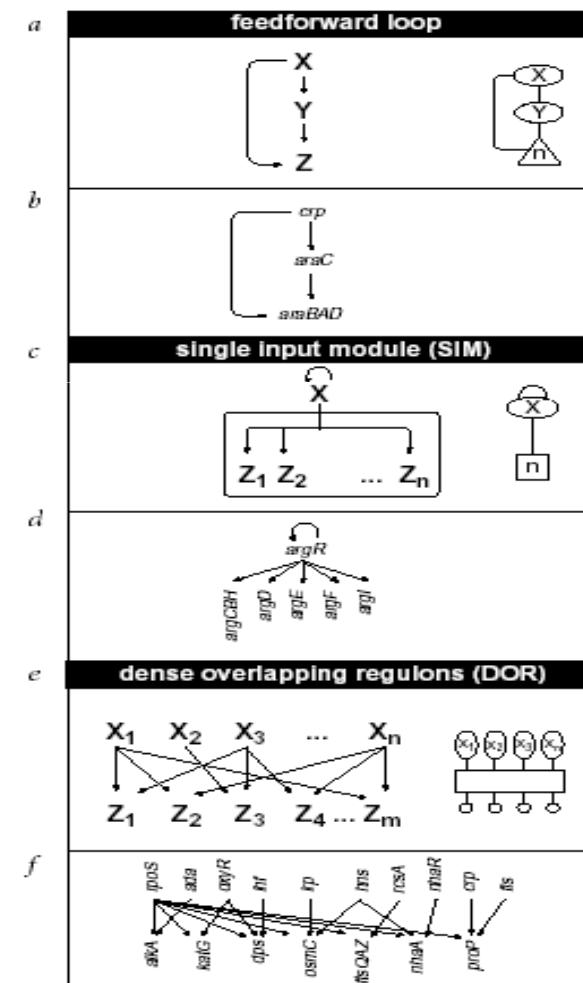
Different networks can be characterized by the frequency of particular subgraphs.

Subgraphs which appear with higher frequency than in random graphs with the same degree distribution are called motifs of the network.

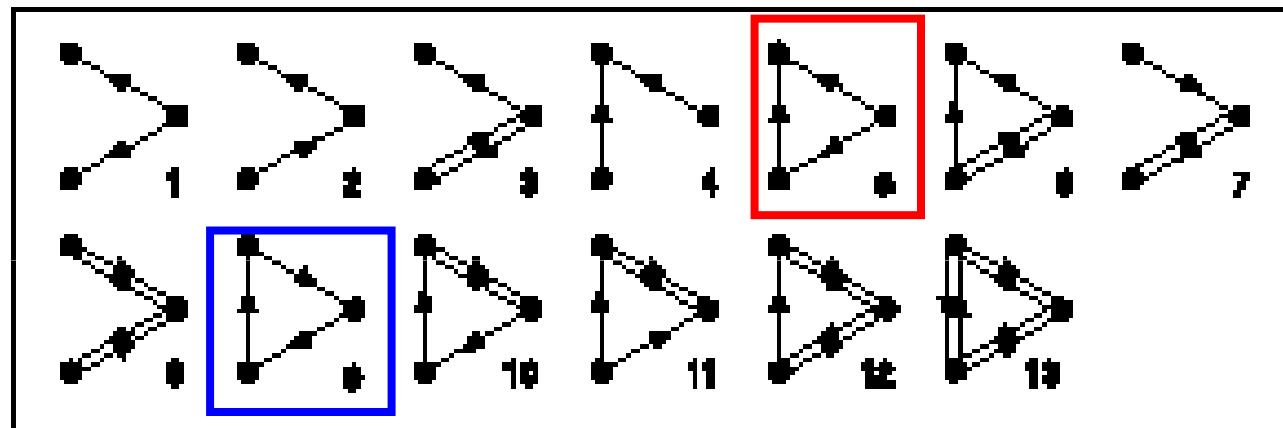
Those motifs usually perform specific functions.

R. Milo et al. *Science* (2002)

S.S. Shen-Orr, et al, *Nature Genetics* 31,64 (2002).



3-nodes subgraphs



Feed-forward loop

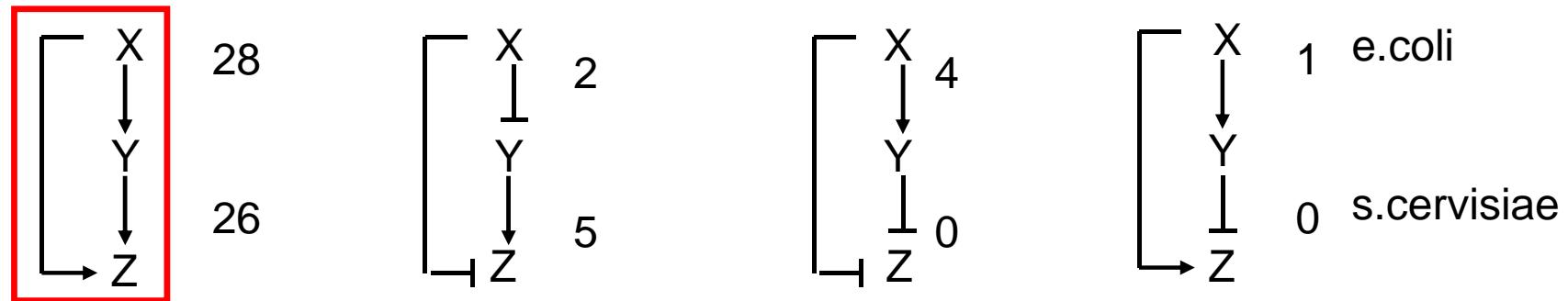
MOTIF

Feedback loop

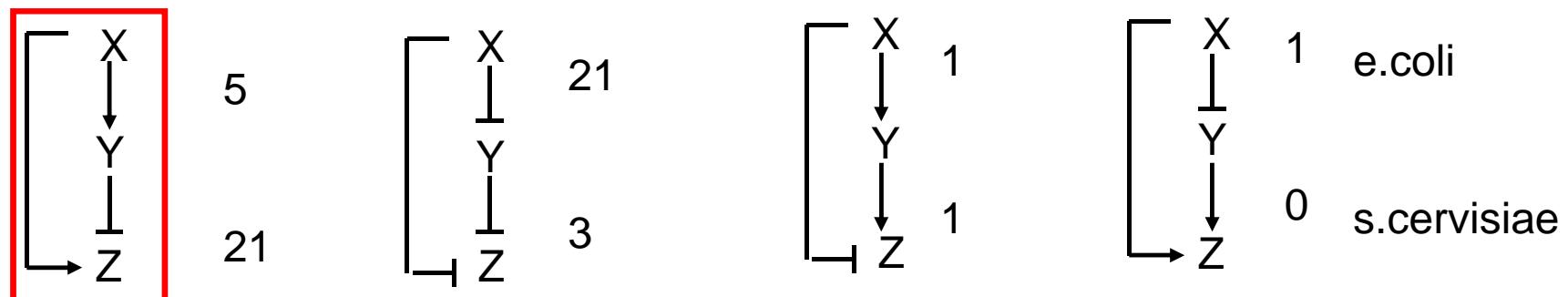
ANTIMOTIF

Abundance of feed-forward loops

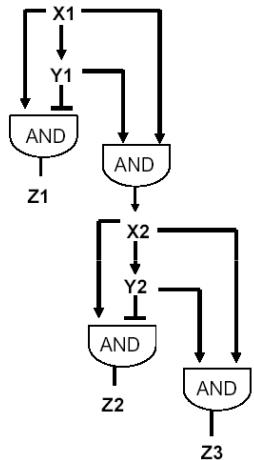
- Coherent FFL



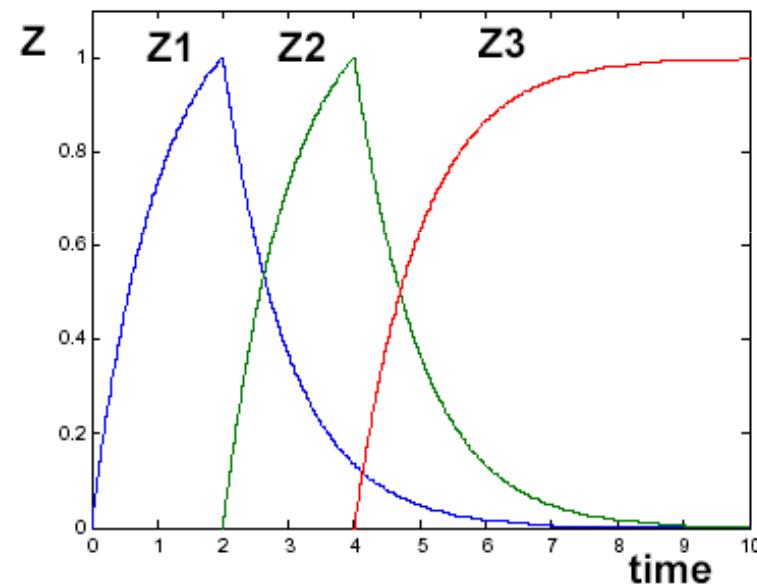
- Incoherent FFL



Feed-forward loops drive temporal pattern of pulses of expression



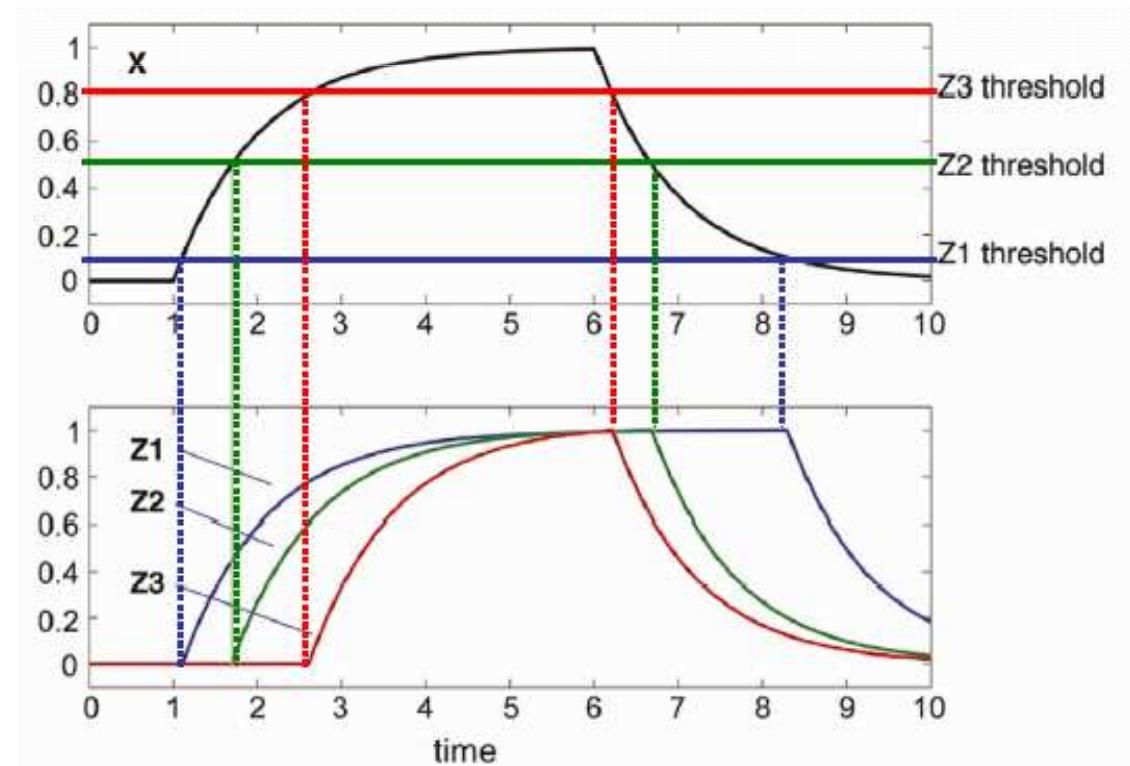
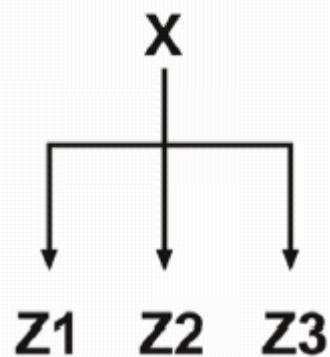
B.*subtilis* sporulation



