EPSRC Symposium workshop

Large Scale Behaviour of Random Spatial Models

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Organisers: Stefan Adams, Stefan Grosskinsky and Roman Kotecký

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Titles and abstracts

Inés Armendáriz (Buenos Aires): *Metastability for zero-range condensation in the thermodynamic limit*

Consider a symmetric zero range process on the one-dimensional discrete torus $\Lambda_L = \mathbb{Z}/L\mathbb{Z}$ with N particles and jump rates $g(n) = 1 + b/n$, $b > 3$. When $N = \rho L$, $\rho > \rho_c$ the critical density, in the stationary regime the excess number of particles $(\rho - \rho_c)L$ accumulates at a randomly distributed location, called the condensate. We show here that in the limit when $L \to \infty$, as we rescale time by L^{1+b} and space by $1/L$, the condensate evolves on the torus $\mathbb{T}$ as a Lévy process with rates determined by the limiting capacities of the underlying random walk. This is joint work with S. Grosskinsky and M. Loulakis.

Alessandra Bianchi (Bologna): *Quasi-stationary measures and metastability*

In this talk we will present some recent results, obtained in collaboration with A. Gaudilliere, concerning the characterisation of metastability in the sense of Lebowitz and Penrose for Markov processes on finite configuration space and in some asymptotic regime. By comparison between the restricted ensemble and the quasi-stationary measure and introducing "soft measures" as interpolation between them, we will derive simple and practical hypotheses to establish metastability. In particular, by means of potential theoretic tools, we will provide sharp estimates on mean exit time and transition time, and prove their asymptotic exponential laws. Related estimates on the relaxation and mixing time, derived via two-sided variational principles, will be also discussed. All our asymptotic are given with explicit quantitative bounds on the corrective terms.

Marek Biskup (UCLA Los Angeles): *Gaussian fluctuations for effective conductance*

In rather broad terms, the effective conductance is the minimal value of the Dirichlet energy on a bounded domain subject to a given natural (Dirichlet or mixed Dirichlet-Neumann) boundary conditions. An interesting question in homogenization theory is to characterise this value when the underlying elliptic operator has random coefficients. It is well known that, for stationary and ergodic coefficients, the (normalised) effective conductance in a sequence of scaled domains tends to a non-random value. A natural next question is thus the size, and the asymptotic distribution, of the fluctuations. I will address this question in the context of the random conductance model with i.i.d. coefficients. In particular, I will show that in this case the effective conductance obeys a central limit theorem. Based on work done jointly with Michele Salvi and Tilman Wolff.

David Brydges (UBC Vancouver): *Combinatorial Cluster Expansion for Almost Gaussian Models*

It is quite common in equilibrium statistical mechanics for a model to have a partition function that can be written as a Gaussian expectation of a polymer system. In such cases cluster expansions

for the pressure based on the Glimm-Jaffe-Spencer cluster expansions are useful. In this talk I will review a small improvement of a theorem that appears in the 2003 PhD thesis of Gianluca Guadagni. According to this theorem, under the hypothesis that the Gaussian expectation has a finite range decomposition in length scales L_j, the pressure and correlations are given by expansions that converge provided the variation from the mean of the polymer activity caused by a typical fluctuation in length scale L_j is small compared with L_j^{-d} for all scales j.

Paul Chleboun (Rome): *Time scale separation in the East model*

The East model is a special example of a one dimensional oriented kinetically constrained particle system. The model has received a lot of attention in the physics literature because at low temperatures (when the equilibrium density of zeros, q, is small) it exhibits glassy behaviour. It has recently been shown that on finite length scales in the low temperature limit (vanishing density of zeros) there is a separation of time scales, which helps to give a rigorous description of the out of equilibrium dynamics [Faggionato, Martinelli, Roberto, Toninelli (2010)]. We show that on the equilibrium length scale $(1/q)$ there is no separation of time scales, contrary to the recently hypothesis of Sollich and Evans (2003), in which continuous time scale separation was used to motivate a "superdomain" model of the dynamics. On intermediate length scales there is time scale separation, but the details are so far less well understood. It is hoped that a further understanding of these issues could help to describe a low temperature continuum limit process.
This is joint work with Fabio Martinelli and Alessandra Faggionato.

Nick Crawford (Haifa): *Random FIeld Induced Order in Low Dimensions*

In this talk we detail recent and ongoing work on randomness induced ordering is statistical mechanics. The paradigmatic, but by no means only, example is given the $O(2)$ model in d dimensions with an additional local field term in the Hamiltonian given by i.i.d. unit vectors pointing in the North/South direction with equal probability. The question is whether, for any sufficiently small local field strength, there is a temperature low enough so that spin variables typically have preferred orientations under the corresponding Gibbs measure. If so, what direction(s) are preferred? In two dimensions, this question had been discussed many years ago in physics literature, with two groups giving contradictory answers due to the effects of strong randomness. We will discuss recent work which provides an answer in dimension 2 and 3.

