Control of Irrelevant terms
in Wilson RG
(with Gordon Slade)

arxiv: 1403.7256

Λ lattice torus, periodic
period L^N

$$V_{g,\nu,z}(\Lambda) = \sum_{x \in \Lambda}$$

$$V_{g,\nu,z}(\Lambda) = \sum_{x\in\Lambda} \left(\frac{z}{2} \varphi_x (-\Delta\varphi)_x + \frac{\nu}{2}|\varphi_x|^2 + \frac{g}{4}|\varphi_x|^4 \right)$$

$$d = 4$$

$$\Xi = \int_{\mathbb{R}^{n\Lambda}} e^{-V_{g,\nu,\overset{1}{\cancel{z}}}(\Lambda)} \, d\phi$$

$$\langle \varphi_a \varphi_b \rangle, \qquad \langle e^{\lambda_a\varphi_a + \lambda_b\varphi_b} \rangle \qquad a, b \in \Lambda$$

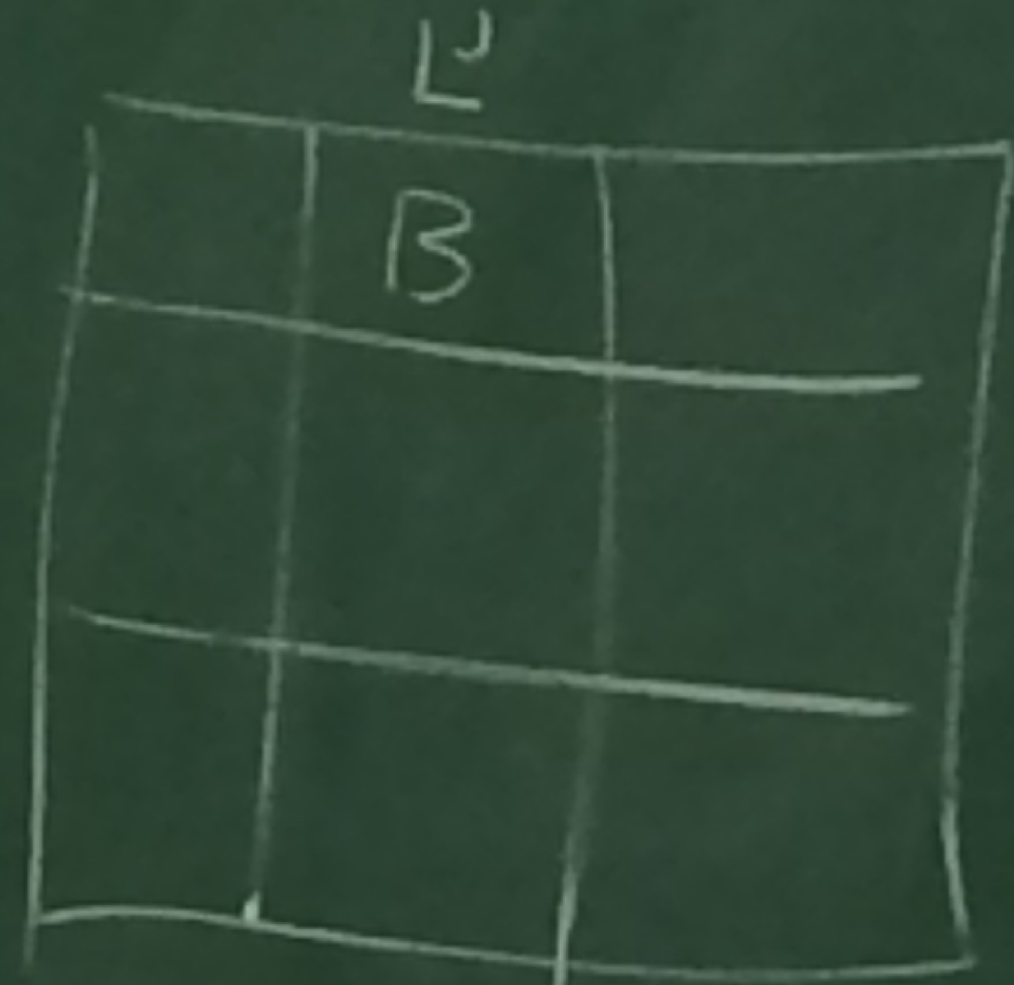
$$\cdots = \cancel{\left(\sqrt{1+z_0}\right)^{|\Lambda|}} \; \mathbb{E}_{GFF(m^2)} \, e^{-\underbrace{V_{g_0,\nu_0,z_0}(\Lambda)}_{V_0(\Lambda)}}$$