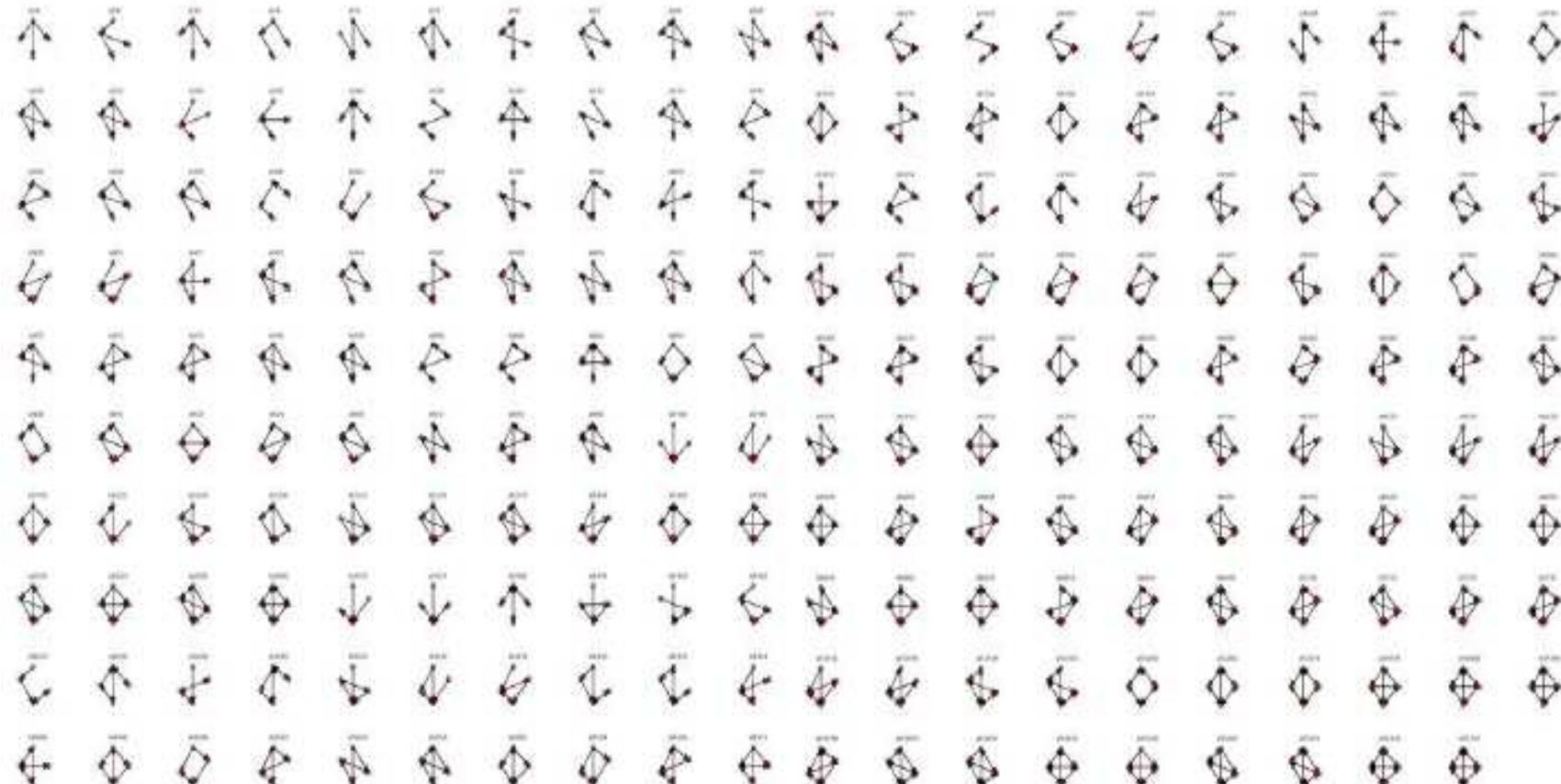
R. Losick et al. PLOS 2004

Single Input Module

Single input module can control timing of gene expression



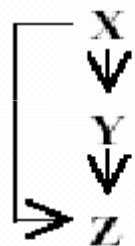
199 4-nodes connected subgraphs



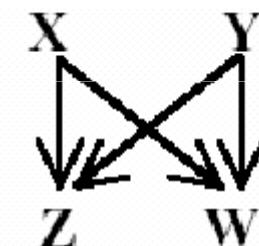
**...and one has 9364 5-node subgraphs,
1,530843 6-node subgraphs**

Yeast and e. coli share the same network motifs

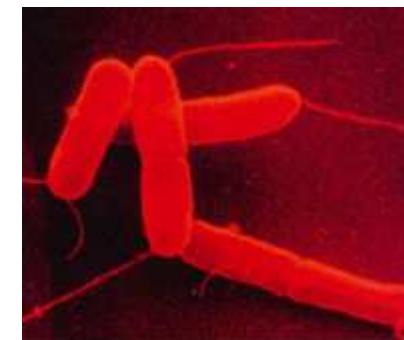
Feed-forward
loop



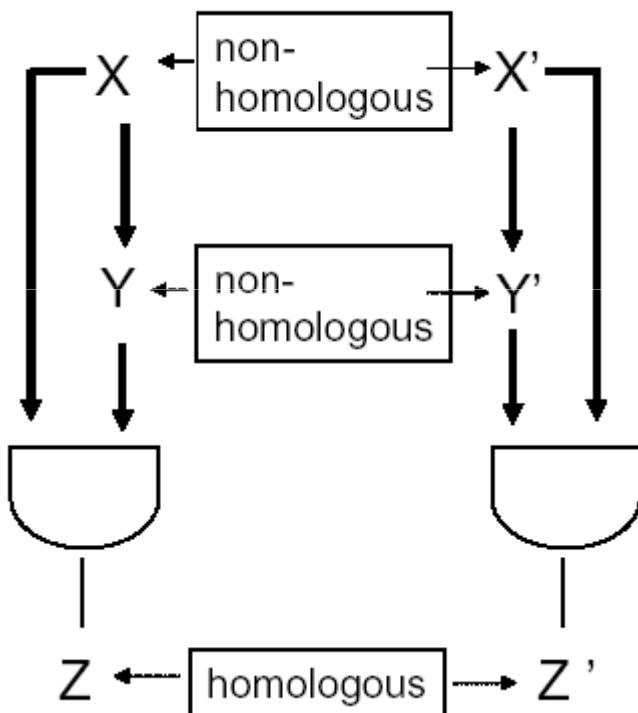
Bi-fan
motif



Although they are
completely different
organism

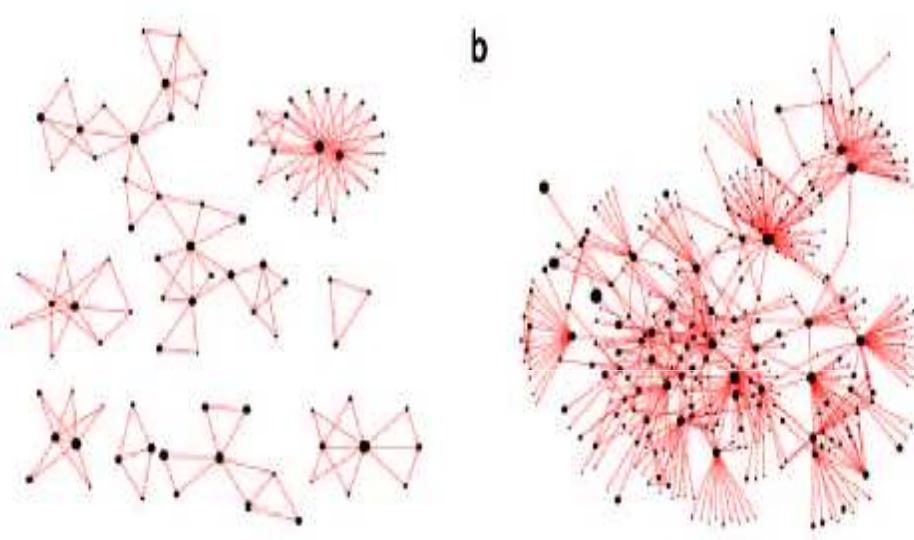


Convergent evolution of transcriptome of yeast and e.coli



G.C Conant , A. Wagner Nature Gen. 34,264 (2003)

Organization (percolation) of motifs



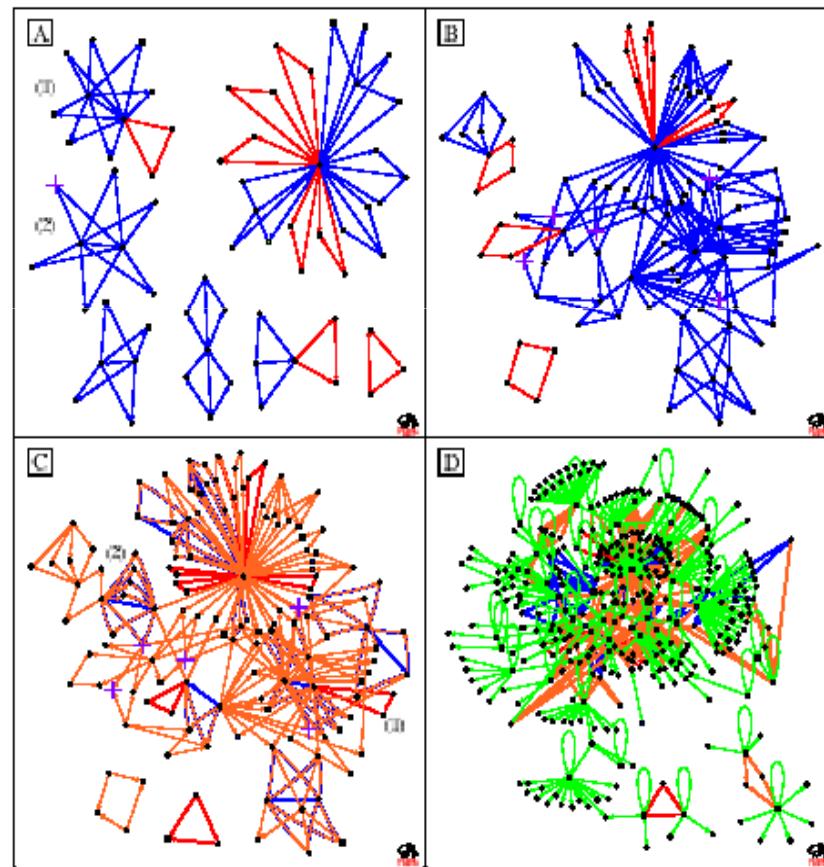
(3,3) subgraphs and (5,5) subgraphs in *s.cervisiae* transcription network

**Motifs do not work
in isolation!**

The percolation properties of the subgraph composed by the intersections of the motifs of a given type depend on the value of the power-law exponent γ and the hierarchical exponent α .

