

DEVELOPMENT OF DYNAMICS IN 1980'S AND EARLY YEARS OF ERGODIC THEORY AND DYNAMICAL SYSTEMS JOURNAL

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ABSTRACT. This is an expanded account of the talk given on September 10, 2010 at the conference dedicated to 30th anniversary of Ergodic Theory and Dynamical Systems journal at the University of Warwick. Various explanations and elaborations have been added to make the piece more suitable in a written form.

Inspired by the work of British historian Arnold Toynbee, popular during the second quarter of the twentieth century, I'd like to suggest (only half-seriously but not as a joke altogether) that areas of mathematical research experience the following phases of development, not necessarily as the full sequence:

- (1) Genesis
- (2) Growth
- (3) Flourishing
- (4) Stagnation
- (5) Decline
- (6) Hibernation or petrification (never death)
- (7) Resurrection

I will allude to this periodization in the course of my story. ¹

1. DYNAMICS BEFORE THE EIGHTIES

Modern theory of dynamical systems in the broad sense was born in the 1950's with two fundamental discoveries made by A. N. Kolmogorov: preservation of invariant tori for perturbations of integrable Hamiltonian systems and entropy as an isomorphism invariant for dynamical systems with invariant measure, see my article *A. Fifty years*

¹After some hesitation I decided not to include bibliography. Obviously a long reference list is not appropriate for this genre. On the other hand, a short list would definitely suffer both from omissions and will be open to charges of partiality. An interested reader could easily construct a list by looking up MathSciNet for the major publications of key people mentioned in the text for the appropriate period.

of entropy in dynamics, 1958-2007, in *Journal of Modern Dynamics* **1**, 2007, pp. 545-596, for an elaboration.

Nineteen seventies, the period immediately preceding creation of the *Ergodic Theory and Dynamical Systems* journal featured major developments that, with some simplification may be described as creation major structural theories, intrinsic to various sub-areas in the subject. Those included

- *Ergodic theory*:
Isomorphism theory (Ornstein with collaborators) and Kakutani equivalence theory (A.K. and Satayev in Moscow; Feldman, Ornstein, B. Weiss, Rudolph in the US) and
Multiple recurrence theory (Furstenberg with collaborators)
- *Hyperbolic dynamics*: It passed through two phases that slightly overlapped in time:
Uniform hyperbolic theory (1960-75) with Smale, Anosov, Sinai, Alexeyev, Ruelle and Bowen making principal contributions, and
Partial (Hirsch-Pugh-Shub and Brin-Pesin) and nonuniform (Pesin) hyperbolicity beginning from the early 70's.
- *Elliptic dynamics*: Here the principal event was passage from local results of the 60's (creators of KAM) to first global results for circle diffeomorphisms (Herman)

2. DYNAMICS IN THE EIGHTIES

In the nineteen eighties, the early years of *Ergodic Theory and Dynamical Systems* journal, emphasis and "action" shifted somewhat toward, on the one hand, interplay between dynamics and other major mathematical disciplines and on the other, on application of structural approach to the study of concrete systems.

At present three areas account for a high percentage of first-rate publications in dynamics and can be viewed as experiencing stage (3) and, according to attitude of the viewer, with certain elements of (4):

- A. *Homogeneous dynamics* with strong interface with *Number theory*
- B. *Teichmuller geodesic flow* with applications to interval exchange transformations, flows on surfaces, flat structures on surfaces, billiards in polygons and some interface with number theory

- C. *Partially hyperbolic dynamics* with particular emphasis on stable ergodicity and interface with nonuniformly hyperbolic theory.

Genesis and crucial early development of the areas A. and B. took place in the nineteen eighties: in the work of Dani, Margulis and Ratner (A) and Veech and Masur (B). Genesis of area C. took place in the nineteen seventies and partial hyperbolicity quickly went into temporary hibernation, from which it reappeared in the nineteen nineties, but nonuniformly hyperbolic dynamics was developing fruitfully with key work done by Mañé, Ledrappier-Young (and, also the author).

Now let me try to list the principal trends of nineteen eighties. Those are the areas that experienced growth and flourished during the period.

2.1. Flourishing of conformal (one dimensional holomorphic) dynamics. This area has peculiar history. Its original development as the theory of iterations of holomorphic functions, took place around 1920 in the classical works of the French school of the time, Fatou, Julia and Montel. It was easily the earliest branch of the theory of dynamical systems where the Poincaré program of studying asymptotic properties rather than looking for explicit solutions was carried out in a serious way. After that period of development the area went into a long hibernation interrupted by sporadic, sometimes brilliant, advances. This situation persisted till the late nineteen sixties where the interest to conformal dynamics was re-awakened as a testing ground of the freshly developed hyperbolic paradigm. It took another decade or so when a proper toolkit was developed and a comprehensive program of studying holomorphic, first of all rational, maps formulated. In the nineteen eighties the area became all the rush and talk of the town. Several major mathematicians, who made their names in different areas, foremost among them, John Milnor, Dennis Sullivan and Adrian Douady, moved to the area and brought both deep insights and enhanced prestige. One may add to that list the brilliant and influential albeit somewhat shallow work by Benoist Mandelbrot. New bright stars appeared.

