THE UNIVERSITY *of* EDINBURGH

Spatiotemporally complete condensation in a non-Poissonian exclusion process

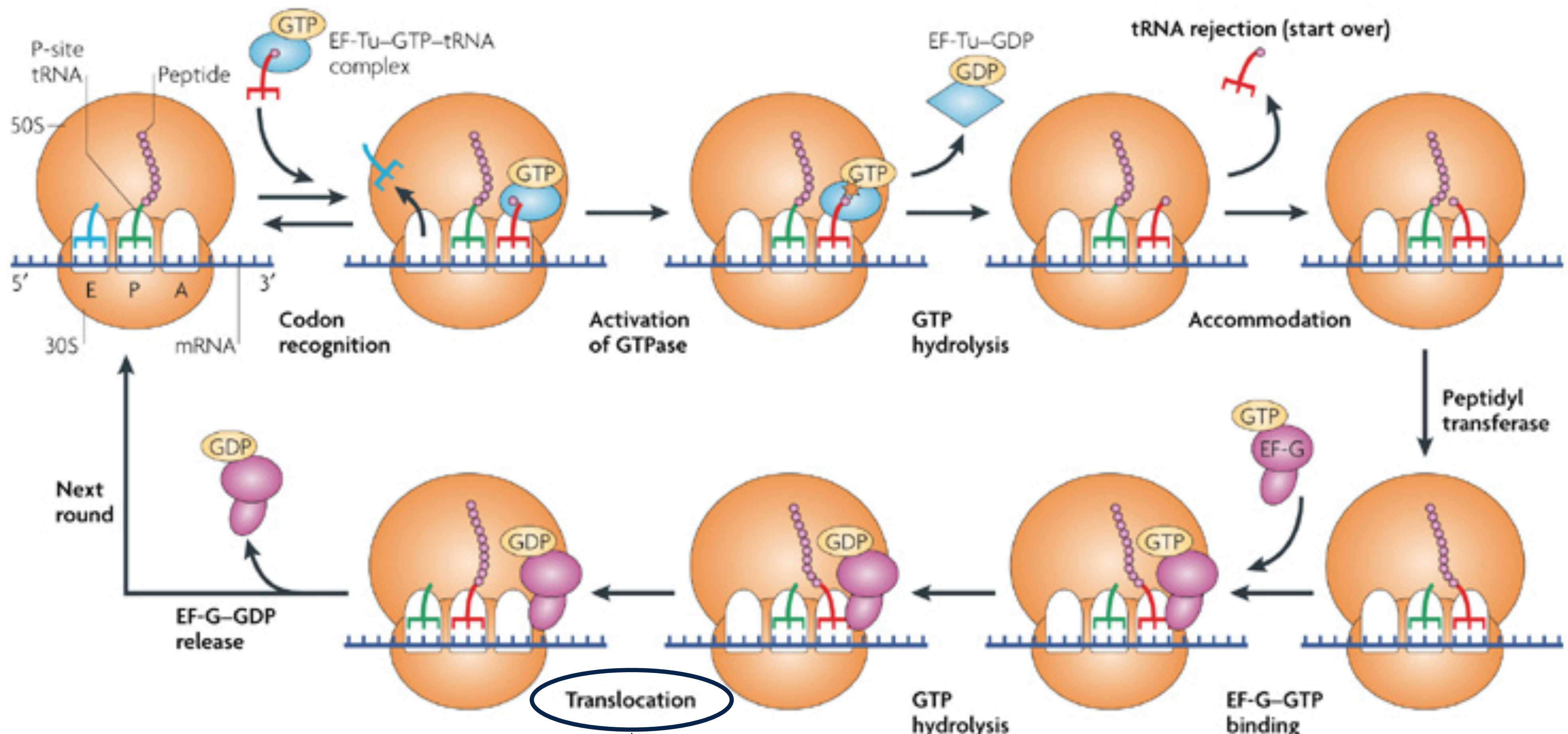
Robert Concannon and Richard Blythe

arXiv:1307.7511

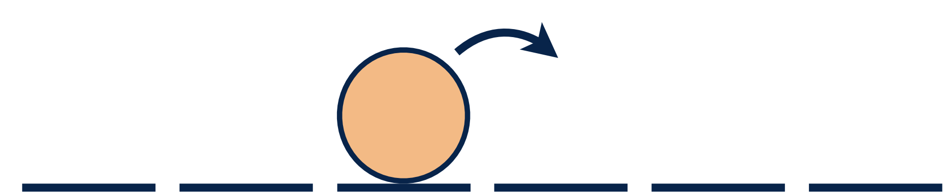
Models from Statistical Mechanics in Applied Sciences ~ 9th September 2013

mRNA translation

Steitz (2008) Nat. Rev. Mol. Cell Biol. **9** 242

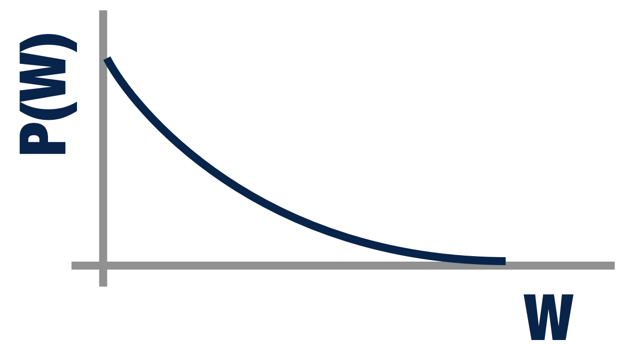


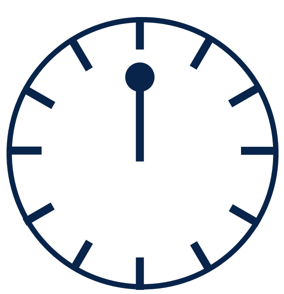
Translocation = moves



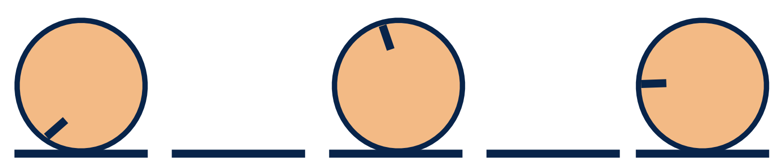
Asymmetric Simple Exclusion Process

Waiting time algorithm



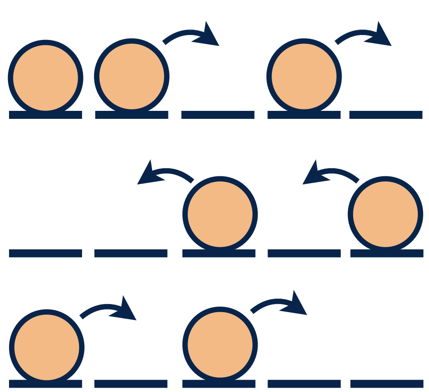


“dumb”