Clustering of motifs: the case of e.coli transcription network

Feed-forward
loops



Feed-forward
loops and bifan
motifs

(1) aerobic-anaerobic switch cluster – (2) flagella cluster

Bifan motifs

Giant
component

Dobrin et al. *BMC bioinformatics* 2004

Modeling cell cycle as a boolean network

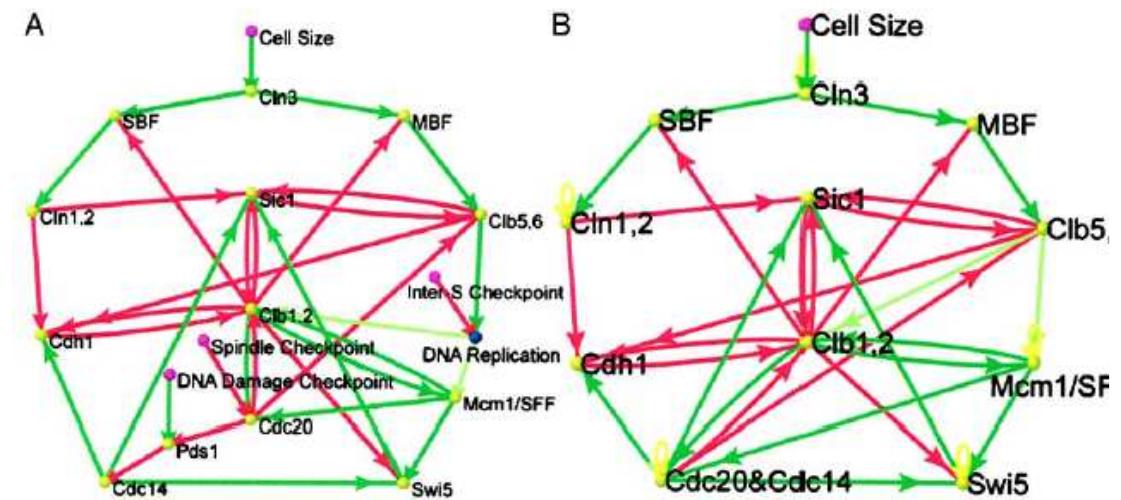
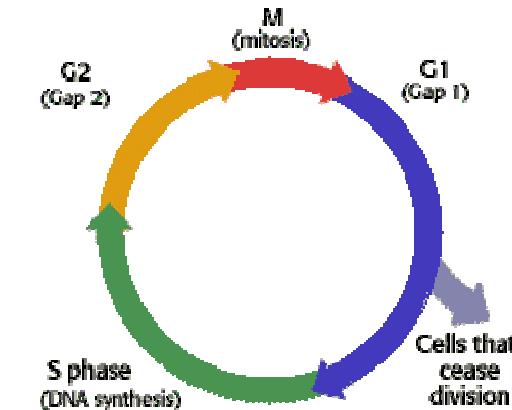
800 genes involved in the cell cycle process in budding yeast

Network constructed from literature analysis

Core of the cell cycle regulatory network:

Cyclins , Inhibitor, degrader and competitors of the cyclin- cdc28 complexes

Checkpoints (cell size, DNA replication and damage spindle assembly)



Li et al. PNAS, 101, 4781 (2004)

Reconstructed network and its simplification

Boolean model for the cell cycle

The dynamic assumed to hold
is the following

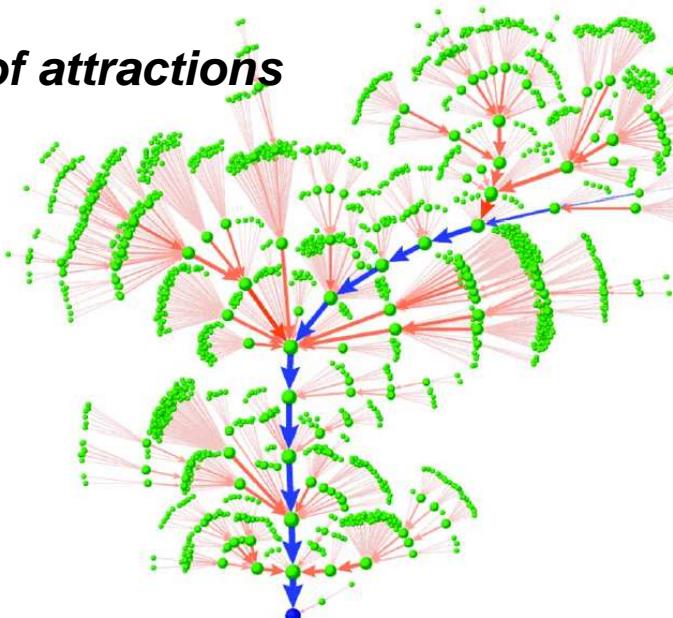
$$S_i(t+1) = \begin{cases} 1 & \sum_j a_{i,j} S_j > 0 \\ 0 & \sum_j a_{i,j} S_j < 0 \\ S_i(t) & \sum_j a_{i,j} S_j = 0 \end{cases}$$

with $a_{ij}=a_g$ for activation
 $a_{ij}=-a_r$ for inhibition
but the results do not
depend strongly on the
values of a_g and a_r

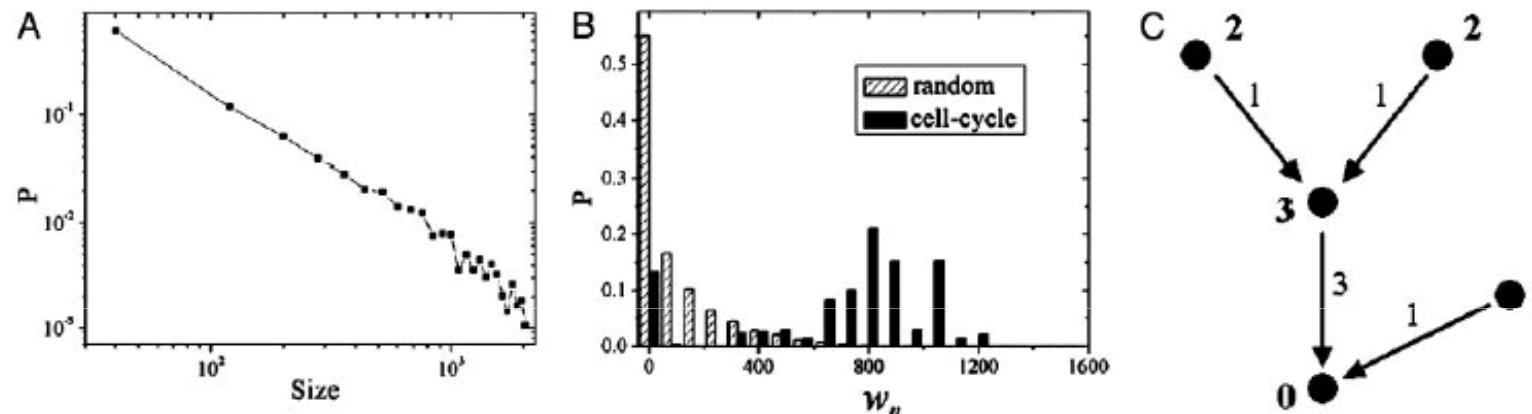
Table 1. The fixed points of the cell-cycle network

Basin size	Cln3	MBF	SBF	Cln1,2	Cdh1	Swi5	Cdc20	Clb5,6	Sic1	Clb1,2	Mcm1
1,764	0	0	0	0	1	0	0	0	1	0	0
151	0	0	1	1	0	0	0	0	0	0	0
109	0	1	0	0	1	0	0	0	1	0	0
9	0	0	0	0	0	0	0	0	1	0	0
7	0	1	0	0	0	0	0	0	1	0	0
7	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	0	0

Basins of attractions



Comparison with random networks



- The cell cycle is compared with a **random network** with the same number of links of the same colors as in the cell cycle
- The distribution of basin sizes is scale-free with a **probability to have a basin of attraction as large as the one find for the model of 10%**
- The total '**traffic**' per arrow measured through the quantity w_n is much larger in the cell cycle than in a random network

From small scale to large scale

Topology

Ex. Regulatory
network

Motifs

Percolation of motifs
and

Modularity

Scale-free (fat tail)
degree distribution

Dynamics

Metabolic network

Regulatory network

Michael-Mentens dynamics

Flux Balance Analysis
(continuous variables)

Boolean networks
(boolean variables)



From random to designed networks

- 
- 
- Fixed connectivity networks
 - Poisson (Erdos and Renyi) networks
 - Scale-free networks Metabolic, protein interaction networks Transcription networks (fat tails)

- 
- Motifs -characteristic subgraphs in real networks which are related to the function of the network.
 - Communities in the networks

The statistical physics and evolutionary dynamics

Raising the challenging question
What is Life?