Jean-Dominique Deuschel (TU-Berlin): *Hydrodynamical Limit for the Ginzburg-Landau model with non-convex potential*

We consider the dynamic of an effective interface model with gradient interaction and derive the corresponding hydrodynamical limit, extending the result of Funaki&Spohn to a certain class of non convex interaction potential. This includes the high temperature regime, where uniqueness of ergodic gradient Gibbs measures has been proved recently. Our proof is based on the identification of the translation invariant stationary measures as gradient Gibbs measures. (joint work with T Nishikawa and Y Vignaud)

Nicholas Dirr (Cardiff): *Deterministic interfaces in a random environment*

We discuss models for interfaces in a random environment, were the law of interface motion is a (deterministic) gradient flow of a randomly perturbed energy. We consider both the static (energy) case and the dynamic (evolution) case under time and space rescaling. Particular attention is given to a simplified dynamical model, the so-called random obstacle model.

Aernout van Enter (Gronningen): *Bootstrap percolation, the role of anisotropy*

Bootstrap percolation models describe growth processes, in which a critical droplet in a metastable situation nucleates. The occurrence of such critical droplets is typically ruled by asymptotic probabilities. We discuss how the scaling asymptotic is modified in the presence of anisotropy. We present sharp results in two dimensions, and discuss preliminary results in three dimensions.
This is based on joint work with Tim Hulshof, Hugo Duminil-Copin and Anne Fey.

Pierluigi Falco (USC Northridge): *Statistical Mechanics of the Two-Dimensional Coulomb Gas*

The Coulomb gas is the prototype of two-dimensional probabilistic model displaying a special kind of phase transition, the 'Kosterlitz-Thouless' one. In this presentation I will review conjectures, results and works in progress on this model, with emphasis on universality aspects.

Tadisha Funaki (Tokyo): *Invariant measure for SPDE related to the KPZ equation*

The particle system approximation to the Cole-Hopf solution of the Kardar-Parisi-Zhang equation due to Bertini and Giacomin shows the invariance of geometric Brownian motion for the stochastic heat equation. I will discuss some direct approach to this problem based on a joint work with Jeremy Quastel.

Alexandre Gaudilliere (Marseille): *Looking for large cliques through spin glasses*

The search problem for the largest cliques in a given graph is an NP-hard problem. Numerical simulations have proven the high efficiency of a recent algorithm for this problem: the cavity algorithm that was introduced by Iovanella, Scoppola and Scoppola. This is a conservative version of a probabilistic cellular automata built on statistical mechanics methods introduced in the study of spin glasses. We will analyse quantitatively the algorithm efficiency for graphs that are generally considered among the more challenging for the largest cliques search problem: Erdös random graphs. We will then have to understand the dynamics of a small cloud of particles in a disordered environment.

Pablo Groisman (Buenos Aires): *A Fleming-Viot process associated to sub-critical branching: a selection principle*

Let X be a continuous time Markov chain in a countable state space S with an absorbing state that we call 0. In this context it is natural to study the distribution of the process conditioned on non-absorption, which we call the conditioned evolution. A quasi-stationary distribution (QSD) is a probability measure that is invariant for the conditioned evolution. These distributions are important since they represent the state of the process at large times for a typical path that has not been absorbed. The Fleming-Viot process associated to X (FV) is a particle system with state space S^N. At the beginning there are N (particles) independent copies of X, but when a particle is absorbed, it jumps instantaneously over one of the N-1 particles that are not absorbed at that time. After that time, each particle evolves independently of the others.
The empirical measure of FV mimics the conditioned evolution of X. This fact makes FV a natural candidate to simulate the conditioned evolution for large times and also quasi-stationary distributions. We consider X to be a sub-critical branching process. In this case there is an infinite number of QSD, but there is a minimal one that has the smallest mean absorption time. We prove that (i) the empirical measure of FV converges to the conditioned evolution in compact time intervals, (ii) FV is ergodic for every N, and (iii) the empirical measure of FV under its own invariant measure

converges to the minimal QSD. That is, FV selects the minimal QSD.
This is a joint work with A. Asselah, P. Ferrari and M. Jonckheere.

Dima Ioffe (Haifa): *Ground states for mean field models with a transverse component: A weak KAM approach.*

For a class of quantum mean field models in a transverse field, a stochastic (path integral) representation and a subsequent large deviation analysis lead to an asymptotic description of ground states in terms of fixed points of the corresponding Lax-Oleinik semigroup. We shall present a general approach along these lines and discuss in more detail the spin-1/2 case with polynomial interactions. Based on a joint work with Anna Levit.