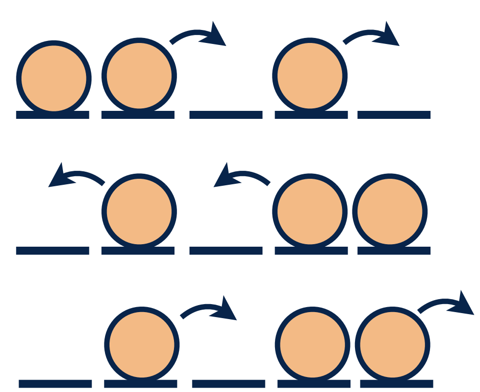


“smart”

“CP” symmetry



“PT” symmetry

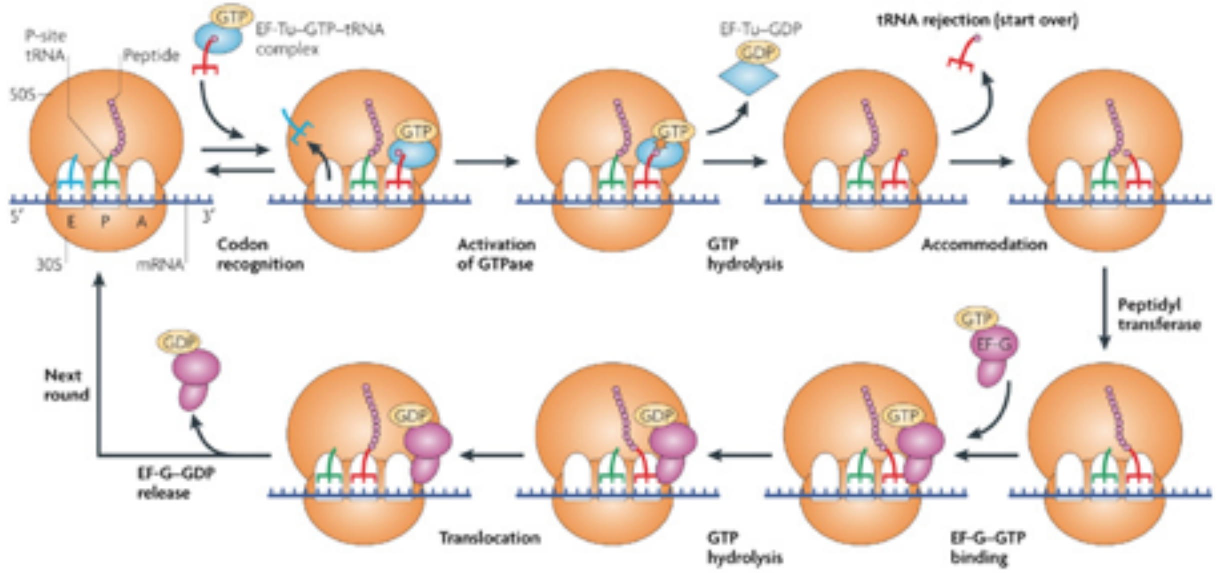


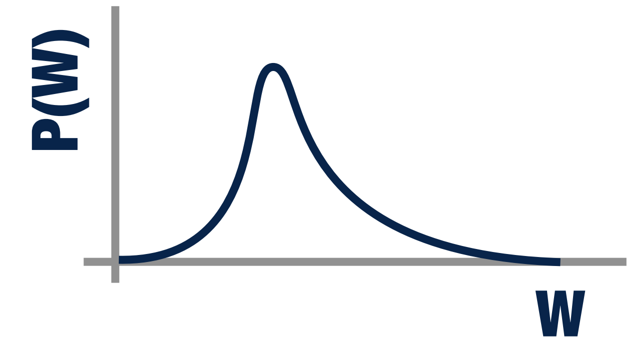
Periodic system

Steady state uniform

“Fluid”

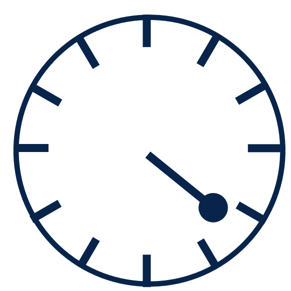
But

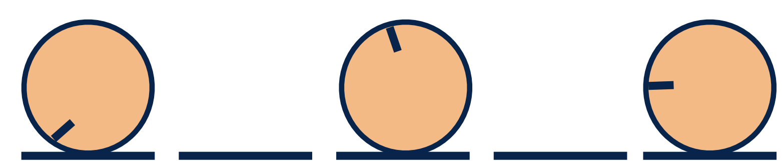




“Reset-on-fail” waiting-time algorithm

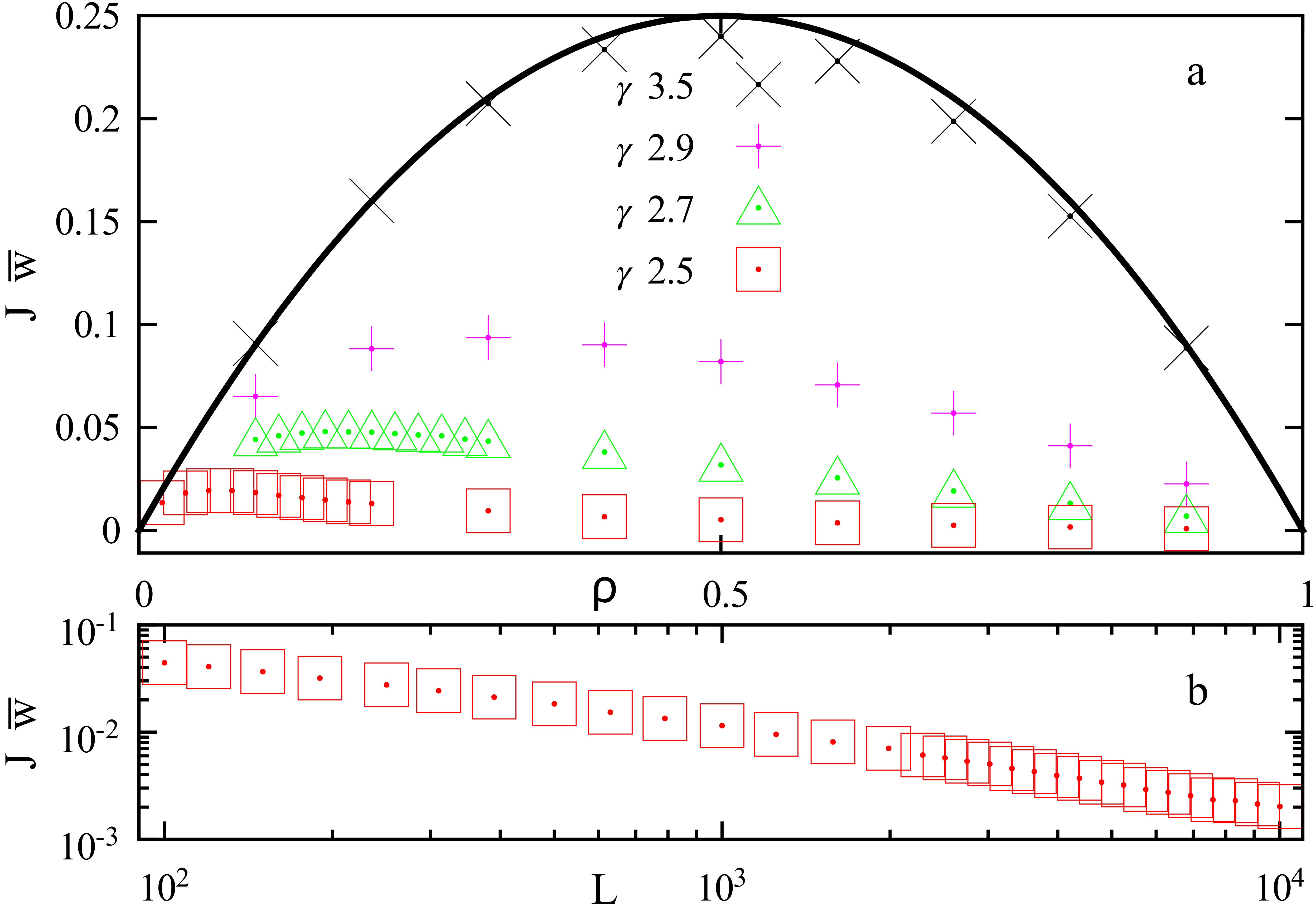
Gorrisen and Vanderzande (2012) J. Stat. Phys. **148** 627