Erwin Schrödinger, 1944

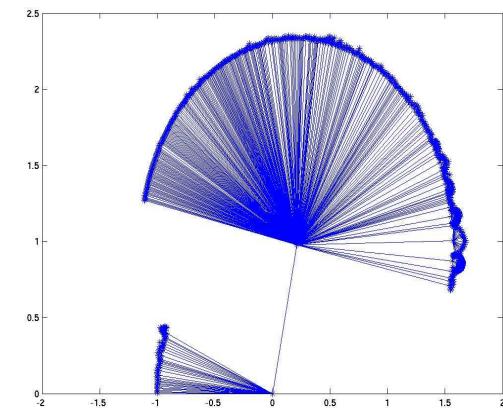
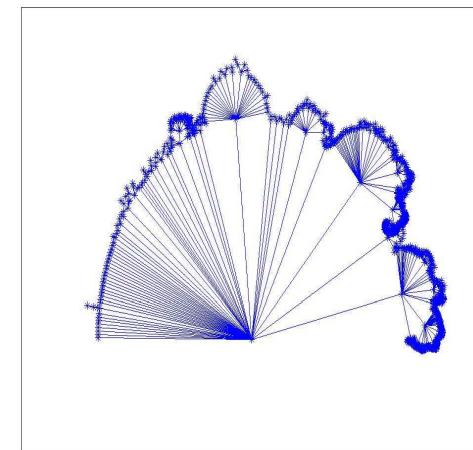
Statistical Mechanics
More is different
P. A. Anderson in Science 1972

Evolutionary Dynamics
Everything in the Universe is the fruit of
chance and necessity

(Democritus Cited by Jacques Monod in "Chance and necessity" 1970)

Bose-Einstein condensation in models of evolution

- Bose-Einstein condensation transition in the Kingman model
(J. F. C. Kingman 1978)
- Bose-Einstein condensation in complex evolving complex networks
(G. Bianconi and A.L. Barabasi PRL 2001)
- Bose-Einstein condensation in evolving ecosystems
(G. Bianconi, L. Ferretti, S. Franz EPL 2009)
- Bose-Einstein condensation in model of asexual evolution and pleiotropy
(S. N. Coppersmith, R. D. Blanck and L. P. Kadanoff 2004)



The Kingman model of asexual evolution

Mutations compete with selection in determining the genetic diversity of the population

- Every individual has a reproductive rate $w = e^{-\beta\varepsilon}$
- A mutation occurs with probability ν
- When a new mutation occurs, the new reproductive rate is drawn at random from a distribution $p(w)$

The Kingman model

$$w = e^{-\beta\varepsilon}$$

Average reproductive number
of a strain

$$\varepsilon$$

Fisher fitness of a strain

$$\beta$$

Selection pressure

$$\nu$$

Rate of mutation

$$p^{t+1}(\varepsilon) = (1 - \nu) \frac{1}{\langle e^{-\beta\varepsilon} \rangle} e^{-\beta\varepsilon} p^t(\varepsilon) + \nu \rho(\varepsilon)$$

Bose-Einstein distribution in the Kingman model

Stationary distribution

$$p(\varepsilon) = \nu \frac{\rho(\varepsilon)}{1 - e^{-\beta\varepsilon} (1 - \nu) / \langle e^{-\beta\varepsilon} \rangle}$$

Normalization condition

$$1 = \int d\varepsilon p(\varepsilon) = \nu \left[1 + \int d\varepsilon \frac{\rho(\varepsilon)}{e^{\beta(\varepsilon - \mu)} - 1} \right]$$

$$e^{-\beta\mu} = \langle e^{-\beta\varepsilon} \rangle / (1 - \nu)$$

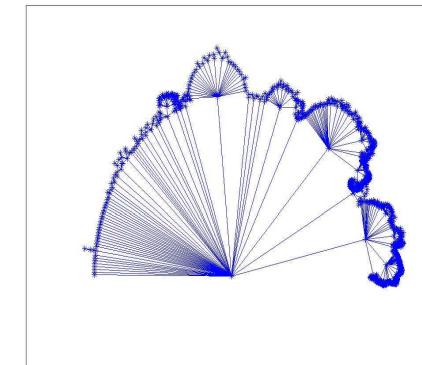
Bose-Einstein condensation in the Kingman model

When the mutation rate is below a critical value and the selection pressure is above a critical value
a finite fraction of individuals in the population have maximal fitness

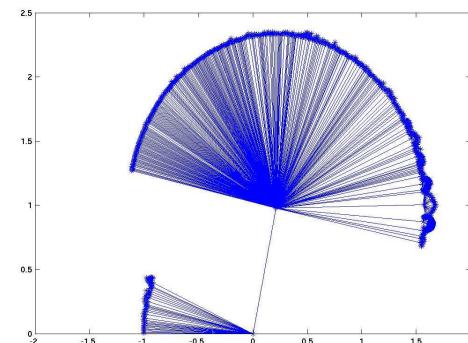
This phase transition it is usually called in the literature quasi-species transition

Bose-Einstein condensation in growing scale-free networks

In evolving network nodes have a fitness indicating their ability to acquire new links



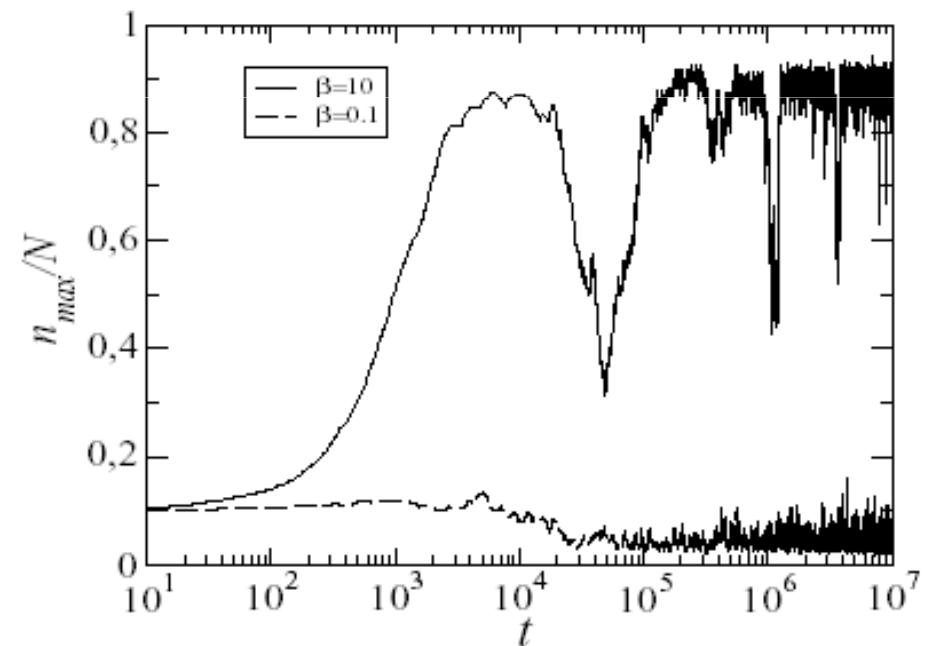
Below the Bose-Einstein condensation transition the most connected node has a finite fraction of all the links



Bose-Einstein condensation in ecology in presence of invasive species

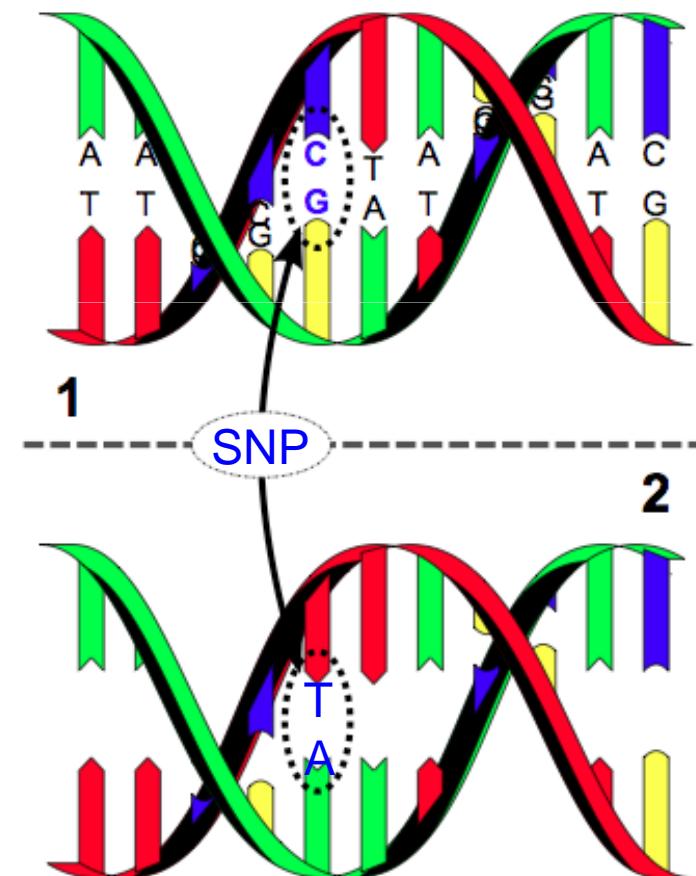
In presence of invasive species
there can be a Bose-Einstein
condensation in ecological system

When the condensation occur
a finite fraction of individuals of
the ecology belongs to the invasive
species



Single nucleotide polymorphisms (SNPs)

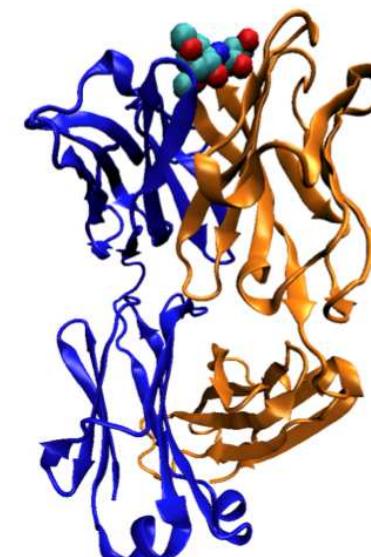
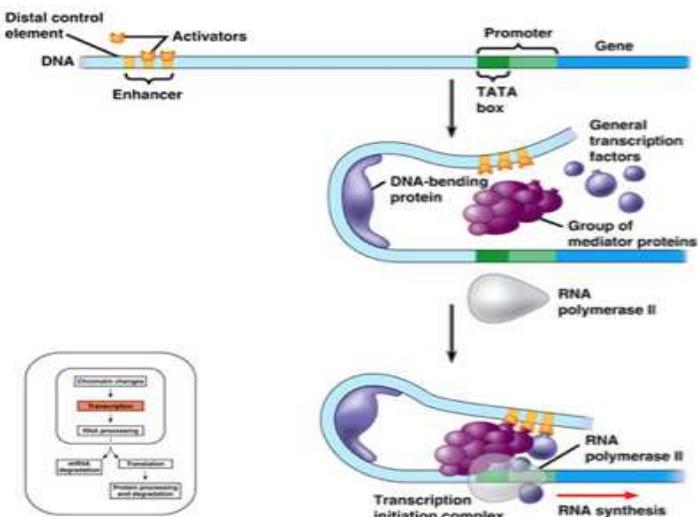
SNPs are single nucleotide (CTAG) variations occurring in more than 1% of the population



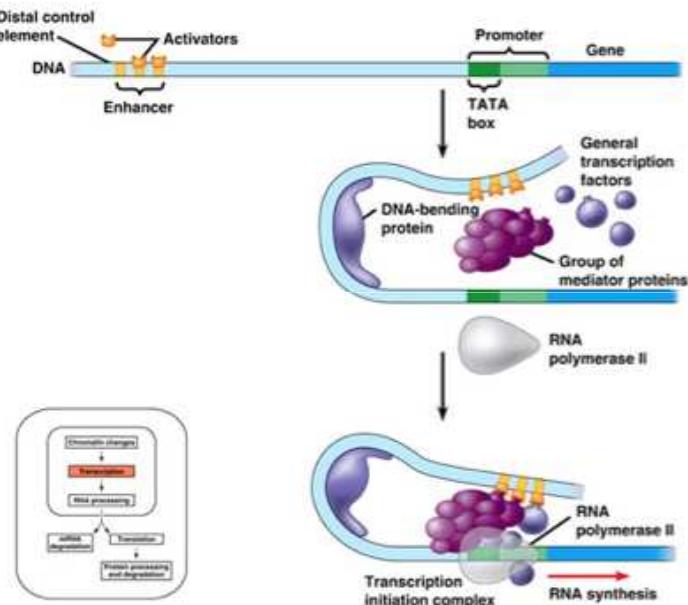
SNPs

- SNPs might occur
 - *in the coding region of a gene,*
 - *in non-coding region of a gene*
- In humans, we have
 - *5 millions SNPs over a genome of 3 billion base-pairs.*
- SNPs
 - *can affect the genetic risk for diseases and the response to pathogens and drugs.*

SNPs interact with the complexity of molecular networks encoding for the map between genotype and phenotype.