Milton Jara (IMPA, Rio de Janeiro): *Quenched scaling limits of trap models*

Let $G = (V, E)$ be a connected, non-oriented graph. Let $W = \{W_x; x \in V\}$ be a sequence of positive numbers. Let $X = \{X_t; t \geq 0\}$ be the following process. At site $x \in V$, the process waits an exponential time of rate $\frac{1}{W_x}$, at the end of which it jumps to one of its neighbors with uniform probability. We interpret W_x as the depth of a trap at site x, and we call X a *trap model*. We obtain sufficient conditions under which trap models defined on a sequence of graphs are metastable in the sense of Beltrán-Landim, and in particular we prove convergence to a generalization of the K-process of Fontes-Mathieu. We prove that these conditions are satisfied by growing sequences of hypercubes, tori, expanders, random d-regular graphs and by the supercritical Erdös-Rényi graph, if the depth of the traps are chosen according to a i.i.d. sequence of positive random variables on the domain of attraction of an α-stable law.
Joint work with C. Landim and A. Teixeira.

Wolfgang König (TU-Berlin): *Large deviations for the local times of random walk among random conductances*

We consider the random walk among random conductances (RWRC) on the d-dimensional lattice, a continuous-time version of a random walk in random environment. The main object of our study is the local times of the walk. We investigate the asymptotics of the annealed probability that the RWRC does not leave a given box (time-dependent and growing with time, respectively) and that its local times approach a given (rescaled) shape. This question is analogous to the famous Donsker-Varadhan large deviations for the local time of simple random walk. Our main assumption on the conductances is that they be arbitrarily small (but stay positive). Then the scale of the principle depends on the tails of the conductances at zero. It turns out that the rate function is essentially equal to the p-norm of the gradient for some characteristic parameter $p \in (0, 2)$, a functional with interesting properties. In particular, its concentration properties display interesting relations with the typical behaviour of the RWRC in a given box.
Joint work with Tilman Wolff and Michele Salvi.

Tobias Kuna (Reading): *Spectral gap for potentials with non-negative parts for an infinite particle system*

We consider a Glauber type dynamics for a system of infinite many particles in the full space of non-zero density for which Gibbs measures are irreversible distributions. A Bochner type formula is derived which yields a spectral gap. Hence exponential ergodicity can be expected in the thermodynamic limit. We define a special class of potentials with non-trivial attractive part and show that there exists temperatures for which no phase transition occurs for any value of the chemical potential. We derive an improved high-temperature regime.

Claudio Landim (Rio de Janeiro): *Metastability of the condensate in zero range processes*

It is well known that in the stationary state of certain zero range processes all particles but a finite number accumulate on one single site. We show in this talk that in an appropriate time scale the site which concentrates almost all particles evolves as a random walk whose transition rates are proportional to the capacities of the underlying random walk.

Stephan Luckhaus (Leipzig): *Many body Hamiltonians for Plasticity*

We propose a many body Hamiltonian that captures phenomenologically microscopic aspects of plasticity. So far we are able to show a lower estimate by another Hamiltonian defined in terms of continuous strains. This can be regarded as a first step in upscaling.

Julián Martínez (Utrecht): *Variational description of Gibbs-non-Gibbs dynamical transitions for the Curie-Weiss Model*

We consider a Curie-Weiss model, with external magnetic field h subject to an independent spin-flip dynamic. For this model, Gibbs-non-Gibbs transitions are shown to be equivalent to the occurrence of bifurcations in the set of global minima of the large deviation rate function for the trajectories of the magnetization conditioned on their endpoint. (joint work with R. Ferndez and F. den Hollander)

Georg Menz (Max-Planck-Institut, Leipzig): *A two-scale proof of the Eyring-Kramers formula*

We consider a diffusion on a potential landscape which is given by a smooth Hamiltonian in the regime of small noise. We give a new proof of the Eyring-Kramers formula for the spectral gap of the associated generator of the diffusion. The proof is based on a refinement of the two-scale approach introduced by Grunewald, Otto, Villani, and Westdickenberg and of the mean-difference estimate introduced by Chafai and Malrieu. The Eyring-Kramers formula follows as a simple corollary from two main ingredients: The first one shows that the Gibbs measures restricted to a domain of attraction has a "good" Poincar constant mimicking the fast convergence of the diffusion to metastable states. The second ingredient is the estimation of the mean-difference by a new weighted transportation distance. It contains the main contribution of the spectral gap, resulting from exponential long waiting times of jumps between metastable states of the diffusion. With this new approach one can also estimate the constant of the logarithmic Sobolev inequality.