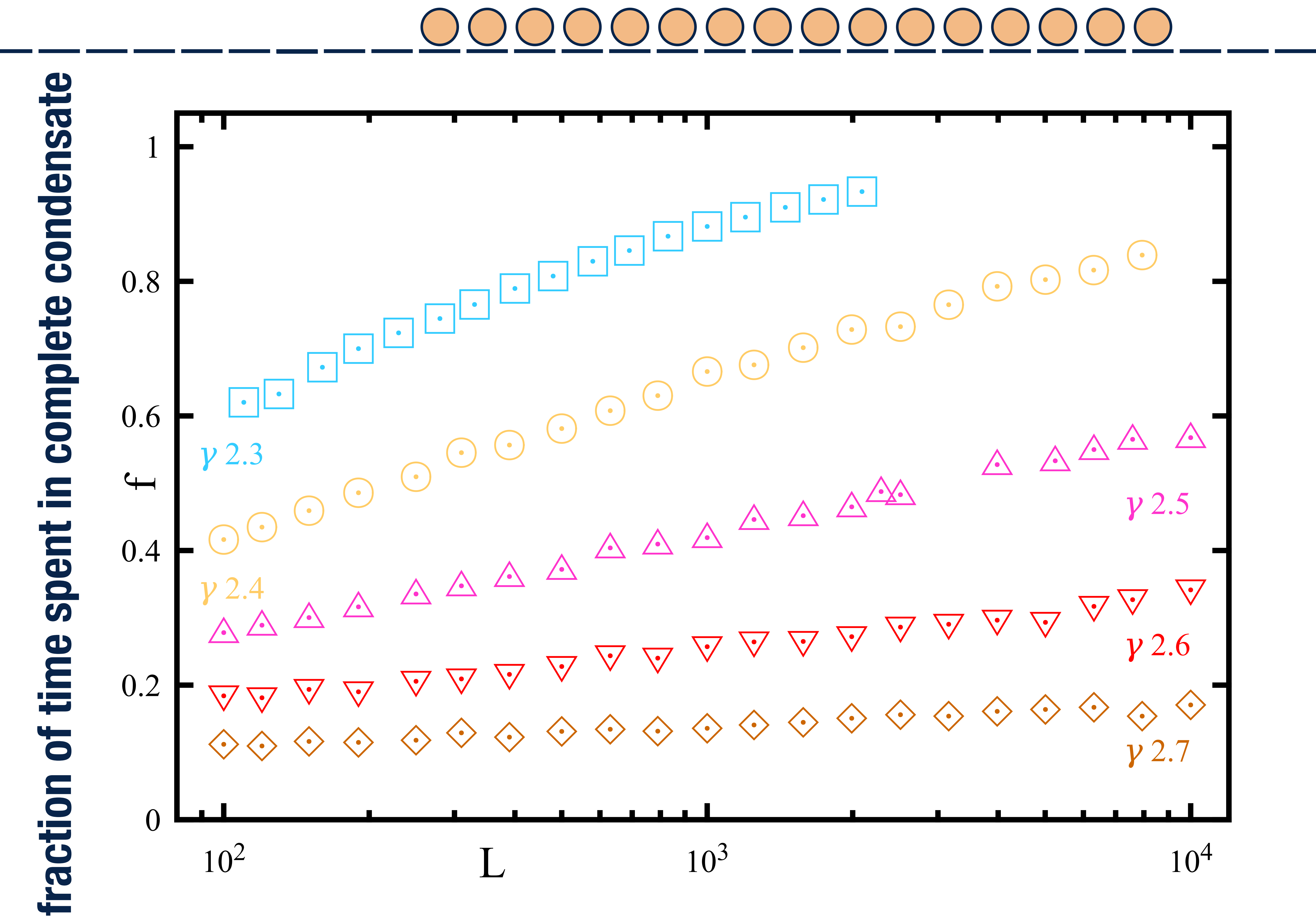


Breaks both “CP” and “PT” symmetries!