$\varphi \overset{D}{=} \varphi_1 + \varphi_2 + \dots + \varphi_N$ indep

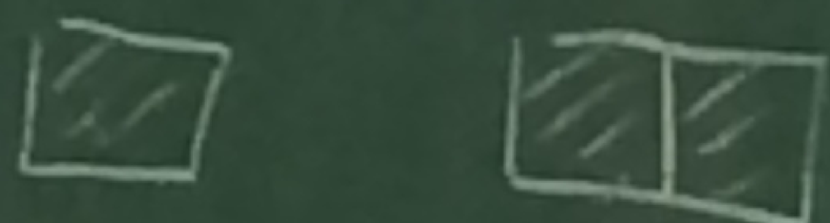
$$= \mathbb{E}_N \theta \cdots \left(\mathbb{E}_1 \theta \, e^{-V_0(\Lambda)} \right)$$

$\varphi \leftarrow \varphi_{>1} + \varphi_1$

Blocks



$B \in \mathcal{B}_j(\Lambda)$



Λ $\quad j = 0, 1, \dots, N$

$X \in \mathcal{P}_j(\Lambda)$ $\quad X = \cup B$

$X \in \mathcal{C}_j(\Lambda)$ connected

$\mathcal{N}(X) =$

$$\mathcal{N}(X) = \{ \text{functions of } \varphi|_X \}$$

$$\mathcal{N} = \mathcal{N}(\Lambda)$$

<u>Example</u>

$$e^{-V(\Lambda)} = \prod_{B \in \mathcal{B}_j(\Lambda)} \underbrace{e^{-V(B)}}_{\in \mathcal{N}(B^{\square})}$$

For a map

$$I : \mathcal{B}_1 \longrightarrow \mathcal{N}$$

$$I^X = \prod_{B \in \mathcal{B}(X)} I(B)$$

<u>Proposition</u>

There exists a sequence $I(X)$ $I_1(V)$ of local maps and a map $V \longmapsto V_{pt}$

