Abstracts for EPSRC Workshop: Topics in Control

November 15, 2011

Optimal trade execution under price-sensitive risk preferences.

Stefan Ankirchner

HCM, Bonn, Germany

We present a discrete time model that allows for a price-sensitive closure of a large asset position, providing thus a device to introduce skewness in the proceeds/costs. By appealing to dynamic programming we derive semi-explicit formulas for the optimal execution strategies. We then present a numerical algorithm for approximating optimal execution rates as functions of the price. We provide error bounds and prove convergence. Finally some numerical experiments illustrate the effciency of the algorithm. The talk is based on joint work with Thomas Kruse.

Multivariate prediction and matrix Szegö theory.

Nicholas H. Bingham

Imperial College, UK

Multivariate prediction goes back half a century, to the work of Winer & Masani and to Helson & Lowdenslager. Prediction theory has many links with the theory of orthogonal polynomials on the unit circle [OPUC], and multivariate prediction has similar links with matrix OPUC [MOPUC]. There has been a lot of recent work on MOPUC; the talk surveys the area.

Sensitivity and Out-of-sample Error in Data Assimilation.

Jochen Bröcker

Max Planck Institute, Dresden, Germany

"Data Assimilation" is one of many names for the following problem: Given a history of observations as well as a dynamical model, find trajectories which are, on the one hand, consistent with the model, and on the other hand, consistent with the observations. Attaining both objectives at the same time is essentially never possible (nor in fact desired) in reality, since our models are invariably simplifications. Any data assimilation algorithm should therefore allow for deviations from the proposed model equations as well as from the observations. How we might trade off between these two is the subject of this talk. It is shown how we can still find "good" trajectories, where the measure of goodness obviously cannot be just the deviation from the observations. Rather, a measure similar to the out-of-sample error from statistical learning is considered. The connection between the out-of-sample error and the sensitivity is elucidated, including some numerical examples.

A statistical approach to data assimilation for hemodynamics.

Marta D'Elia

Emory University, Georgia, USA

Cardiovascular applications recently gave a strong impulse to numerical methods for fluid dynamics. Furthermore, thanks to new precise measurement devices and efficient image processing techniques, medicine is experiencing a tremendous increment of available data, inevitably affected by noise. Beyond validation, these data can be combined with numerical simulations in order to develop mathematical tools of clinical impact. Techniques for merging data and mathematical models are known as data assimilation (DA) methods. In the context of hemodynamics the reliability of assimilated solutions is crucial and it is mandatory to quantify the uncertainty of numerical results.

In this talk we discuss a DA technique for hemodynamics based on a Bayesian approach to inverse problems for the estimation of statistical properties of the blood velocity and related variables. This method is formulated as a control problem where a weighted misfit between data and velocity is minimized under the constraint of the Navier-Stokes equations. The optimization problem is solved with a discretize-then-optimize approach relying on the finite element method. The result of the inversion is the probability density function of the normal stress at the inflow section of the vessel; using such distribution we derive statistical estimators (namely the maximum a posteriori and the maximum likelihood estimators) and confidence intervals for the velocity.

We present numerical results on 2-dimensional and 3-dimensional axisymmetric geometries approximating blood vessels, we compare statistical and deterministic estimators and we draw confidence regions for quantities of interest.

Some applications of the Stochastic Maximum Principle.

Boualem Djehiche

KTH, Stockholm, Sweden

I will highlight some aspects (old and new) of the Stochastic Maximum Principle for the characterization of optimal controls of diffusion processes of mean field type in the pre-commitment and the non-pre-commitment cases.

Mean field games on graph.

Olivier Guéant

Universit Paris-Diderot, Paris, France

We will present mean field games on graphs. Existence and uniqueness will be discussed in a very general framework. In the absence of congestion we will present a global problem whose solutions are the same as the equilibria of the mean field game.

Differentiable Approximation of Lévy and Fractional Processes.

Astrid Hilbert

Linnaeus University, Vaexjoe, Sweden

Regularity of the sample paths is crucial, e.g. when constructing solutions to stochastic differential equations on manifolds. Another reason might be that the field of applications suggests it as for Newton equations with a stochastic perturbation, the natural generalizations of Ornstein Uhlenbeck processes where physicists desire a dynamical theory.

Using a scaling limit we derive a differential approximation of Fractional Brownian motion. This generalizes joint results with Haidar Al Talibi and V. Kolokoltsov for Lévy processes.