SNPs and transcription networks



A SNP in the promoter region of a gene might change the binding affinity of the transcription factor and introduce a perturbation in the transcription network

SNPs and metabolic networks



SNPs in the coding region might change the amino acid composition of the coded protein and introduce perturbations in the katalysis of chemical reactions or binding of other proteins

More is different:

The importance of epistatic interactions

Linkage disequilibrium

In a given population,
for each pair of SNPs, (i,j)
with joint allelic frequency $p_{ij}(x_i, x_j)$,
linkage disequilibrium is defined as

$$LD_{ij} = p_{ij}(x_i, x_j) - \sum_{x, x'} p_i(x_i x') p_j(x, x_j)$$

Epistatic interactions

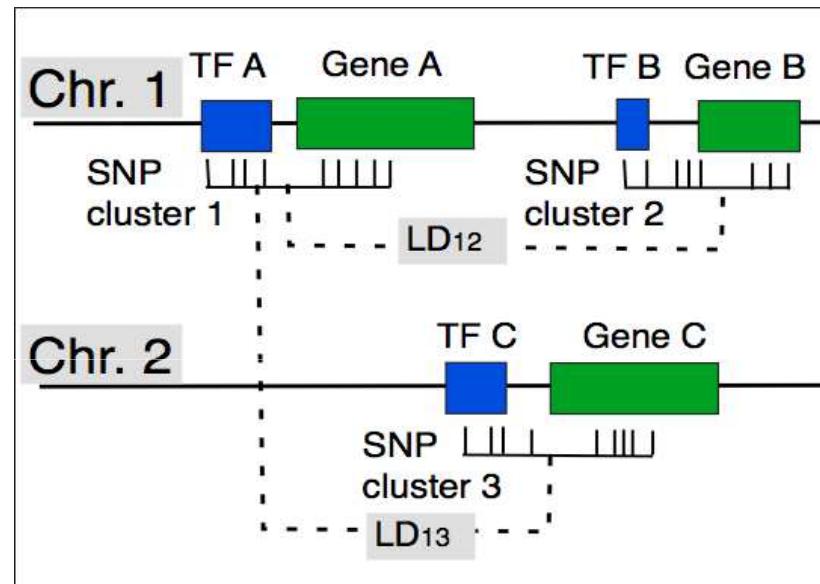
Definition

Epistatic interactions are the non additive contributions that two mutations or genetic variations have on the fitness of one organism

Sign of the epistatic interaction

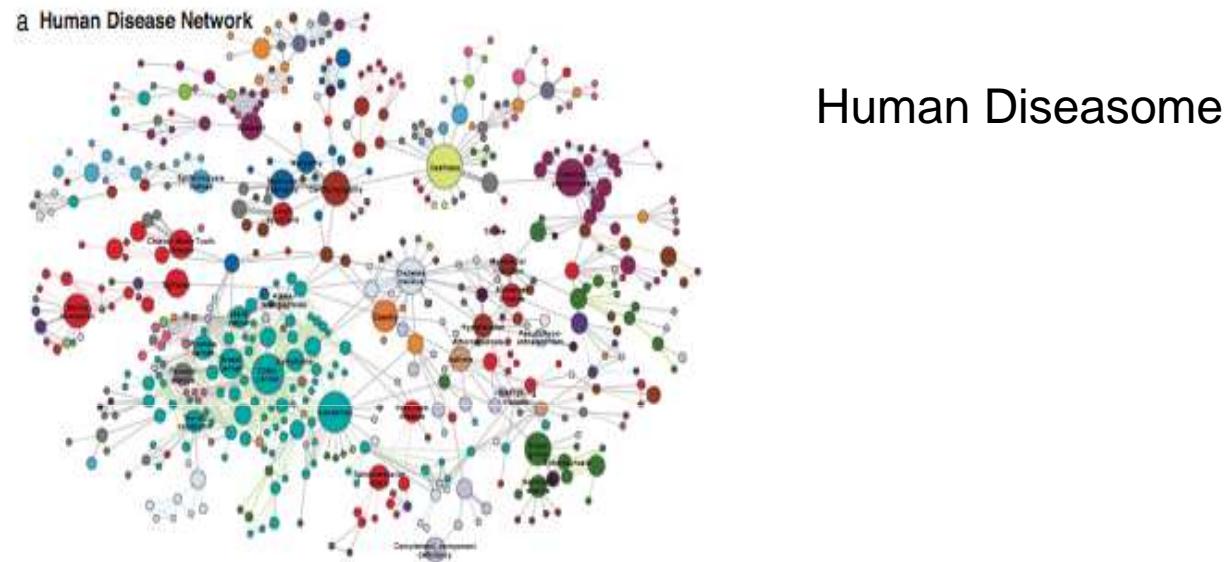
Epistatic interactions can have synergistic, neutral or antagonistic effects.

Epistatic interactions between SNPs



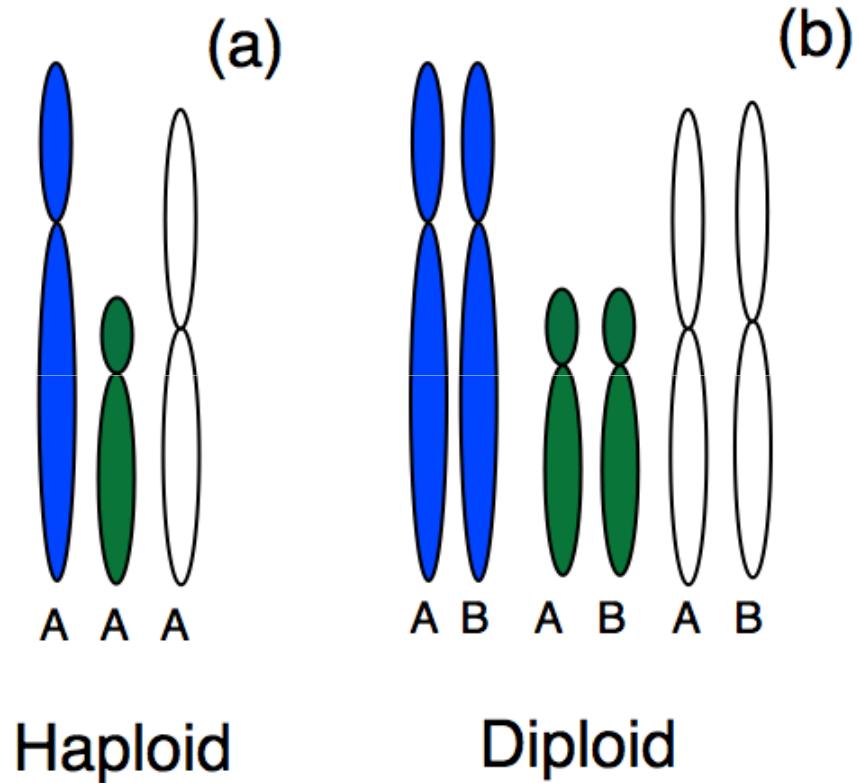
- SNP are organized in cluster in complete linkage equilibrium
- Different cross-over rates cannot explain linkage disequilibrium between very distant SNPs along the same chromosome or in different chromosomes

Phenotypes and diseases in Humans



- Most of the disease are complex, i.e. they are due to a large set of genetic loci
- Each gene might contribute to the risk of developing different diseases
- Uncovering the epistatic network in humans might be essential to prevent diseases and advance in the personalized medicine

Diploid species



Diploid species have two copies of each chromosome one coming from the father gamete and one coming from the mother gamete

Fitness function without epistasis

$\{x\} = x_1, x_2, \dots, x_N$

$x_i = 1, 2, 3, 4$ (C, G, T, A)

Gametes with N genetic loci

$\{x^A\}, \{x^B\}$

Parental gametes

$$W\{x^A, x^B\} = \prod_i \varphi_i(x_i^A, x_i^B)$$

Fitness function with pairwise epistatic interactions

$\langle i, j \rangle$ Pair of genetic loci in epistatic interaction

β Selective pressure

$$W\{x^A, x^B\} = \prod_{\langle i, j \rangle} e^{-\beta U_{ij}(x_i^A, x_j^A, x_i^B, x_j^B)}$$

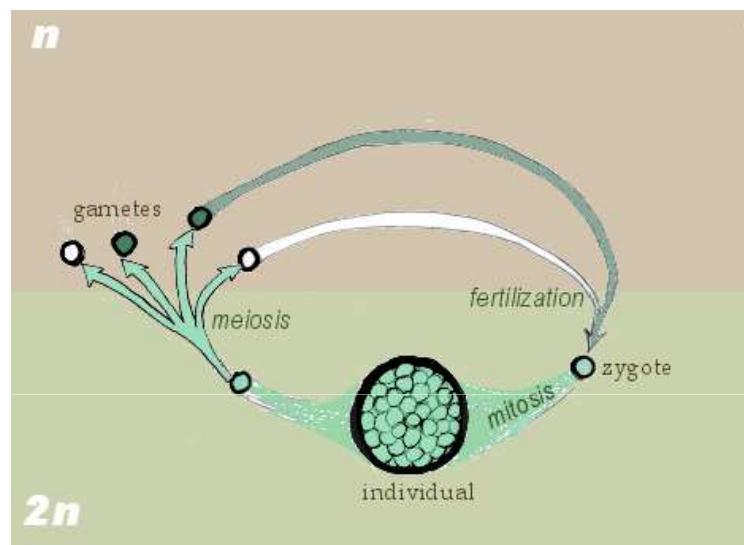
Symmetries also called ‘Robbins proportions’

$$U_{ij}(x, \bar{x}, x', \bar{x}') = U_{ij}(x', \bar{x}', x, \bar{x})$$

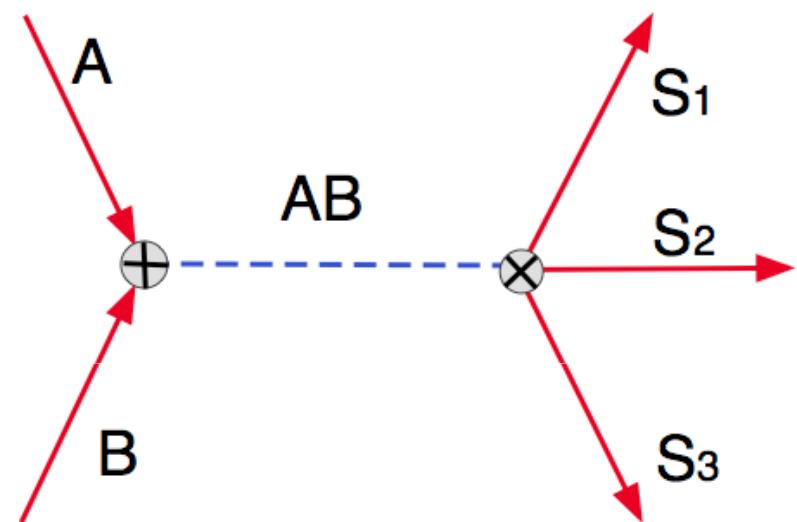
$$U_{ij}(x, \bar{x}, x', \bar{x}') = U_{ij}(x, \bar{x}', x', \bar{x})$$