such that

$$\mathbb{E}_{j+1}\, \Theta\left(I_j(V)\right)^{\wedge}$$

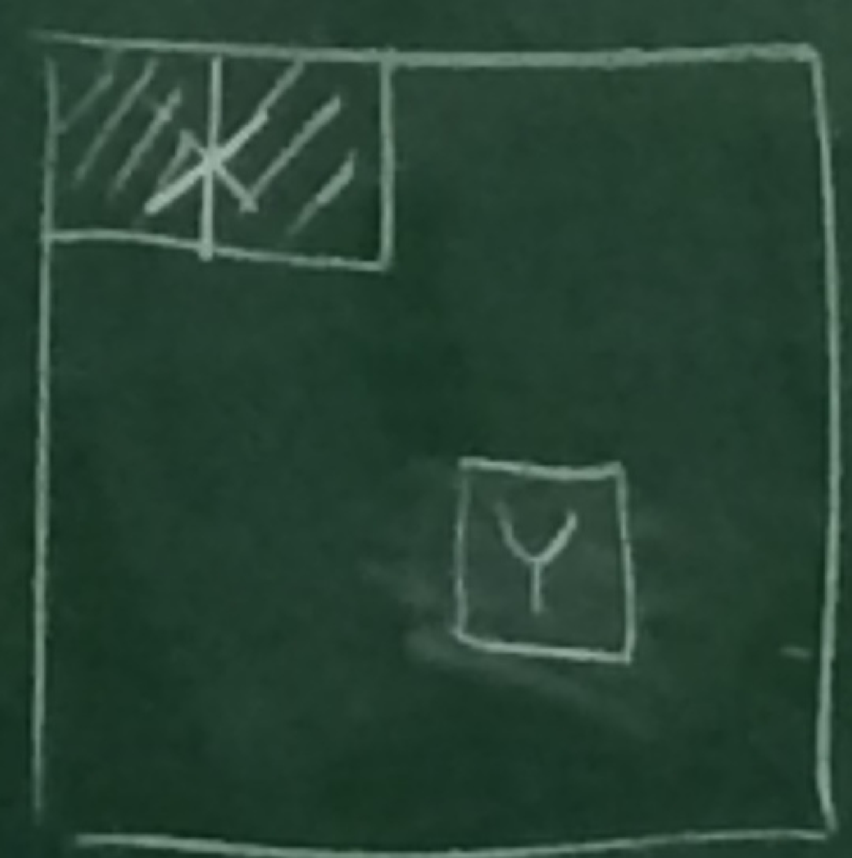
$$= I_{j+1}(V_{pt})^{\wedge} + O(V^3)$$

$V_0(\Lambda)$

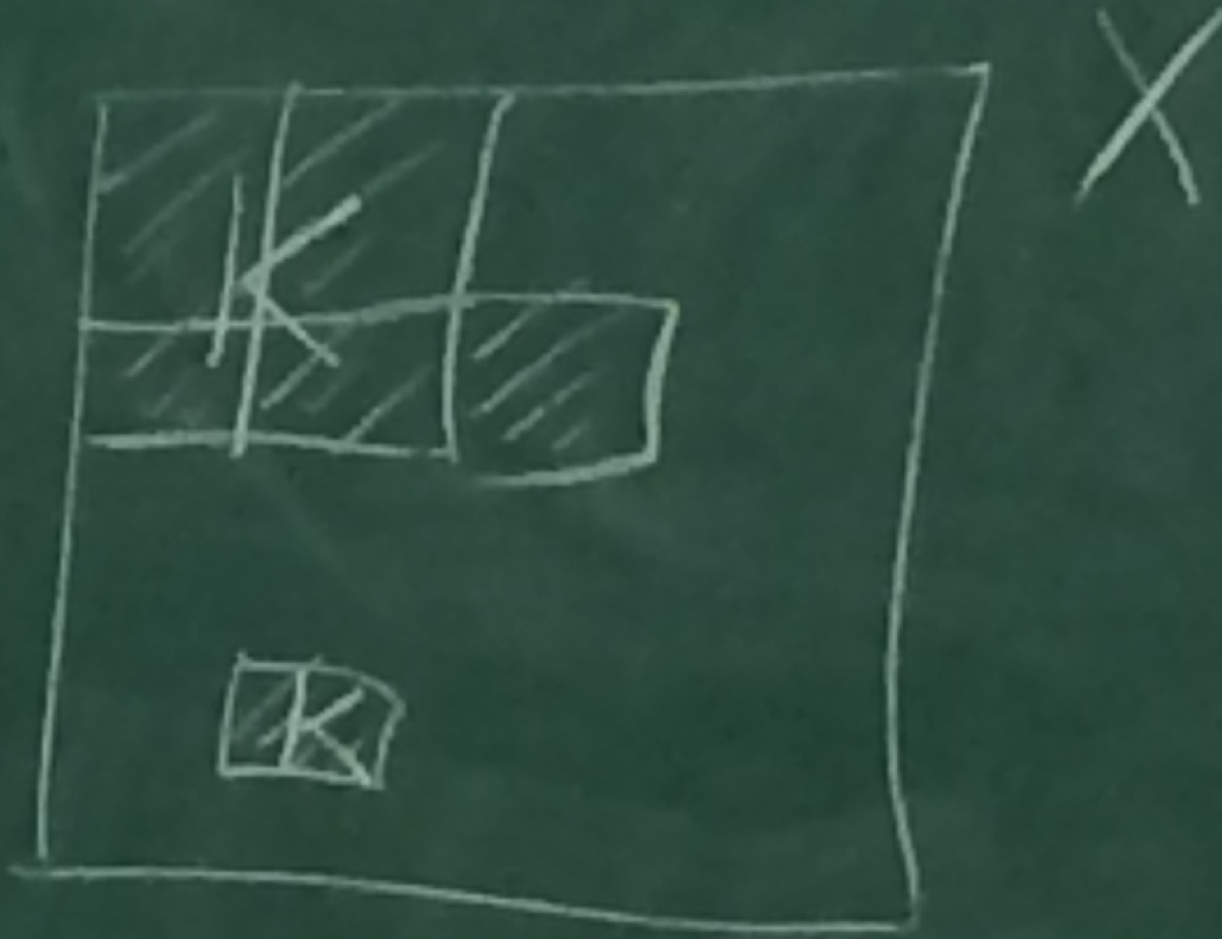
Define

$\mathcal{K}_j = \{$ all local $K : \mathcal{P}_j \to \mathcal{N}$ that factor across components $\}$

$K(X \sqcup Y) = K(X)\,K(Y)$



$$(K \circ F)(X) = \sum_{Y \in P_j(X)} K(Y) \; F(X \setminus Y)$$



Example

$$F(X \setminus Y) = I^{X \setminus Y}$$

Define

$$\widetilde{\mathcal{F}}_j = \{ \text{all } K : \mathcal{C}_j \to \mathcal{N} \text{ local} \}$$