Stefano Olla (Paris): *Dispersion, diffusion (and super-diffusion) of energy in a chain of coupled oscillators*

I will review some results on the dispersion and diffusion of energy in a chain of oscillators whose hamiltonian dynamics is perturbed by stochastic conservative terms. On one dimensional unpinned case the energy super-diffuse, but we still do not understand the nature of this super-diffusion.

Ron Peled (Tel Aviv): *Random surfaces and graph homomorphisms*

I will discuss connections between certain discrete models of random surfaces and the notion of graph homomorphisms coming from combinatorics. I will then explain a notion of strong localization conjectured to occur for graph homomorphisms and discuss its implications for the random surface models. Finally, I will tell about a recent development in the context of expander graphs which is joint work with W. Samotij and A. Yehudayoff.

Frank Redig (Delft): *Large deviations for non-conventional averages and lattice spin systems*

For an iid Bernoulli sequence of Ising spins we consider averages along arithmetic progressions, i.e., of the form $S_n := (1/n) \sum_{i=1}^n s_i s_{2i}..s_{ki}$ Such sums appear in the ergodic theory of multiple ergodic averages, which plays an important role in Furstenberg's proof of Szemeredi's theorem. In this talk, we look at the large deviation properties of S_n. For $k = 2$ we explicitly compute the large deviation function of S_n, and show that it is a weighted average of one-dimensional Ising model partition functions. For $k > 2$, we show that the large deviations of S_n are related to free energy of higher dimensional lattice spin systems. We also show an underlying structure of "multiplicatively invariant Gibbs measures", which we believe is of independent interest.

Based on joint work with G. Carinci, J.R. Chazottes and C. Giardina,

Eris Runa (Bonn): *Sets of finite perimeter in Wiener spaces*

The space of functions of bounded variation, BV, is defined as the collection of functions whose distributional derivative is a finite measure. These spaces arise naturally in a lot of applications. A very remarkable result, due to De Giorgi, says that if $\chi_E \in \mathrm{BV}$ then $D\chi_E = \nu_E \cdot \mathcal{H}^{n-1} \llcorner \mathcal{F}E$, where $\mathcal{H}^{n-1}$ is the Hausdorff measure, $\mathcal{F}E$ is rectifiable and ν_E is a suitable vector field which represents the inner normal. Let (X, H, γ) be an abstract Wiener space. Using the Malliavin Calculus it is possible to introduce weakly differentiable functions, hence Sobolev spaces and functions of bounded variations. It is natural to ask if there exist some equivalent extension of the De Giorgi's Theorem with the appropriate definitions. One of the main difficulties is the lack of the so called differentiation of measures theorems in Wiener spaces. There have been few results in this direction, which will be briefly discussed. Then I will explain some recent results obtained in collaboration with L. Ambrosio and A. Figalli. Among these, we establish a blow-up principle, a vector field which represents the inner normal in the infinite dimensional setting and an extension of De Giorgi's Theorem.

Senya Shlosman (Marseille): *Rotating States*

I will explain the construction of an interacting particle system which has a unique stationary state but which is not ergodic. In other words, the dynamics started from different initial states never converge to each other, while the stationary state of the dynamics is unique. The result can be interpreted as the case of the non-uniqueness of the steady state.

Joint work with Christian Maes.

Martin Slowik (Berlin): *Metastability in stochastic dynamics: Upper and lower bounds on the small eigenvalues of the generator*

In the context of reversible stochastic dynamics, the potential theoretic approach to metastability has proven to be a powerful tool to derive sharp estimates for quantities characterizing the metastable behavior, e.g. metastable exit times. A key idea is to express objects of interests through capacities and to use variational principles to compute the latter.

In this talk, I will present an approach to derive matching upper and lower bounds on the small eigenvalues of the generator. This approach is specifically aimed at studying systems in which the entropy of microscopic paths plays a crucial role such that sufficiently precise computations of microscopic point to point capacities are not possible. In particular, I will discuss the relation between the low-lying part of the spectrum and mean exit times of metastable sets. As an illustration, this approach is applied to one of the simplest non-trivial model of this type: Curie-Weiss-Potts model at finite temperature and in an iid continuously distributed random field.

Dimitris Tsagkarogiannis (Bonn): *Cluster expansion in the canonical ensemble*

We shall present the validity of the cluster expansion for the canonical partition function in the low

density regime. Working in the context of classical particles confined in a box and interacting via a stable and tempered potential we prove (in a joint work with E. Pulvirenti) that the convergence is uniform in the volume and that in the thermodynamic limit it reproduces Mayer's virial expansion providing an alternative and more direct derivation.