Physical ageing in systems without detailed balance

Malte Henkel

Groupe de Physique Statistique, Institut Jean Lamour (CNRS UMR 7198)
Université de Lorraine **Nancy**, France

Symposion University of Warwick:
Models from statistical mechanics in applied sciences,
13^{th} of September 2013

MH, J.D. Noh and M. Pleimling, Phys. Rev. **E85**, 030102(R) (2012)

N. Allegra, J.-Y. Fortin and MH, `arxiv:1309.1634`

MH, Nucl. Phys. **B869**, 282 (2013); MH & S. Rouhani, J. Phys. A (2013) `arxiv:1302.7136`

Remerciements :

N. Allegra, F. Baumann, C. Chatelain, X. Durang, J.-Y. Fortin — U Nancy I (France)

M. Pleimling — Virginia Tech. (É.U.A.)

J.D. Noh, H. Park — KIAS Seoul (Corea)

F. Sastre — U Guanajuato (Mexico)

S. Rouhani — U Teheran (Iran)

M. Hase, T. Tomé, M.J. de Oliveira — U São Paulo (Brazil)

S. Stoimenov — Sofia (Bulgarie)

T. Enss, U. Schollwöck — TU & LMU Munich (Germany)

J. Ramasco, M.A. Santos, C. da Silva Santos — U Porto (Portugal)

Overview :

1. Ageing phenomena
2. Interface growth (KPZ universality class)
3. Form of the scaling functions
   & **L**ocal **S**cale-**I**nvariance (LSI)
4. Logarithmic conformal & ageing invariance (LLSI)
5. Numerical experiments
   (KPZ and DP classes in $1D$, majority voter in $2D$)
6. Outlook : growth on semi-infinite substrates
7. Conclusions

1. Ageing phenomena

known & practically used since prehistoric times (metals, glasses)
systematically studied in physics since the 1970s STRUIK '78

discovery : ageing effects **reproducible** & **universal** !
occur in widely different systems
(structural glasses, spin glasses, polymers, simple magnets, . . .)

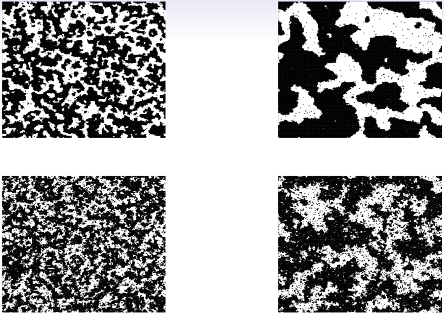
Three defining properties of **ageing** :

1. slow relaxation (non-exponential !)
2. **no** time-translation-invariance (TTI)
3. dynamical scaling without fine-tuning of parameters

Most existing studies on 'magnets' : relaxation towards equilibrium

Question : what can be learned about intrisically **irreversible** and/or **complex** systems by studying their ageing behaviour ?

$t = t_1$ $\qquad$ $t = t_2 > t_1$

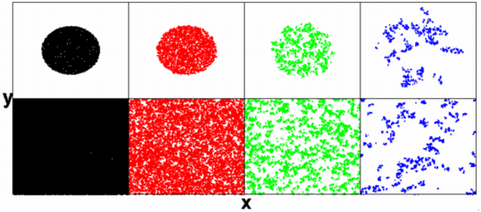


magnet $T < T_c$

$\longrightarrow$ ordered cluster

magnet $T = T_c$

$\longrightarrow$ correlated cluster



critical contact process

$\Longrightarrow$ **cluster dilution**

voter model, contact process,. . .

common feature : growing length scale

$$L(t) \sim t^{1/z}$$

z : dynamical exponent

Two-time observables : analogy with 'magnets'

time-dependent order-parameter $\phi(t, \mathbf{r})$

two-time **correlator** $\quad C(t,s) := \langle \phi(t,\mathbf{r})\phi(s,\mathbf{r})\rangle - \langle \phi(t,\mathbf{r})\rangle \langle \phi(s,\mathbf{r})\rangle$

two-time **response** $\quad R(t,s) := \left.\frac{\delta \langle \phi(t,\mathbf{r})\rangle}{\delta h(s,\mathbf{r})}\right|_{h=0} = \left\langle \phi(t,\mathbf{r})\widetilde{\phi}(s,\mathbf{r})\right\rangle$

t : observation time, s : waiting time

a) system **at** equilibrium : **fluctuation-dissipation theorem**

$$R(t-s) = \frac{1}{T}\frac{\partial C(t-s)}{\partial s} \quad , \quad T \text{ : temperature}$$

b) **far from equilibrium** : C and R **independent** !