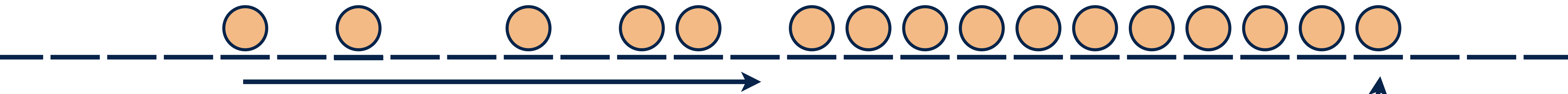
Hereafter: $P(W) = (\gamma - 1)W^{-\gamma}$ $W > 1, \gamma > 1$



0.25
0.2
0.15
0.1
0.05
0
J $\overline{w}$
a
γ 3.5
γ 2.9
γ 2.7
γ 2.5
0
ρ
0.5
1
10^{-1}
10^{-2}
10^{-3}
J $\overline{w}$
b
10^2
L
10^3
10^4


fraction of time spent in complete condensate
f
1
0.8
0.6
0.4
0.2
0
L
10^2
10^3
10^4
γ 2.3
γ 2.4
γ 2.5
γ 2.6
γ 2.7

Condensate forms by a particle receiving anomalously large W

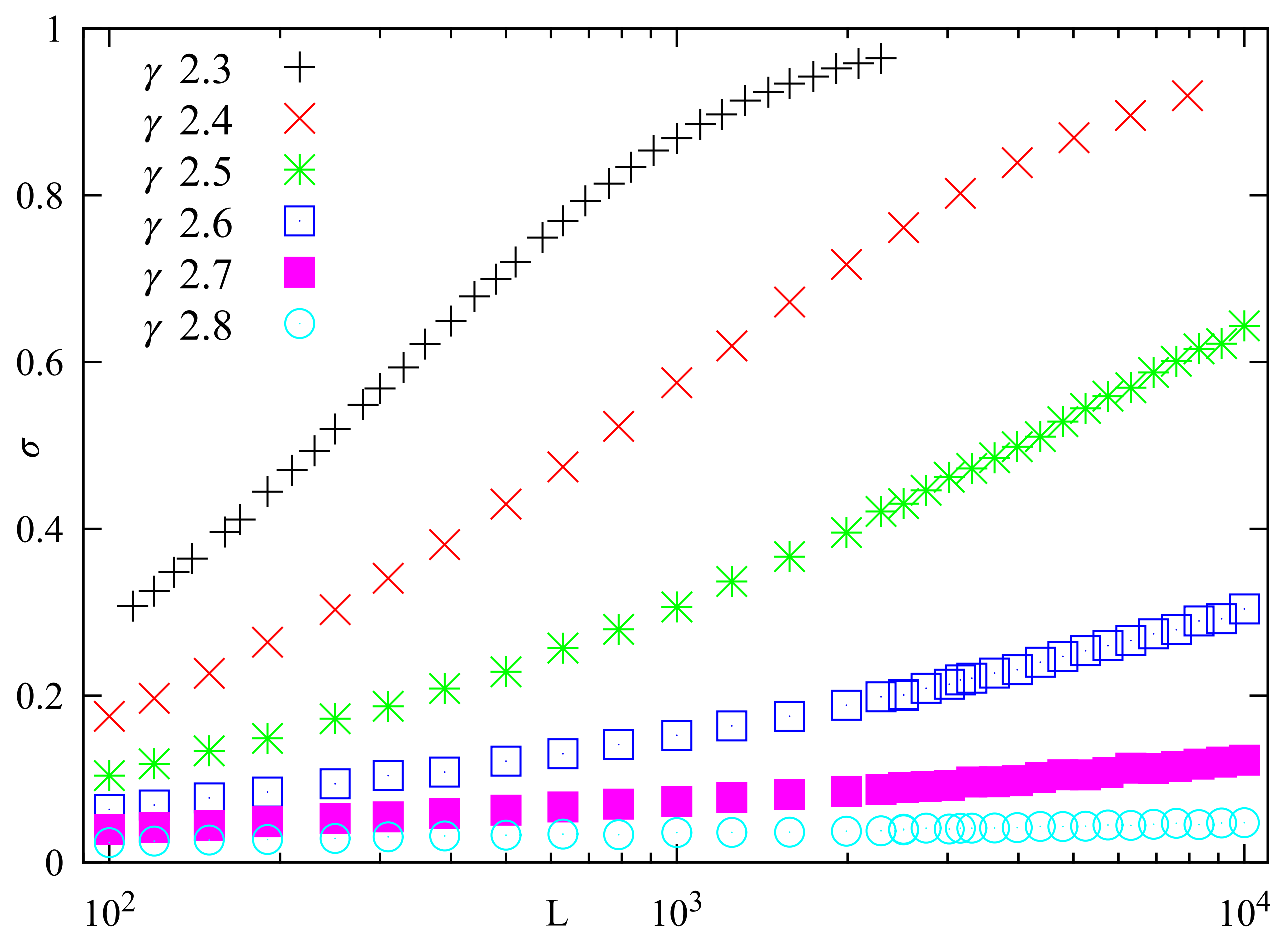


Travels distance **~L**, formation time **~L**

$P\left[W > O(L)\right] \sim L^{1-\gamma}$ nucleation time **~$L^{\gamma-2}$**

Fluid lifetime $\sim L^{\max\{1,\gamma-2\}}$

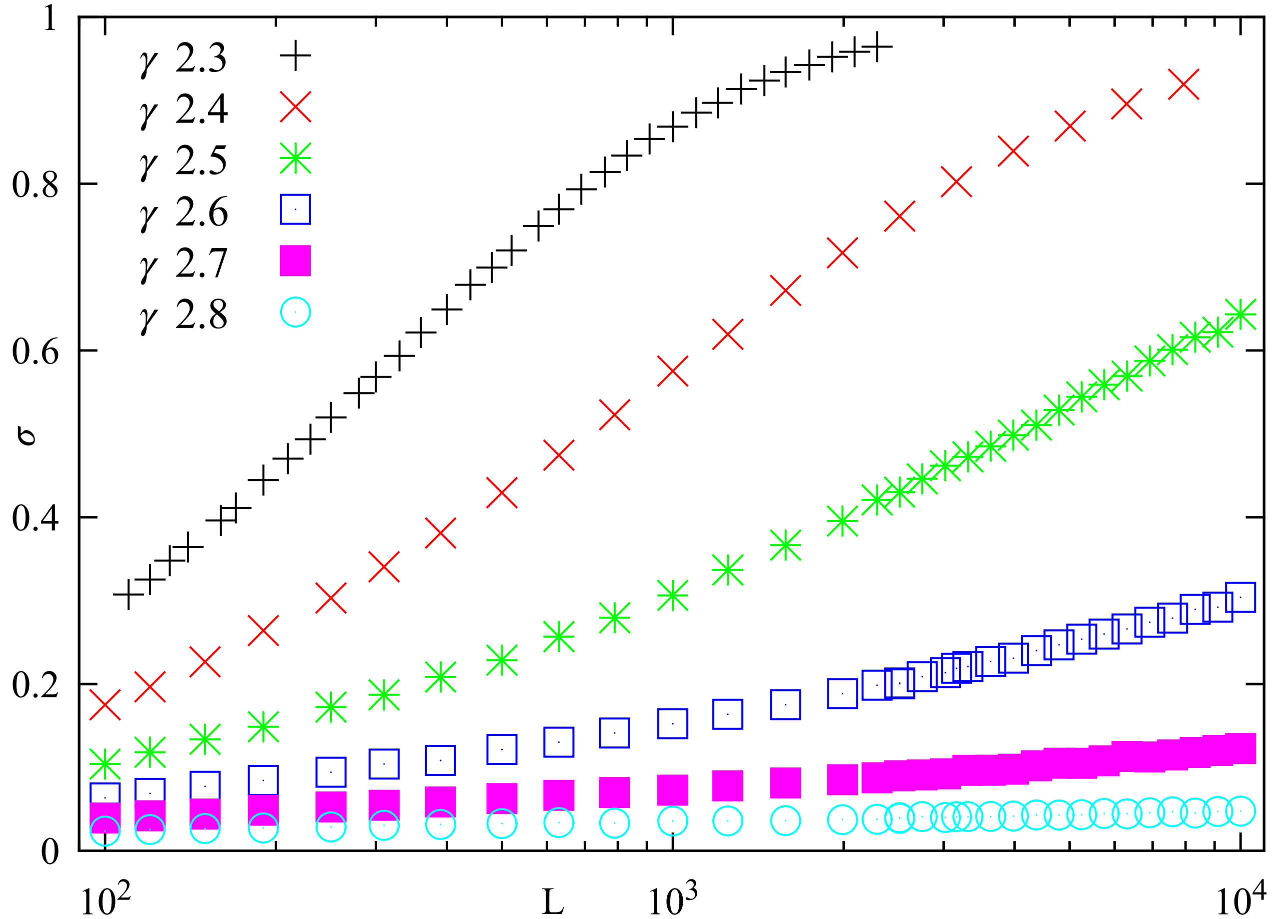
Solid lifetime $\bar{W}|_{W>O(L)} \sim L$ if **γ>2**