Gametic cycle

Biological view



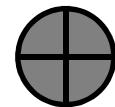
Physical view



Gamete



Zygote



Individual



Meiosis

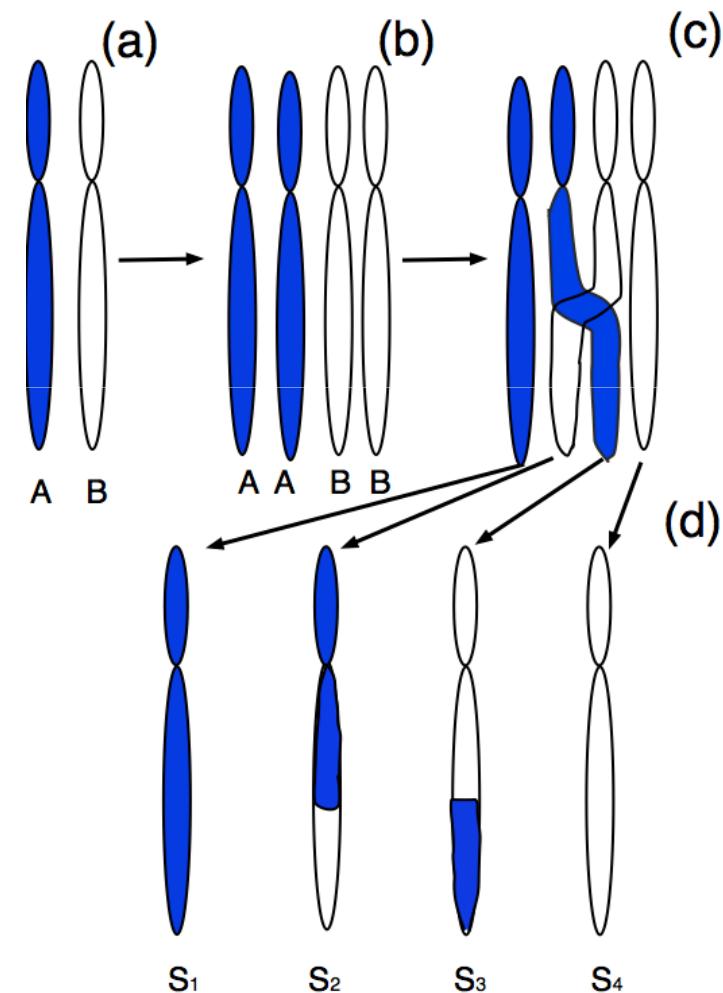


Chance and Necessity

*Recombination processes
versus selection*

Meiosis

During meiosis
the gametes
of each individual are
formed by a combined
process of
crossing-over and
recombination

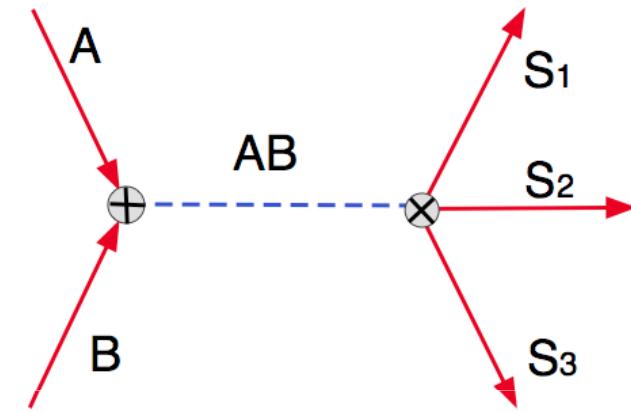


Evolution of diploid populations

$P(\{x\})$

Frequency of gametes with
allelic configuration $\{x\}$

Evolutionary dynamics



$$\frac{dP(\{x\})}{dt} = M_{\{x\}|\{x^A, x^B\}} \left[\frac{W(\{x^A, x^B\}) P(\{x^A\}) P(\{x^B\})}{\langle W \rangle} \right] - P(\{x\}) \quad \text{with}$$

$$M_{\{x\}|\{x^A, x^B\}} [f(\{x^A, x^B\})] = \sum_{\{x^A, x^B\}} \prod_i \left(\frac{1}{2} \delta(x_i, x_i^A) + \frac{1}{2} \delta(x_i, x_i^B) \right) f(\{x^A, x^B\})$$

Steady state equation

The steady state equation is therefore given by

$$P(\{x\}) = M_{\{x\} \mid \{x^A, x^B\}} \left[\frac{W(\{x^A, x^B\}) P(\{x^A\}) P(\{x^B\})}{\langle W \rangle} \right]$$

With the marginals defined as in the following

$$p_{ij}(x, x') = \sum_{\{x\}} P(\{x\}) \delta(x_i, x) \delta(x_j, x')$$

Structure of the solution on a locally tree-like epistatic network

On a tree-like network the general solution of the evolutionary dynamics will be of the type

$$P(\{x\}) = \sum_h \prod_{\langle i,j \rangle} b_{ij}^{(h)}(x_i, x_j)$$

To solve the problem by analytic methods we look for solution of the stationary distribution of the type

$$P(\{x\}) = \prod_{\langle i,j \rangle} b_{ij}(x_i, x_j)$$

Multiple solutions of the self-consistent equation

The cavity equations and the self consistent equation can be used to find the functions $b_{ij}(x_i, x_j)$

$$b_{ij}(x_i, x_j) = \frac{G_{ij}(x_i, x_j)/F_{ij}(x_i, x_j)}{\langle W \rangle Z_{i|j}(x_i)Z_{j|i}(x_j)/F_{ij}(x_i, x_j) - 1}$$

**These equations have multiple solutions
Therefore the asymptotic state of the population depends on the initial conditions**

Bose-Einstein distribution of the marginal probability of pairs of genetic loci

$$p_{ij}(x_i, x_j) = \frac{1}{\langle W \rangle} G_{ij}(x_i, x_j) [1 + n_B(\varepsilon_{ij}(x_i, x_j))]$$

$$n_B(\varepsilon) = \frac{1}{e^{\beta[\varepsilon(x_i, x_j) - \mu]} - 1}$$

When $\varepsilon_{ij}(x_i, x_j) = \mu$ the pair of linked loci go to fixation

Condensation as a function of the selective pressure

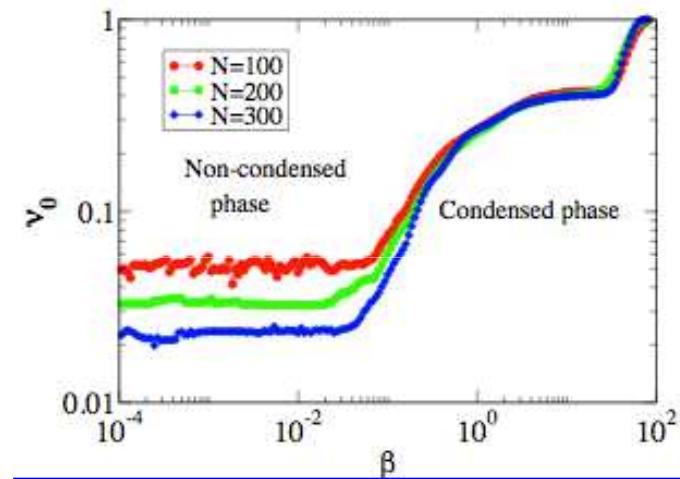
Normalization condition

$$\sum_{x_i, x_j} p_{ij}(x_i, x_j) = 1$$

$$(1 - \nu_{ij}) = \frac{1}{\langle W \rangle} \sum_{x_i, x_j} G_{ij}(x_i, x_j) [1 + n_B(\epsilon_{ij}(x_i, x_j))]$$

Averaging over all the pairs of SNPs

$$1 - \nu_0 = \frac{1}{\langle W \rangle} \int d\epsilon g(\epsilon) [1 + n_B(\epsilon)]$$



A finite fraction ν_0 of linked pairs of SNPs go to fixation for high selection pressure

Conclusions

Biological networks structure and dynamics constitute the richness of living systems.

The complexity of biological networks is reflected at different scales and in their dynamical behavior.

Understanding the interplay between genomic information and biological network can shed light on the genotype-phenotype mapping with relevant future application for devising a personalized medicine.

New developments of the theory of evolution will include the general principles of complex system evolution and the recent new finding about biological and epistatic networks.

Binding of the inducer to the repressor



Steady state:

$$X S_x k_{\text{on}} = [XS_x] k_{\text{off}}$$

Michaelis-Menten equation

$$[XS_x] = X_T \frac{S_x}{S_x + K_X}$$

$$K_X \approx 1,000$$

Inducer molecules /cell

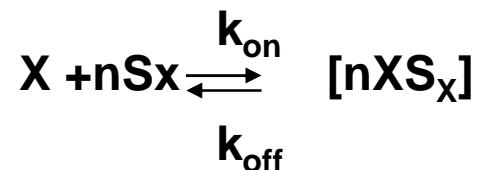
$$X^* = X_T \frac{1}{1 + \frac{S_x}{K_X}}$$

Active repressor

Cooperativity of inducer binding

Hill equation

Most transcription factors are composed by several repeated subunits for example dimers or tetramers. For activating a transcription factor usually all subunits must bind to the inducers.



$$[nXS_X] = X_T \frac{S_X^n}{S_X^n + K_X^n}$$

Hill equation

$$X^* = X_T \frac{1}{1 + \left(\frac{S_X}{K_X}\right)^n}$$

Active repressor

Input-function of a gene regulated by a repressor

The input function describes
the rate of transcription as a
function of the inducer S_x

$$f(S_x) = \beta \frac{K_d}{K_d + X^*} = \beta \frac{K_d}{K_d + X_T \frac{1}{1 + (S_x / K_x)^n}}$$

X^* -X active-
not bound to S_x

The input function reaches half maximal value at

$$S_{1/2} \approx (X_T / K_d)^{1/n} K_x$$

$S_{1/2}$ can be significantly **larger** than K_x

Input-function of a gene regulated by an activator

The input function describes
the rate of transcription as a
function of the inducer S_x

$$f(S_x) = \beta \frac{X^*}{K_d + X^*} = \beta \frac{1}{K_d[1 + (K_x/S_x)^n]/X_T + 1}$$

X^* -X a
active-
bound to S_x

The input function reaches half maximal value at

$$S_{1/2} \approx (K_d/X_T)^{1/n} K_x$$

$S_{1/2}$ can be significantly **smaller** than K_x

Negative auto-regulation



- 34 negative auto-regulations occurrences in e. coli transcription network
- Why is negative auto-regulation a motif?

$$\frac{dX}{dt} = \beta \frac{K_d}{K_d + X} - \alpha X \approx \beta \frac{K_d}{X} \quad Y \gg K_d$$