This is joint work with Haidar Al Talibi.

Decentralized Decision Making in Mean Field Stochastic Systems.

Minyi Huang

School of Mathematics and Statistics, Carleton University Ottawa, Canada

Mean field interaction phenomena arise in a broad range of fields including engineering, economics and social science. We consider stochastic control systems with a large but finite population of players which are coupled via their individual dynamics and/or costs and which in the infinite population limit are individually insignificant. We then present a paradigm for obtaining decentralized equilibrium strategies in the non-cooperative games and cooperative social optimization cases. Inspired by statistical mechanics, we use infinite population mean field approximations to decompose the original large scale Nash game (resp., social optimization problem) into localized optimal control problems, which leads to the so-called Nash (resp., Social) Certainty Equivalence Principle. In the resulting solution, each individual uses controls depending only on (i) local information for stochastic feedback control, and (ii) on a separate process which is a function of the infinite population macroscopic behaviour of the system. For nonlinear diffusion models, the analysis involves a coupled system of McKean-Valsov equation and Hamilton-Jacobi-Bellman (HJB) equation. In the linear-quadratic-Gaussian (LQG) case, explicit solutions can be obtained and performance of the decentralized strategies may be estimated. Some related works by other authors will also be discussed.

We further extend the above methodology to models with major players and a large number of minor players, and examine the informational impact of major players.

(Based on joint work with Profs. Peter E. Caines, Roland P. Malhame, and Dr. Son Luu Nguyen).

New results on (F)BSDE of quadratic growth

Peter Imkeller

HU Berlin

A financial market model is considered on which agents (e.g. insurers) are subject to an exogenous financial risk, which they trade by issuing a risk bond. They are able to invest in a market asset correlated with the exogenous risk. We investigate their utility maximization problem, and calculate bond prices using utility indifference. In the case of exponential utility, this hedging concept is interpreted by means of martingale optimality, and solved with BSDE with drivers of quadratic growth in the control variable.

For more general utility functions defined on the whole real line we show that if an optimal strategy exists then it is given in terms of the solution (X, Y, Z) of a fully coupled FBSDE. Conversely if the FBSDE admits a solution (X, Y, Z) then an optimal strategy can be obtained. In the complete market case, an assumption on the risk aversion guarantees that the FBSDE admits a solution for any finite time horizon. As a particular example of our approach we recover the BSDE for exponential utility, and are able to treat non-classical utility functions like the sum of exponential ones.

For utility functions defined on the half line we also reduce the maximization problem to the solution of FBSDE connected with the ones obtained by Peng (1993). For power utility and convex constraints, a compactness criterion by Delbaen and Schachermayer provides solvability. This is joint work with U. Horst, Y. Hu, A. Réveillac, and J. Zhang.

Minimizing the shuttle time for a diffusion.

Saul Jacka

University of Warwick, UK

We consider the problem of controlling the drift of a diffusion (which reflects at 0 and 1) so as to minimise the time it takes to travel from 0 to 1 and back again. Once the drift has been set at a level it remains fixed. Versions of the problem are solved in both discrete (time-permitting) and continuous set-ups.

Game theoretic analysis of incomplete markets: emergence of probabilities, nonlinear and fractional Black-Scholes equations.

Vassili Kolokoltsov

University of Warwick, UK

Expanding the ideas of the author's paper 'Nonexpansive maps and option pricing theory' (Kibernetica, **34:6**, 1998, 713-724) we develop a pure game-theoretic approach to option pricing, by-passing stochastic modeling. Risk-neutral probabilities emerge automatically from the robust-control evaluation. This approach seems to be especially appealing for incomplete market encompassing extensive, so to say untamed, randomness, when the coexistence of an infinite number of risk-neutral measures precludes one from unified pricing of derivative securities. Sometimes, we can specify even a unique risk-neutral selector. Our method is robust enough to be able to accommodate various market rules and settings including path-dependent payoffs, American options and transaction costs. On the other hand, it leads to rather simple numerical algorithms. The continuous-time limit is described by nonlinear and/or fractional Black-Scholes type equations.

Filtering the Navier Stokes Equations.