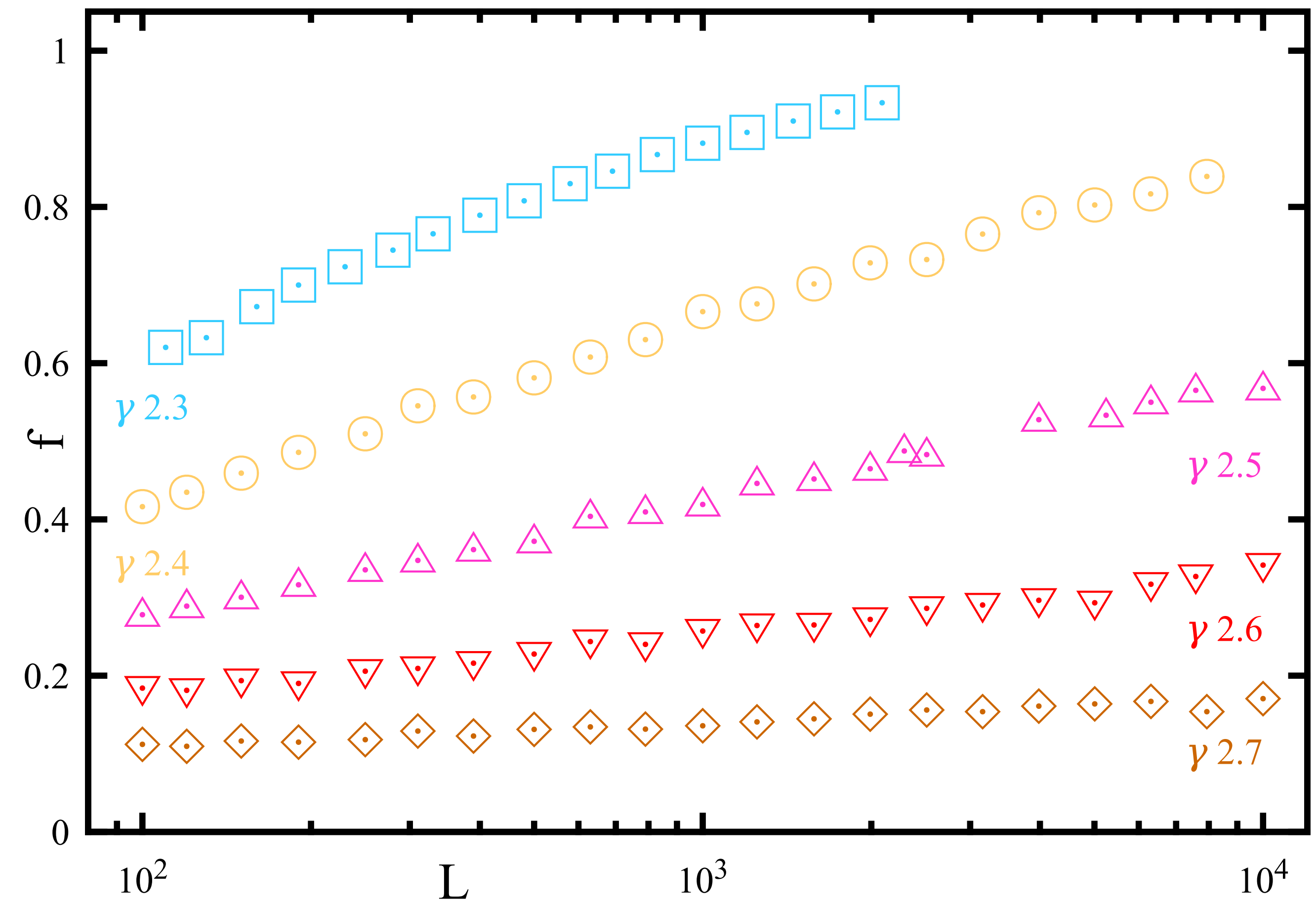


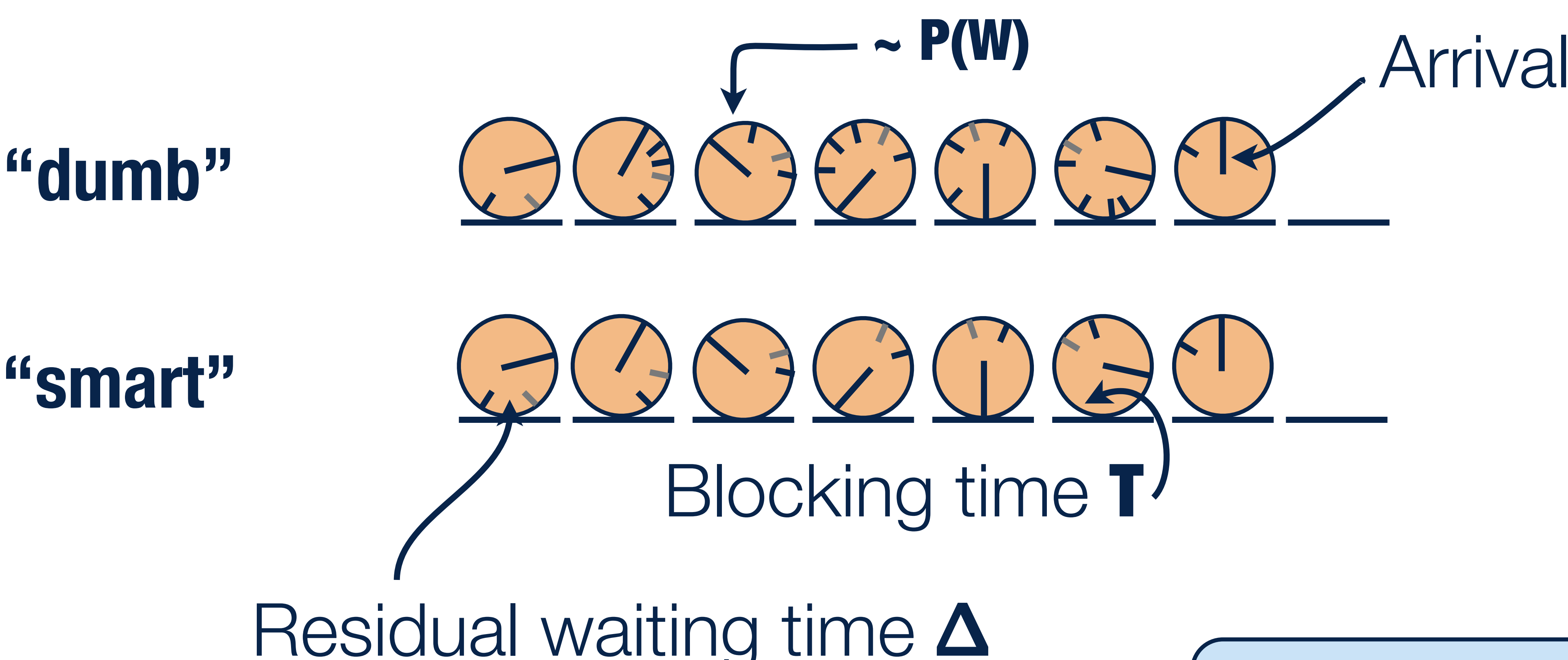
γ>3: no condensation

2<γ<3: condensation
but for how long?

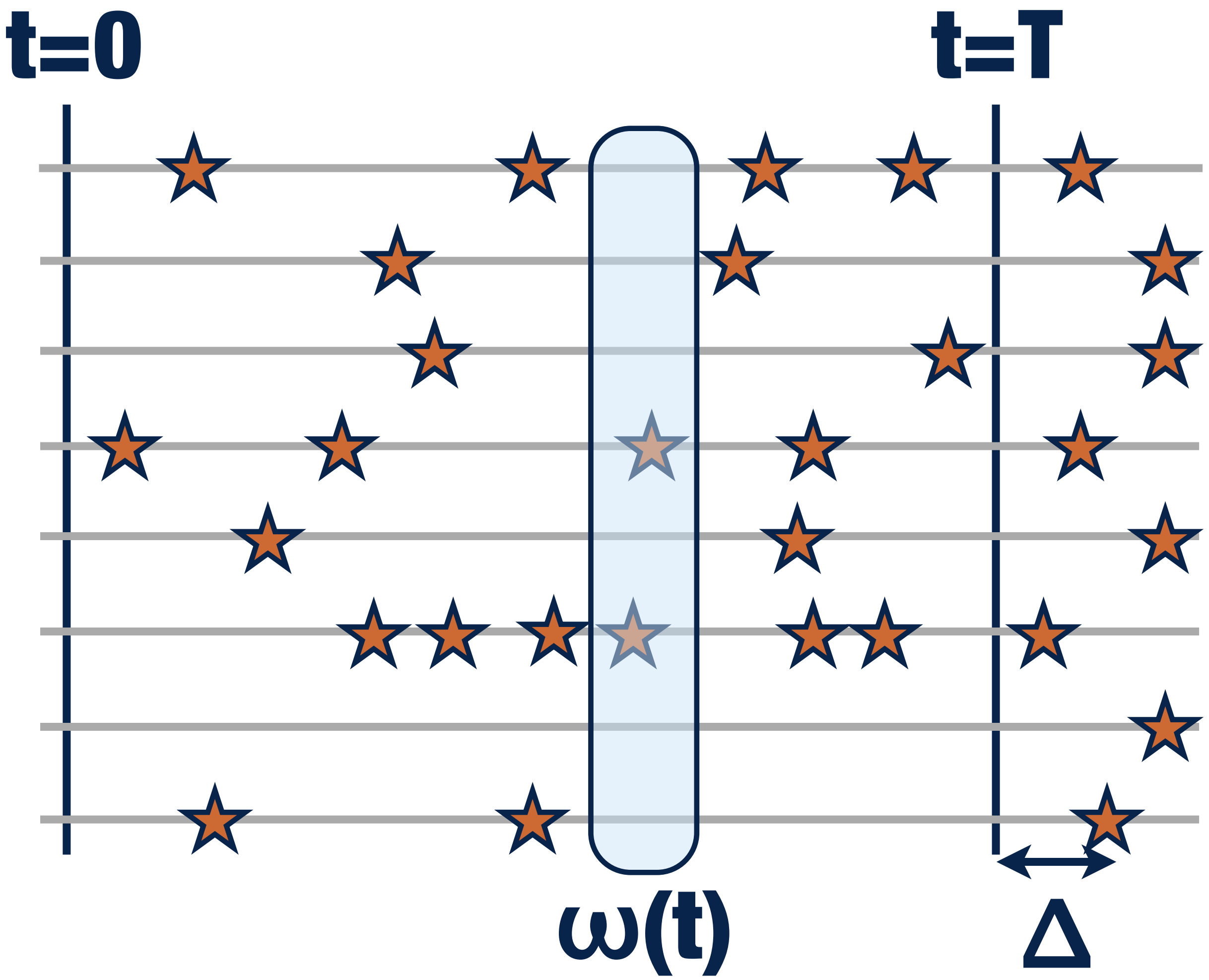
γ<2: “black hole” condensate


γ 2.3
γ 2.4
γ 2.5
γ 2.6
γ 2.7
γ 2.8
σ
L
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4


f
L
0
0.2
0.4
0.6
0.8
1
10^2
10^3
10^4
γ 2.3
γ 2.4
γ 2.5
γ 2.6
γ 2.7


~ P(W)
Arrival
“dumb”
“smart”
Blocking time **T**
Residual waiting time **Δ**

Seek P(Δ|T)