2.2. Development of “parameter exclusion” method that allowed to begin finding the elusive not structurally stable nonuniformly hyperbolic behavior in “real systems”. Here the fundamental advance was made by Yakobson at the beginning of the period followed by work of Benediks-Carleson and others.

2.3. Flourishing of global one-dimensional theory and further development of KAM. Herman remained the central figure here and his brilliant student Yoccoz emerged as another major star.

Interestingly, all three areas listed above deal with low-dimensional systems and aim at a comprehensive analysis of concrete classes of systems. Two “poster children” of this approach are quadratic families, real and complex, given by simple formulas and exhibiting a fascinating variety and complexity of asymptotic behavior with intriguing parallel between the asymptotic pictures in the face space (Julia set) and bifurcation diagrams in the parameter space (“Mandelbrot bug”). Toward the end of the period producing colorful pictures of those and related phenomena became veritable industry and exhibited clear signs of decadence. Along with serious researchers and young people fascinated by the beauties and complexities of the subject, it provided fertile ground for sham operators who boosted their reputations with the general public and, unfortunately, in some academic circles as well.

Now let me mention two developments related to the tendency of integrating dynamics with other major mathematical disciplines.

2.4. Foremost among those is the synthetic area that goes under the name of “rigidity” often with various adjectives. Critical developments in rigidity that have taken two forms (i) Zimmer program of the study of actions of “large” and “rigid” groups (ii) hyperbolic dynamical systems with invariant geometric structures.

2.5. Resurrection and flourishing of variational methods that started with the pioneering works of Aubry and Mather. The key step here was transition from using variational methods to produce periodic orbits, a venerable enterprise going back to the earlier part of twentieth century and never abandoned, to considering complex infinite-dimensional variations problems that produced as critical points, various kinds of interesting non-periodic orbits and invariant sets.

3. HOW THE JOURNAL CAME ABOUT

Here my account is definitely incomplete. Peter Walters gave a short account from the Warwick perspective at the conference reception.

I left the Soviet Union for good in February of 1978, spent six months in Europe (in Vienna, Rome and Paris, in the latter place being based in IHES) and arrived to the US in the late August. During my first few years in the free world I travelled quite extensively in the great contrast with the first 33 years of my life confined to the communist “paradise” of the Soviet Union with the only diversion being two trips to Poland toward the end of period (1975 and 1977) that made deep impression on me. So my impressions of the color and variety of life in general and the professional scene in the West were fresh and deep.

I believe that idea of starting a journal dedicated to the growing field of dynamical systems came independently to Bill Parry and to me; in my case the influence of those impressions in the late nineteen seventies was essential.

Let me explain my motivation. I realized how different the publication scene in the free world was compared to the Soviet one. In the Soviet Union there were no more than half a dozen serious journals and the group of people who controlled publication in those through editorships, recommendations and refereeing, was quite small. During the Golden Age of 1960s and 70s overall quality of at least three or four leading journals was quite high, despite in-fighting of various cliques, discrimination of Jews and other attendant factors. And the field of dynamical Systems was already were represented in those leading journals: Uspehi, Doklady, Izvestija, Sbornik and then-new Funkcionalnyj Analiz.

In the West the scene was much greater in volume and much more diffuse. Interesting papers in dynamics kept appearing in a variety of journals, both general and more specialized. It looked natural to try to create a journal dedicated mostly to the field that, as I explained above, progressed greatly in the previous decade or so.

My early exchanges with Bill Parry related to the subject took place during my visits to Warwick in 1979 and 1980. According to recollections of Peter Walters and others, several crucial developments took place at the London Mathematical Society Symposium on Ergodic Theory in Durham in the summer of 1980. That was quite a remarkable gathering that certainly influenced the scene in dynamics for a number of years to come. I presume (although I do not remember the specifics) that it was there that Michel Herman became involved into the discussions and was brought on board as one of founding editors.

Projected enterprise took shape quickly and pretty painlessly. I was totally unexperienced in publication matters and when it transpired that Cambridge University Press on the initiative of indomitable David Tranah was interested in setting up the journal and anchoring it at Warwick, I was happy to be freed from thinking about administrative matters. It was agreed that editorial work would be divided between three offices with equal rights and responsibilities with respect to handling: at Warwick, in the US and in the continental Europe. Warwick office was given sole responsibility for the production phase including composition of individual issues. In retrospect I consider complete independence in editorial matters as a mistake. One should take into account though that this was before email, not speaking about internet and the only means of fast communication available was telephone.

I doubt that at the time even conference telephone calls with more than two parties were easy to arrange. But, on the other hand, none of the three principal founding fathers of the journal, Parry, Herman and myself, would pay attention to such mundane issues. Bill Parry, who played a central role in bringing the journal to life, stepped aside and for first five years of journal's life was not involved with it directly. Warwick was very ably represented by Klaus Schmidt and Peter Walters.