Kody J. H. Law

University of Warwick, UK

Data assimilation refers to methodologies for the incorporation of noisy observations of a physical system into an underlying model in order to infer the properties of the state of the system (possibly including parameters). The model itself is typically subject to uncertainties, in the input data and in the physical laws themselves. This leads naturally to a Bayesian formulation in which the posterior probability distribution of the system state, given the observations, plays a central conceptual role. We use this Bayesian posterior probability distribution as a gold standard against which to evaluate various commonly used data assimilation algorithms.

We study the 2D Navier-Stokes equations in a periodic geometry and evaluate a variety of sequential filtering approximations based on 3DVAR and on extended and ensemble Kalman filters. The performance of these data assimilation algorithms is quantified by comparing the relative error in reproducing moments of the posterior probability distribution.

The primary conclusions of the study are that: (i) with appropriate parameter choices, approximate filters can perform well in reproducing the mean of the desired probability distribution; (ii) however these filters typically perform poorly when attempting to reproduce information about covariance; (iii) this poor performance is compounded by the need to modify the filters, and their covariance in particular, in order to induce filter stability and avoid divergence. Thus, whilst approximate filters can be a useful tool in predicting mean behaviour, they should be viewed with caution as predictors of uncertainty. We complement this study with (iv) a theoretical analysis of the ability of the filters to estimate the mean state accurately, and we derive a non-autonomous (S)PDE for this estimator in the continuous-time limit.

This is joint work with A. M. Stuart, C. E. A. Brett, K. F. Lam, D. S. McCormick, M. R. Scott.

Control of complex systems.

Robert S. MacKay

University of Warwick, UK

A major issue in complex systems is to control not the state but the probability distribution for the state. In the simplest context of Markov chains we present a formula for how the natural probability distribution (non-autonomous version of stationary probability) depends on time-dependent control of the transition probabilities.

Suppression of bad news in markets: Explicit equilibrium analysis of correlated optimal data censors.

Adam J. Ostaszewski

LSE, UK

We generalize to a multi-firm setting the no-arbitrage theory of voluntary disclosure due to Dye (1985). Here n firms, simultaneously and voluntarily, release at an interim-report date 'partial' information concerning their 'common operating conditions'. Each of the firms has, as in the Dye model, some (known) probability of observing a signal of their end of period performance, but (unlike in Dye) this signal includes noise determined by a firm-specific precision parameter. The co-dependency of the firms results entirely from their common operating conditions.

Each firm has a disclosure cutoff, which is a best response to the cutoffs employed by the remaining firms. To characterize these equilibrium cutoffs explicitly, we introduce n new/hypothetical firms/, which are operationally independent but are assigned refined precision parameters and amended means related to the corresponding actual firms. This impounds all existing correlations arising from conditioning on the other potentially available sources of information.

In the model the actual firms' equilibrium cutoffs are geometric weighted averages of the above-mentioned hypothetical firms.

We uncover two countervailing effects. Firstly, there is a /bandwagon effect/, whereby the presence of other firms raises each individual cutoff relative to what it would have been in the absence of other firms. Secondly, there is an /estimator-quality effect/ whereby individual cutoffs are lowered, unless the individual precision is above average.

Planar dynamical systems perturbed by heavy-tailed Levy noise.

Ilya Pavlyukevich

Friedrich-Schiller-Universität, Jena, Germany

We study metastable behaviour of planar dynamical systems perturbed by small multiplicative Levy noise with heavy tails. Special attention is paid to the dynamics near stable limit cycles. As examples we consider Duffing and van der Pol oscillators in presence of stable Levy perturbation. This is a joint work with M. Hgele (U Postdam, Germany).

A new ν-metric in control theory.

Amol Sasane

LSE, UK

The need for measuring the distance between control systems is basic in control theory. For example, in robust control theory, one knows that the unstable control system P to be stabilized is just an approximation of reality, and so one would really like the stabilizing controller to not only stabilize the nominal control system P, but also all sufficiently close systems P' to P. The question of what one means by "closeness" of control systems thus arises naturally. So one needs a metric on the set of unstable systems which is amenable to computation and it has good properties in the robust stabilization problem. Such a desirable metric was introduced by Glenn Vinnicombe in 1993, and is called the ν-metric. There essentially the ring R of transfer functions of stable control systems was taken to be the set of the rational functions without poles in the closed unit disk. The problem of what happens when R is some other ring of stable transfer functions (for example the Hardy algebra) was left open. In this talk, we address this issue, and give an extension of the ν-metric.