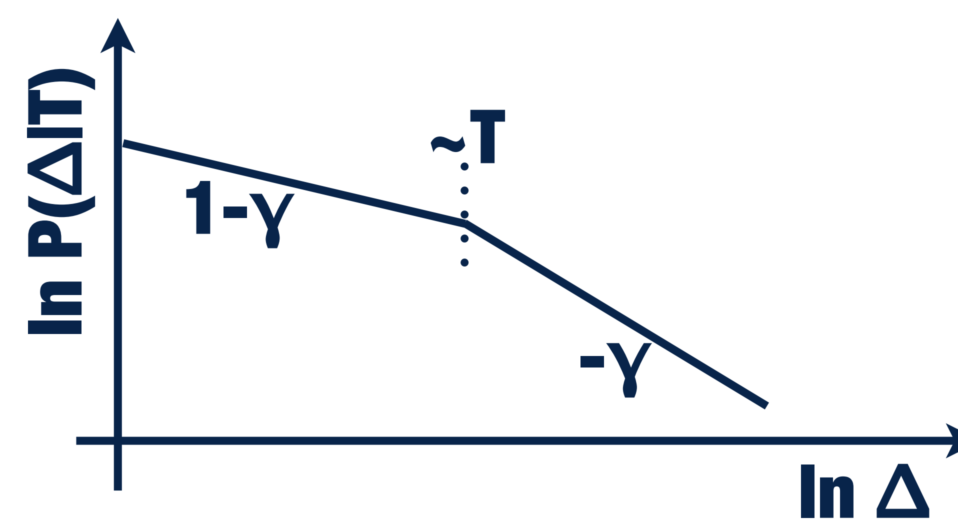
$$P(\Delta|T) = \int_0^T dt\, \omega(t) P(T + \Delta - t)$$

$$\omega(t) \overset{t\to\infty}{\longrightarrow} \frac{1}{\bar{W}}$$

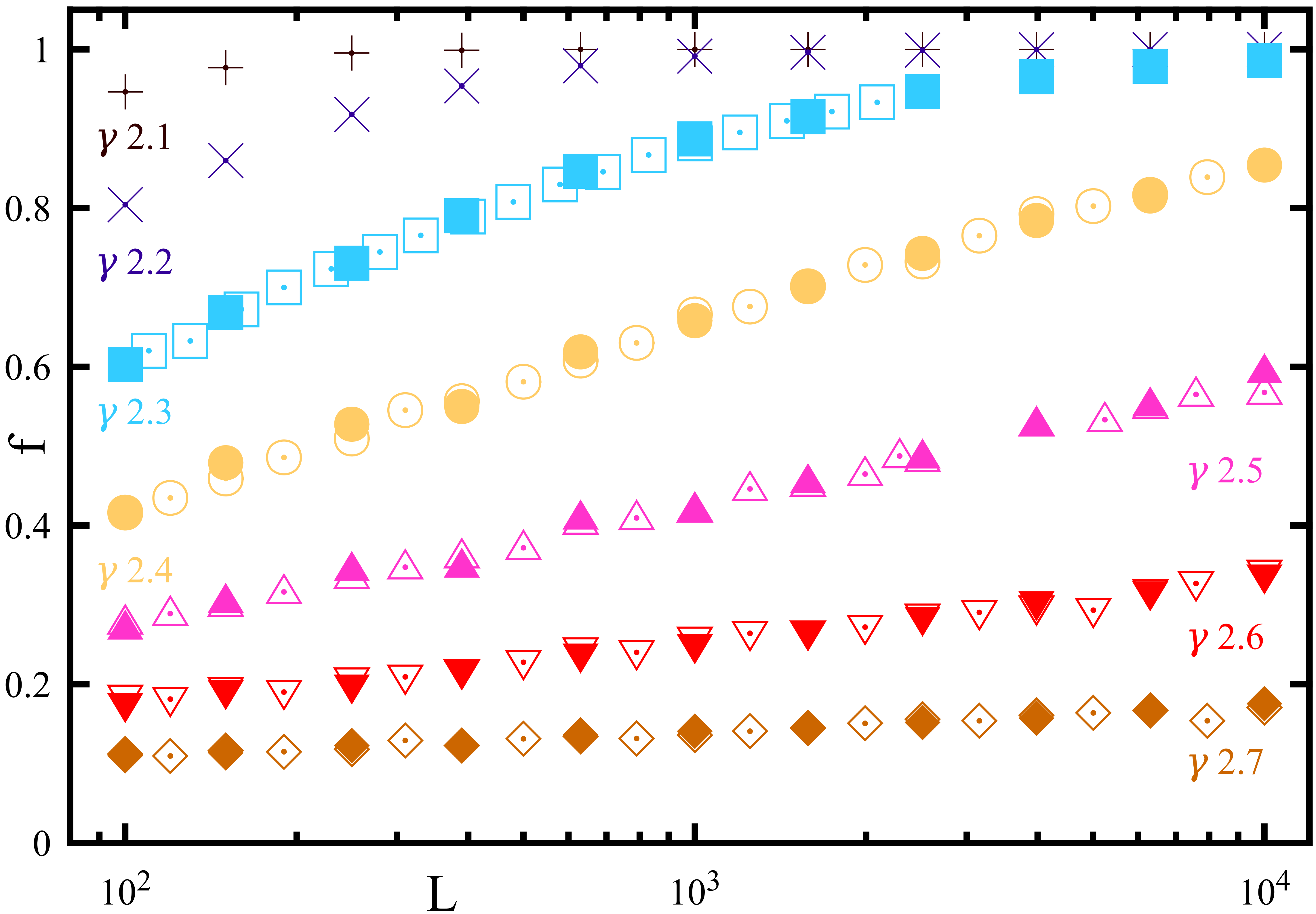
$$P(\Delta|T) \sim \frac{P(W > \Delta) - P(W > T + \Delta)}{Z}$$

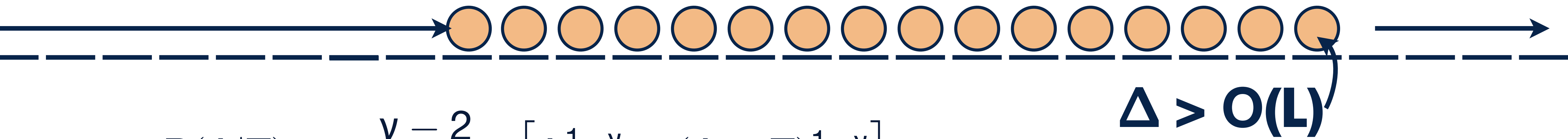
Power law $P(W) = (\gamma - 1) W^{-\gamma}$

$$P(\Delta|T) = \frac{\gamma - 2}{1 - T^{2-\gamma}} \left[\Delta^{1-\gamma} - (\Delta + T)^{1-\gamma} \right]$$



fraction of time spent in complete condensate revisited





$$P(\Delta|T) = \frac{\gamma - 2}{1 - T^{2-\gamma}} \left[\Delta^{1-\gamma} - (\Delta + T)^{1-\gamma}\right]$$