This is a Banach space with norm

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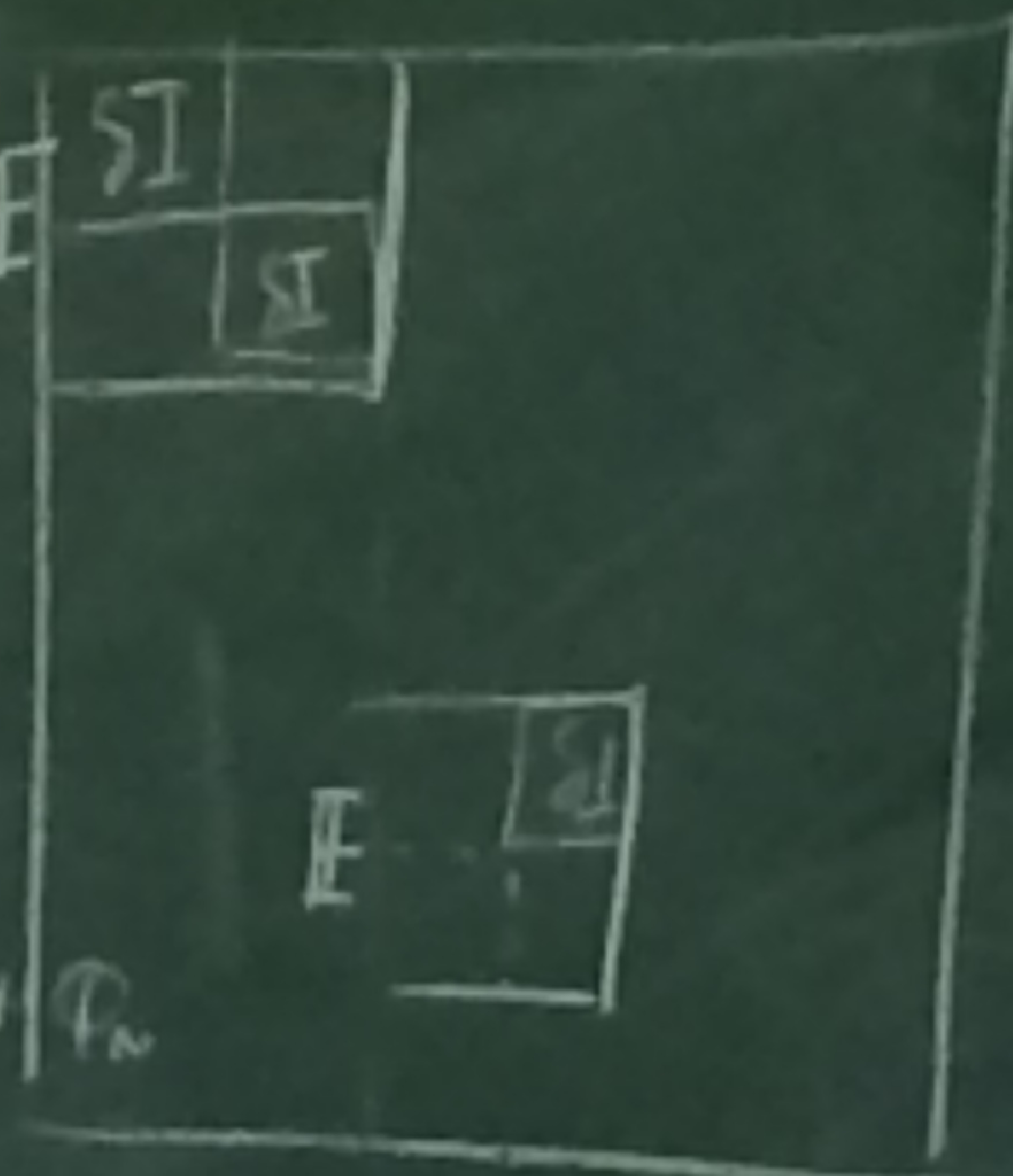
$L=2$

$\mathbb{E}_1 \, \Theta \, I_0^{\wedge}$

$\varphi_2 + \cdots + \varphi_N$ $\mathbb{E}$

$$\Theta I_0(B) = I_1(B) + \delta I(B)$$

$\varphi_1, \varphi_2 + \cdots + \varphi_N$



$$= \mathbb{E}_1 \left(I_1 + \delta I \right)^{\wedge} =$$

$$e^{-V(\Lambda)} = \prod e^{-V(B)}$$

Dem of RG

$$(V, K_j)$$

$$\downarrow$$

$$(\mathbb{I}(V) \circ K_j)(\Lambda)$$

$$\parallel$$

$$(\mathbb{I}(\hat{V}) \circ \hat{K}_j)(\Lambda) \qquad\qquad (\hat{V}_{pt}, K_{j+1})$$

$$\downarrow \qquad\qquad\qquad\qquad \uparrow$$

$$\mathbb{E}_{j+1}\, \theta\, (\mathbb{I}(\hat{V}) \circ \hat{K}_j)(\Lambda) = (\mathbb{I}(\hat{V})_{pt} \circ K_{j+1})(\Lambda)$$

Thm

For L large, $\exists$ domain D and a ball $B_{\mathcal{F}_j}(\epsilon)$ s.t

$$K_+ : D \times B_{\mathcal{F}_j}(\epsilon) \longrightarrow B_{\tilde{\mathcal{F}}_{j+1}}(\kappa\epsilon)$$

$$K_+(V, 0) \in B_{\tilde{\mathcal{F}}_{j+1}}(\epsilon^3)$$

s.t K_+ is analytic and $\kappa = O\left(\frac{1}{L}\right)$