4. EARLY YEARS OF THE JOURNAL

As far as physical bulk is concerned, beginnings of the journal were modest. The first two volumes were correspondingly 520 and 540 pages total and volumes 3–7 contained between 640 and 650 pages. This period coincides with the tenure of the two non-Warwick founding editors, Herman and myself.² I do not remember any discussions about limitations of the journal size. Our concerns were rather to keep it going. By some reasons Cambridge insisted on officially starting the journal at the beginning of the calendar year 1981 and there was not enough time to assemble initial issues. As the result of this, during first two years at least, maybe even later, the issues were appearing considerably later than the nominal date on the cover; I remember delays of six months or more.

Let be jump ahead and describe how the journal's size changed with time. Regular issues of volume 8 still totaled 650 pages but there was also a special issue labelled 8*³ 400 pages long. After that volumes 9 through 14 stabilized at 800–830 pages. A dramatic change took place in 1995 with journal going from four to six issues per year and size jumping to 1240 pages. That was followed by a period of steady growth with 2000 pages reached in 2003 (volume 23); after that the size of the journal stabilized at that level.

In the early years the ratio of papers accepted by Warwick/US/Europe editors was about 3:2:1. This reflected the volume of papers submitted to different editors that was undoubtedly affected by perception that Herman was especially tough. Peter and Klaus acted as a team unlike the later time when, as Tony Manning told me, he and Parry just divided papers submitted to the Warwick office and handled them separately. The only coordination between editors was exchange of lists of submitted papers. My perception is that overall quality of published papers was quite high. It is interesting that five out of seven

²Parry and Anthony Manning replaced Schmidt and Walters after five years.

³Conley memorial issue, see below

papers in the anniversary collection of landmark papers published on the occasion of the journal thirtieth anniversary came from the first four volumes of the journal. I do not know exactly how many papers in regular issues were solicited by various editors but there must have been a fair number.

To what extent the journal reflected or catalyzed the principal trends of the period that I tried to describe above is for the reader to judge. My more or less thorough inspection of the first ten volumes led me to the following conclusion: not very large, but significant number of the key papers in those areas appeared in the journal, and it also played an essential role in consolidating and broadening those trends.

5. SPECIAL ISSUES OF THE JOURNAL

Aside from the regular issues that contain papers submitted to the journal in a customary fashion the journal publishes special memorial issues, or collections of papers dedicated to outstanding mathematicians shortly after their untimely deaths. Those special tributes were paid to people who died relatively young: Vladimir Mihailovich Alexeyev (46), Charles Conley (51), Vladimir Abramovich Rokhlin (65), Jurgen Moser (71), Michel Herman (58), William Parry (72); a volume is planned in memory of Dan Rudolph who died earlier this year at 60.

Let me briefly recall the story of the first of those issues, the double issue NN3-4 of volume 2, published in 1982 and dedicated to the memory of V.M.Alexeyev. Alexeyev, a leading figure in the Moscow school of dynamics, along with Anosov, Arnol'd and Sinai, but less known in the West at the time, died in December of 1980 at the time when publication of the journal has been decided but before its first issue appeared. Alexeyev never travelled to the West; like myself before the emigration, his only foreign trip was to Poland. There was however, a difference: I was automatically handicapped as a Jew, and I was also quite junior. Official mathematical establishment at the time was notoriously anti-Semitic and also clannish: I worked in an economics institute and on top of the main handicap was an outsider.⁴ Alexeyev on the other hand, was a pure ethnic Russian, and a professor of Moscow State University, the second institution, after the Steklov Institute, in the mathematical hierarchy of the time. So he did not suffer from "built-in" handicaps. But he was a Russian "intelligent"

⁴Were I interested in advancing my career under the circumstances, I would have paid more attention to research and "landscape" in economics and would have been able to overcome the main handicap at least to the extent of being allowed to travel to the West occasionally.

of the purest and highest brand, and this did not sit comfortably with the party hacks at the university and elsewhere. I felt it to be my duty as a friend and colleague to create a fitting tribute to the outstanding mathematician and remarkable man. Most people who knew and appreciated Alexeyev's work were Russians, either in the USSR or emigrants. The moment was the last bout of the Cold War, when even limited emigration for the Soviet Union virtually stopped and emigrants were officially treated as non-persons. And at this time we managed to assemble an issue with contributions from top mathematicians still in the Soviet Union (Arnol'd, Pesin, Sinai), émigrés (Brin Ratner, myself) as well as few Western mathematicians. Quality of the issue was very high, some of the papers that appeared there turned out to be quite influential, but the human aspect was at least as important.

Alexeyev memorial issue set the standard and the subsequent special issues although produced in a more conventional fashion, with a special set of editors, produced remarkable collections of papers as well as excellent biographical pieces. Needless to say, decisions to publish special issues were made by consensus of all active editors. I'd like to mention also that at some point a proposition to honor on rare occasions advanced anniversaries of outstanding living mathematicians was discussed and turned down.

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