Dissolution probability

$$\prod_{i=2}^{N} P(\Delta_i < O(L)|T_i) = \prod_{i=2}^{N} \left[1 - \int_{O(L)}^{\infty} d\Delta_i \, P(\Delta_i|T_i)\right] \sim \exp\left[-\sum_{i=2}^{N} \int_{O(L)}^{\infty} d\Delta_i \, P(\Delta_i|T_i)\right]$$

Lower bound: all $\mathbf{T_i \to \infty}$

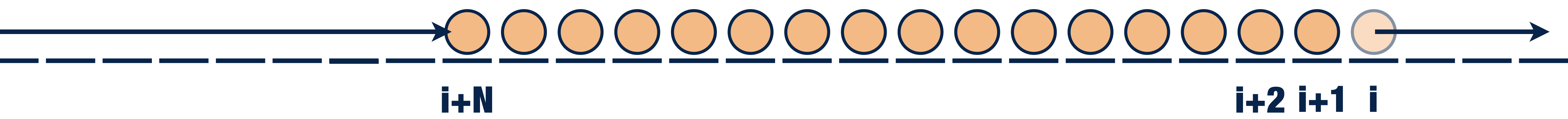
$$\exp\left[-aL^{3-\gamma}\right]$$

Upper bound: all $\mathbf{T_i \sim O(L)}$

$$\exp\left[-bL^{3-\gamma}\right]$$

For γ<3 the condensate does not dissolve once formed

(NB: argument holds for any finite particle density)



Blocking time **T_{i+N} = depart(i+N-1) - arrive(i+N)**

~~depart(i)~~ + catchup(i)

depart(i+N-2) + Δ_{i+N-1}

depart(i+N-3) + Δ_{i+N-2}

~~depart(i)~~ + Δ_{i+1}

$$T_{i+N} = \sum_{j=i+1}^{i+N-1} \Delta_i + C_i$$

Steady state

$$\overline{T} = (N-1)\overline{\Delta} + \overline{C}$$

$$\overline{\Delta}\big|_T \sim T^{3-\gamma}$$

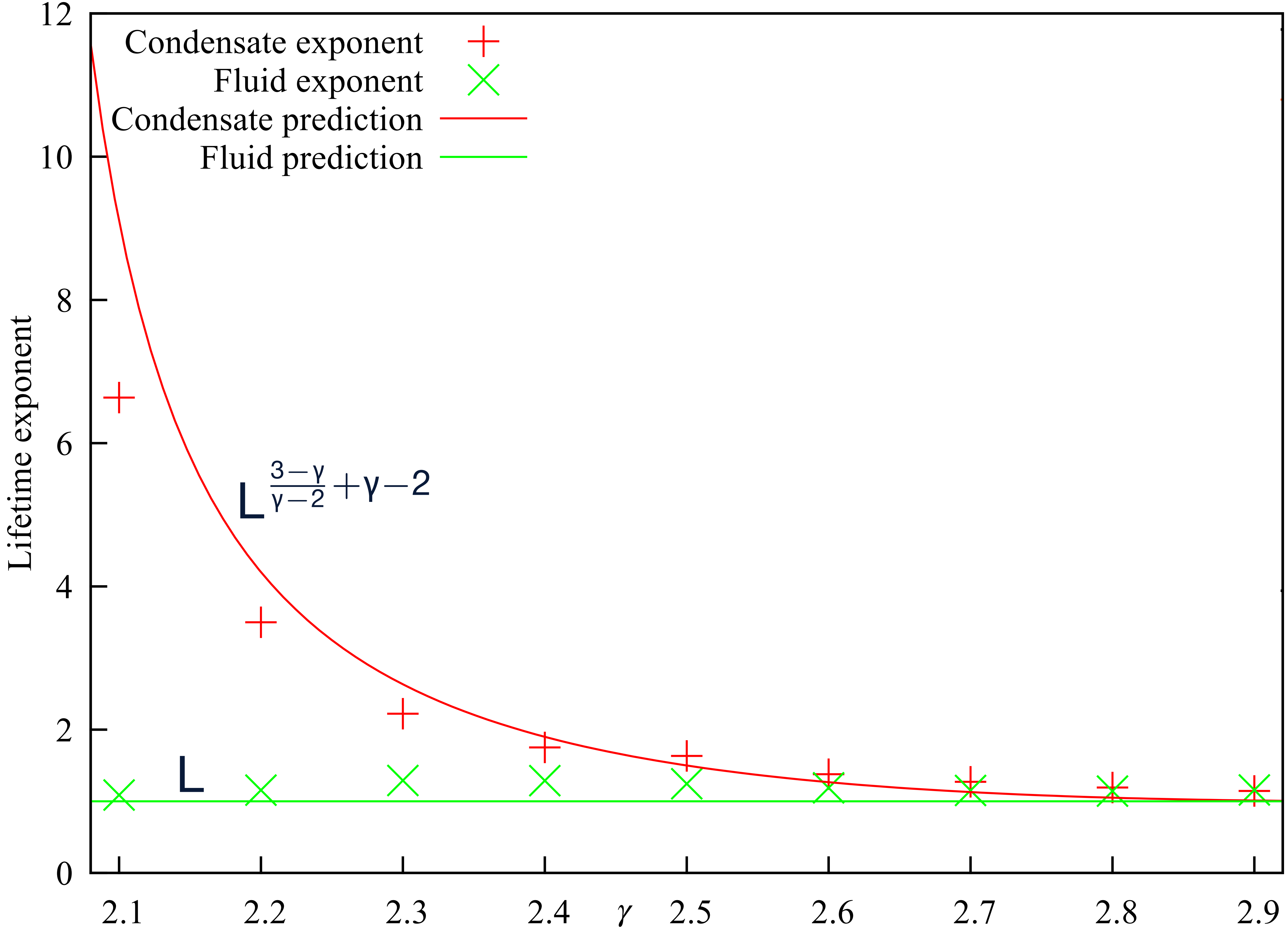
"Mean field"

$$\overline{\Delta} \sim \overline{T}^{3-\gamma}$$

$$\overline{T} \sim L^{\frac{1}{\gamma-2}}$$

Pack leader

$$\overline{\Delta}\big|_{\Delta > O(L)} \sim L^{\frac{3-\gamma}{\gamma-2}+\gamma-2}$$



Condensate exponent
Fluid exponent
Condensate prediction
Fluid prediction
$L^{\frac{3-\gamma}{\gamma-2}+\gamma-2}$
L
Lifetime exponent
γ
0
2
4
6
8
10
12
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9

Evidence for a condensate

complete in space and time

in a homogeneous exclusion process

with short-range (?) interactions

whose stability arises due to an “aging” of the blocking time

that exists at all densities

but relies on “reset-on-fail” dynamics and infinite-variance waiting time

which seems to be related to PH (and PT??) symmetries

Merely a theoretical curiosity?