Fluid approximation to controlled Markov chains with local transitions

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Fluid scaling is widely used in queuing theory, inventory, population dynamics etc because it usually leads to relatively simple deterministic models which describe the underlying stochastic systems accurately enough. In this talk, the idea of this approach will be illustrated on simple examples:
– firstly, for a fixed control strategy, the trajectories of the stochastic process converge almost surely to the (deterministic) solution of an ordinary differential equation;
– secondly, the solution of the original Bellman equation converges to the Bellman function of the corresponding deterministic optimal control problem.

We shall discuss the rates of convergence and estimate the error terms for a special case of absorbing model.

It should be emphasized that the straightforward fluid approximation sometimes fails to work. A concrete example will show that the deterministic Bellman fucntion can be finite and well defined, whereas in the underlying stochastic model the Bellman function equals infinity. To avoid such situations, we propose the 'refined' fluid approximation and show that it works for a wider class of models.

If time permits, we can also discuss applications of this theory to information transmission, mathematical epidemiology and inventory.

Several recent publications of mine on the topic:
A.Piunovskiy. Controlled jump Markov processes with local transitions and their fluid approximation. WSEAS Trans. On Systems and Control, 2009, V.4, N.8, p.399-412.
A.Piunovskiy. Random walk, birth-and-death process and their fluid approximations: absorbing case. Math. Meth. Oper. Res., 2009, V.70, N.2, p.285-312.
A.Piunovskiy and Y.Zhang. Accuracy of fluid approximation to controlled birth-and-death processes: absorbing case. Math. Meth. Oper. Res., 2011, V.73, N.2, p.159-187
K.Avrachenkov, A.Piunovskiy and Y.Zhang. Asymptotic fluid optimality and efficiency of tracking policy for bandwidth-sharing networks. J. Appl. Prob., 2011, V.48, N.1, p.90-113.
A.Piunovskiy and Y.Zhang. On the accuracy of fluid approximations to a class of inventory-level dependent EOQ and EPQ models. Adv. In Oper. Research, 2011, Article ID 301205, 23 pp.

Of course, I would be happy to provide these articles to the interested people.

ABSTRACT

PASSIVE REALIZATIONS OF STATIONARY STOCHASTIC PROCESSES

Natalia Rozhenko

We study stationary regular vector processes with spectral densities which are the boundary values of matrix-functions with bounded Nevanlinna characteristic. Such types of stochastic processes give rise of natural and widely used class of realization models. These models lead to simple recursion estimate algorithms called filters. Specifically the processes with rational spectral densities admit minimal realizations with finite-dimensional state spaces which take on the role of such filters.

We are going to present a new approach to study stochastic processes, linear stochastic systems and corresponding prediction, filtering, stability problems in discrete time case. This approach is based on the connection between the theory of stochastic realizations ([1]–[3]) and the theory of passive impedance systems ([4]–[6]).

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Optimal control of SDEs associated with general Lévy generators.

Jiang Lun Wu

Swansea Universtiy, UK

In this talk, I will consider an optimal control problem for SDEs associated with general Lévy generators. The problem has its root from finanacial optimization. We will discuss maximum principle and related HJB equations with a special focus on the case of polar-decomposed Lévy measures.

Mean field games and nonlinear Markov processes.

Wei Yang

University of Warwick

We present mean field games methodology in a fully general setting, based on the theory of nonlinear Markov processes. The realization of this methodology includes three steps. First, we prove the well-posedness of general kinetic equations with coupled functional parameters. Second, we proved the well-posedness and sensitivity for the HJB equation. Finally, we justify the mean field games as a proper approximation of large population systems as N goes to infinity.

Mean field games appear in the papers by P. L. Lions and co-authors and P. Cains and co-authors, where the underlying processes are all Brownian motions. Our aim is to extend this methodology to a fully general setting with arbitrary underlying Markov processes of interactions. This is a joint work with Vassili Kolokoltsov and Jiajie Li.

Singular control of SPDEs and backward SPDEs with reflection.

Tusheng Zhang

Manchester University, UK

We first consider general singular control problems for random fields given by a stochastic partial differential equation (SPDE). We show that under some conditions the optimal singular control can be identified with the solution of a coupled system of SPDE and a kind of reflected backward SPDE (RBSPDE). Secondly, we establish the existence and uniqueness of solutions of RBSPDEs, which is of independent interest.