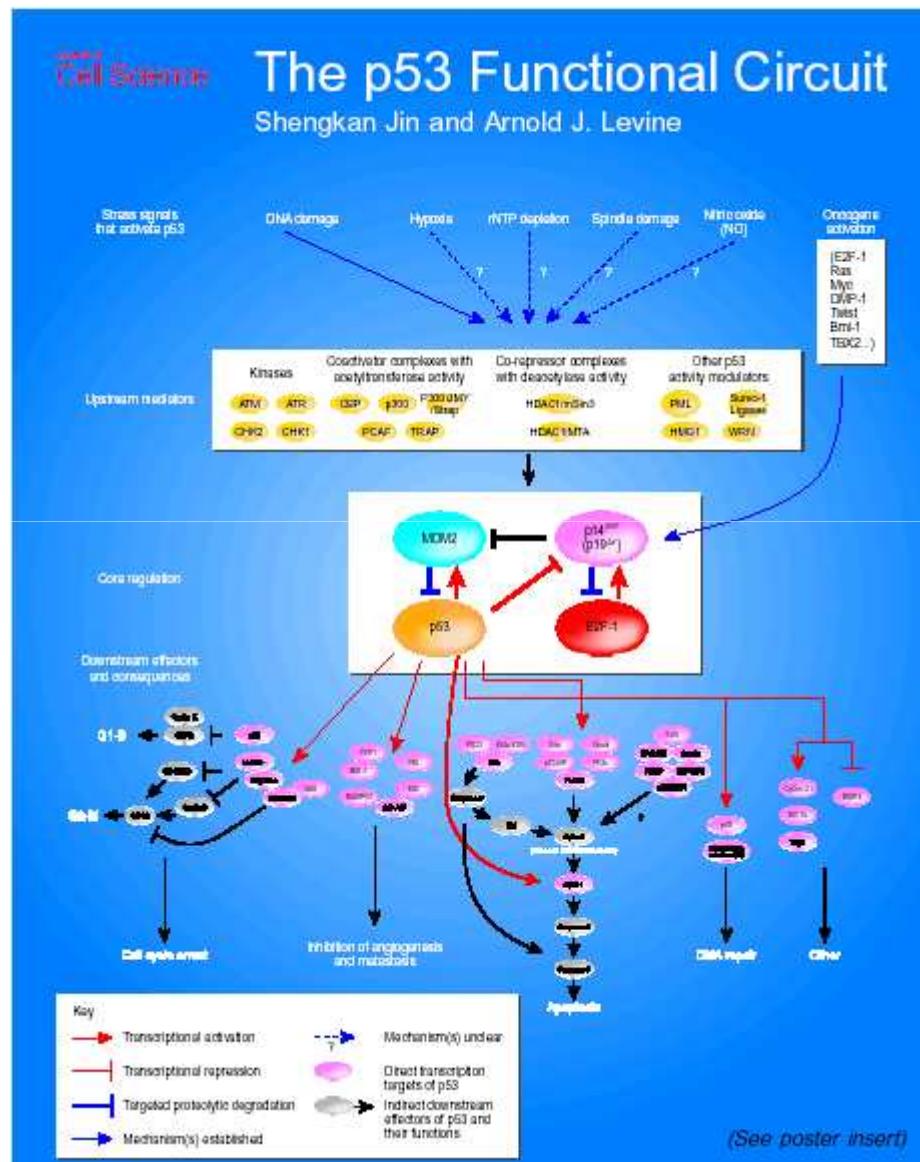
The dynamic of this simplified equation is

$$X(t) = X_{st} \sqrt{1 - e^{-2\alpha t}} \quad X_{st} = \sqrt{\beta \frac{K_d}{\alpha}}$$

The response time is reduced in a relevant way!

$$T_{1/2} = \log(4/3)/2\alpha \approx 0.2 T_{1/2}^{\text{simple}}$$

P53 network



p53 is a tumor suppressor gene that plays the role of safeguarding the integrity of the genome.

Is inactivated in almost all cancers

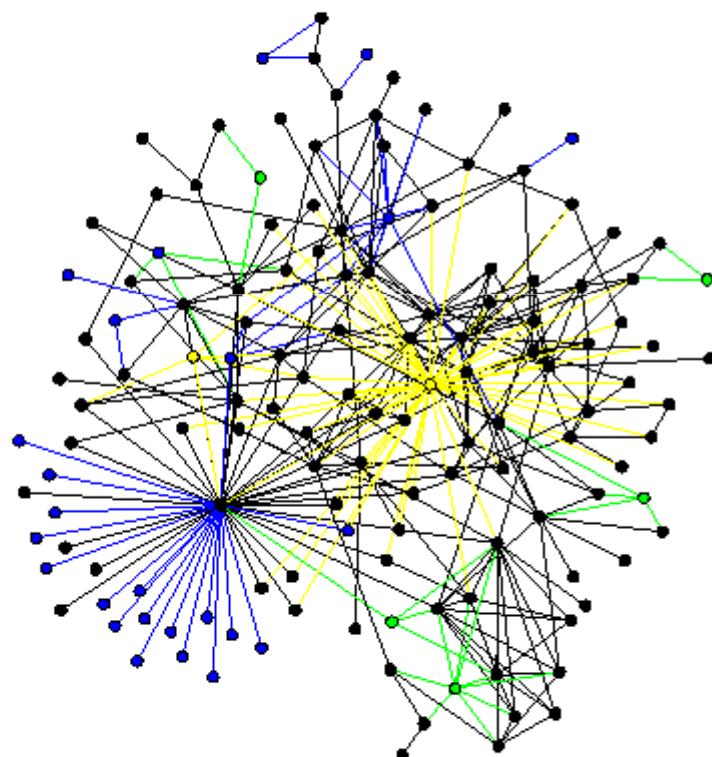
p53 is a binding TF kept at low level in cells under normal conditions

Various **stress signals** like DNA damage can activate p53

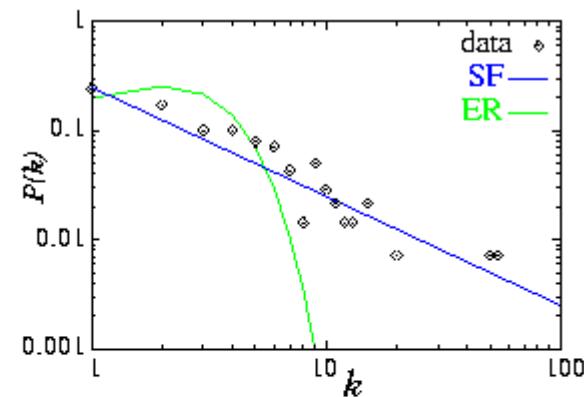
p53 activates tumor suppressing functions as **cell cycle arrest, apoptosis, DNA repair ,inhibition of angiogenesis and metastasis**

Two feedback loops are the core of the network

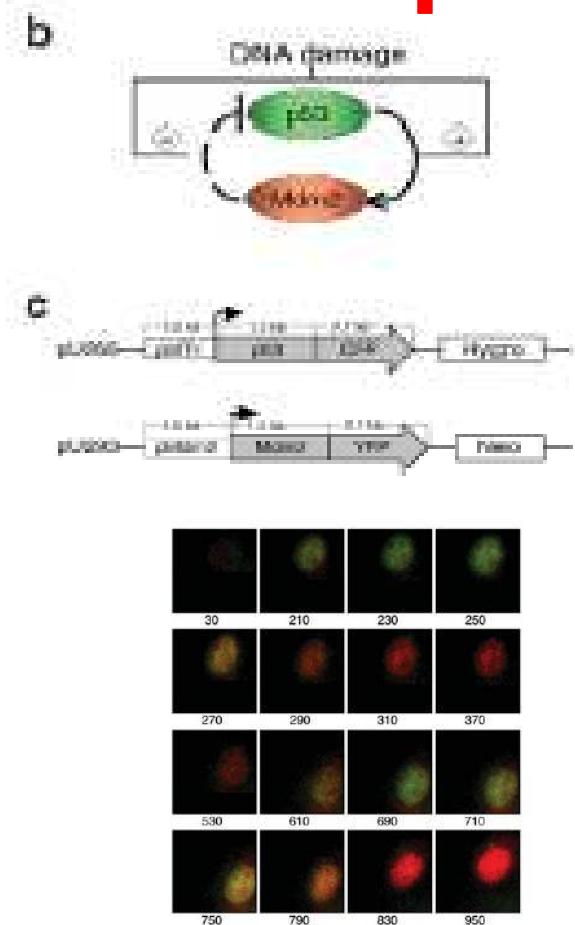
p53 network (mammals)



Degree distribution of the p53 network



Dynamics of p53-Mdm2 feedback loop in individual cells



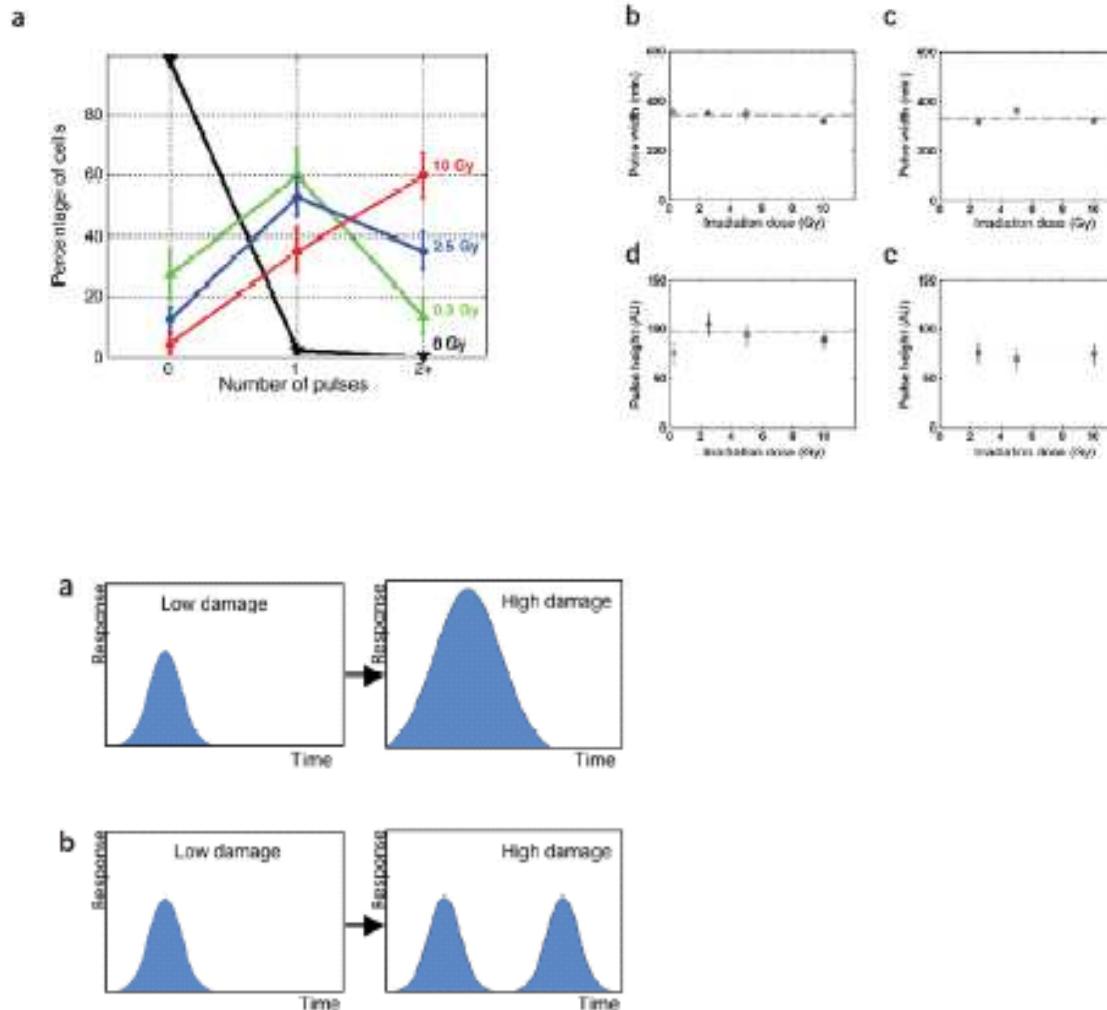
The authors generated stable cells lines that expressed p53 and Mdm2 fused to fluorescent proteins

P53 fused with CFP (cyan fluorescent promoter) Mdm2 fused with yellow fluorescent protein (YFP)

They were able to observe damped oscillations of expression of p53-mdm2.

These oscillations could be one or more

Non-analogic behavior under stress

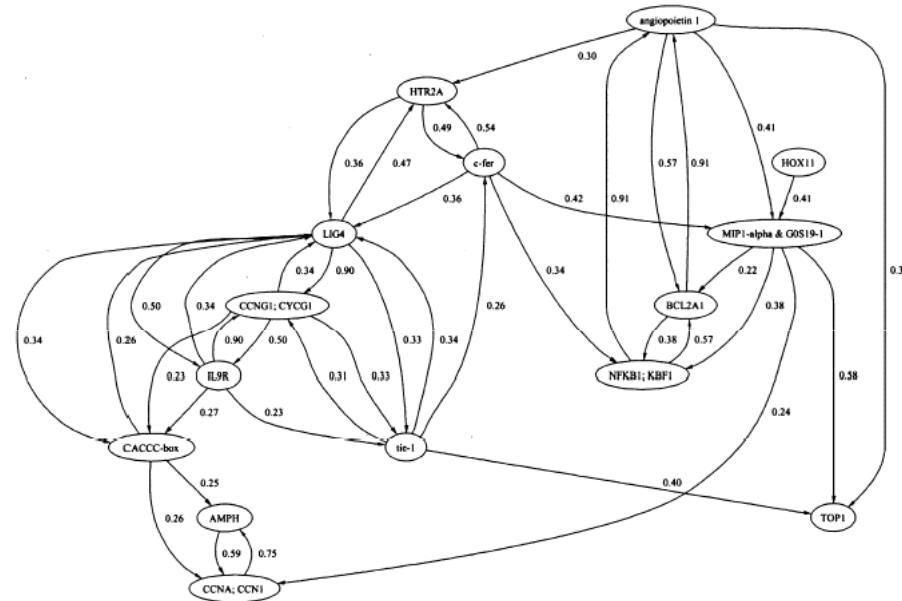


Fraction of cells with zero, one, two or more pulses as a function of g irradiation.

The irradiation dose dependency of the width of first and second pulse and the height of the first and second pulse is compatible with a digital behavior of the p53 oscillations as response to stress.

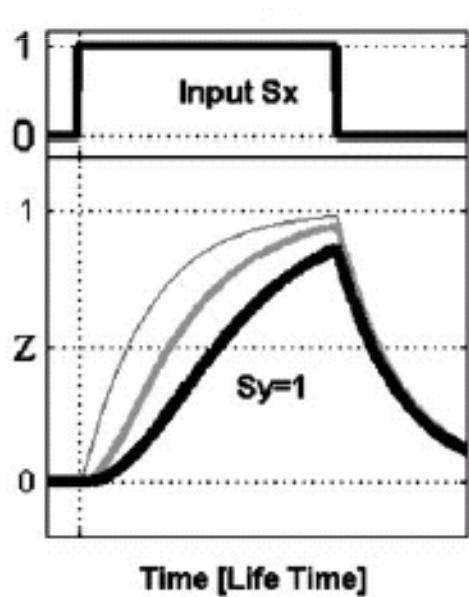
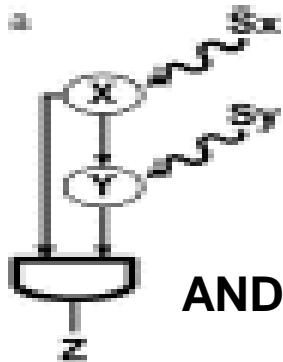
Glioma network

- This technique would be useful to **design target drugs** to perturb the state of the cell (for example a tumor cell) in the desired state.
- In particular probabilistic boolean networks would provide the information about the **timing of the effect of a perturbation on one particular gene** which is a crucial problem in many pharmaceutical applications



For a review I. Schmulevic et al. Proc. IEEE, 90 (2002)

Type 1 FFL-AND



S_x increases $\implies X^*$ at saturation
Y increases exponentially in time and
only when it cross the activation value
 K_{dzy} it activates Z



delay in Z production
when increasing S_x

When S_x goes to zero X^* goes to zero

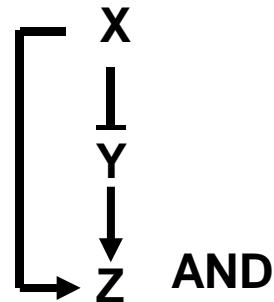


immediately Z
production goes to
zero

The type 1 FFL is a sign-sensitive delay element

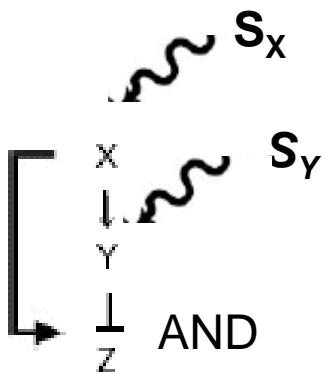
Some FFL are badly malfunctioning

For example the 4-Type IFFL is insensitive to the presence of the inducer S_Y



S_X	S_Y	4IFFL
0	0	0
0	1	0
1	0	0
1	1	0

Type 1 IFFL-AND

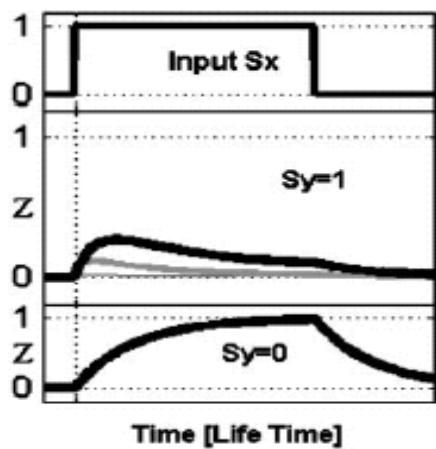


Dynamics with $S_y=1$

X^* is at saturation.

While $Y(t)$ increases $Z(t)$ increases also.

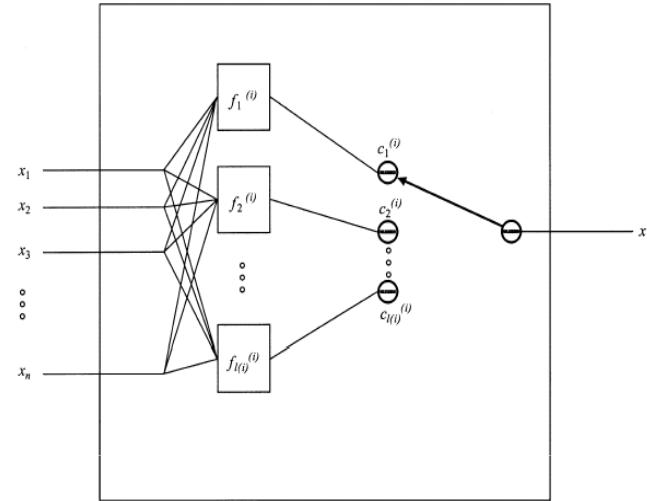
When $Y(t) > K_{dzy}$ $Z(t)$ start to decrease.



The Type1IFFL-AND acts as a weak pulse generator

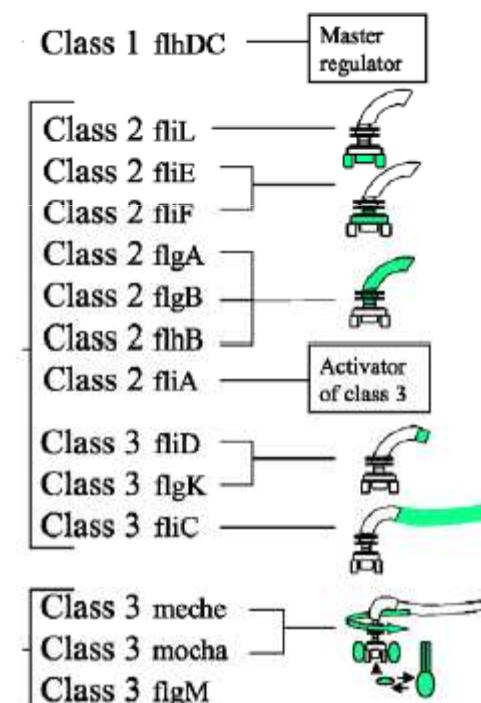
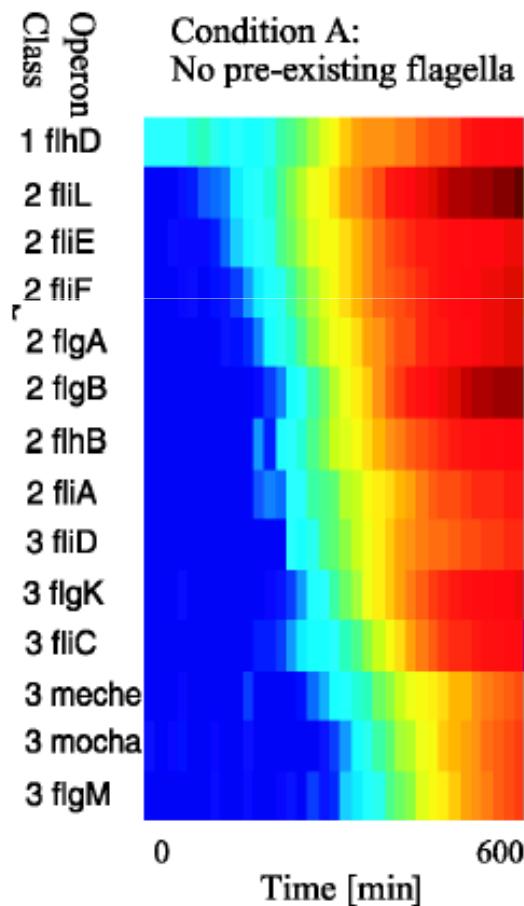
Probabilistic boolean networks

- Given the data in a probabilistic boolean network one wants to find the nature of the boolean functions and their wiring which gives the results closer to the real data.
- The goal of probabilistic boolean network is to infer minimal dependencies between the genes which is still able to describe the data and infer form a bottom-up approach the structure and nature of the regulatory interaction with reasonable error



To make the convergence of the search faster one doesn't look for the best boolean function for each gene but for a set of plausible boolean functions to use with probability c_i

Single input motif is responsible for the timing of flagella assembly



Kalir et al. Science 2001