The **fluctuation-dissipation ratio** (FDR) Cugliandolo, Kurchan, Parisi '94

$$X(t,s) := \frac{TR(t,s)}{\partial C(t,s)/\partial s}$$

measures the distance with respect to equilibrium : $X_{\rm eq} = X(t-s) = 1$

Scaling regime : $t, s \gg \tau_{\rm micro}$ **and** $t - s \gg \tau_{\rm micro}$

$$C(t,s) = s^{-b} f_C\left(\frac{t}{s}\right) \;\; , \;\; R(t,s) = s^{-1-a} f_R\left(\frac{t}{s}\right)$$

asymptotics : $f_{C,R}(y) \sim y^{-\lambda_{C,R}/z}$ for $y \gg 1$

λ_C : autocorrelation exponent, λ_R : autoresponse exponent,
z : dynamical exponent, a, b : ageing exponents

ex. : critical particle-reaction model (contact process),
initial particle density > 0 — Baumann & Gambassi 07

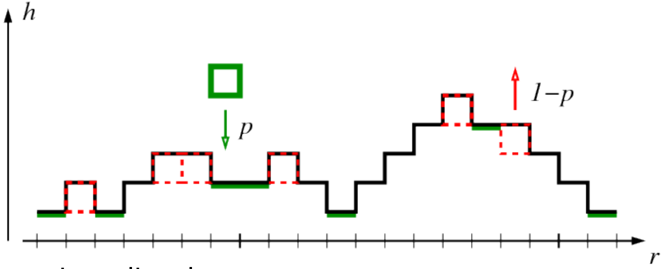
$$\lambda_C = \lambda_R = d + z + \frac{\beta}{\nu_\perp} \;\; , \;\; b = \frac{2\beta'}{\nu_\|}$$

$\longrightarrow$ stationary-state critical exponents $\beta, \beta', \nu_\perp, \nu_\| = z\nu_\perp$

2. Interface growth

deposition (evaporation) of particles on a substrate $\rightarrow$ height profile $h(t, \mathbf{r})$

generic situation : RSOS (**r**estricted **s**olid-**o**n-**s**olid) model — Kim & Kosterlitz 89



p = deposition prob.

$1 - p$ = evap. prob.

here $p = 0.98$

some universality classes :

(a) KPZ $\partial_t h = \nu \nabla^2 h + \frac{\mu}{2} (\nabla h)^2 + \eta$ — Kardar, Parisi, Zhang 86

(b) EW $\partial_t h = \nu \nabla^2 h + \eta$ — Edwards, Wilkinson 82

(c) MH $\partial_t h = -\nu \nabla^4 h + \eta$ — Mullins, Herring 63 ; Wolf, Villain 80

η is a gaussian white noise with $\langle \eta(t, \mathbf{r}) \eta(t', \mathbf{r}') \rangle = 2\nu T \delta(t - t') \delta(\mathbf{r} - \mathbf{r}')$

Family-Viscek scaling on a spatial lattice of extent L^d : $\overline{h}(t) = L^{-d} \sum_j h_j(t)$

Family & Viscek 85

$$w^2(t;L) = \frac{1}{L^d} \sum_{j=1}^{L^d} \left\langle \left(h_j(t) - \overline{h}(t) \right)^2 \right\rangle = L^{2\zeta} f\left(tL^{-z} \right) \sim \begin{cases} L^{2\zeta} & ; \text{if } tL^{-z} \gg 1 \\ t^{2\beta} & ; \text{if } tL^{-z} \ll 1 \end{cases}$$

β : growth exponent, ζ : roughness exponent, $\boxed{\zeta = \beta z}$

two-time correlator :

limit $L \to \infty$

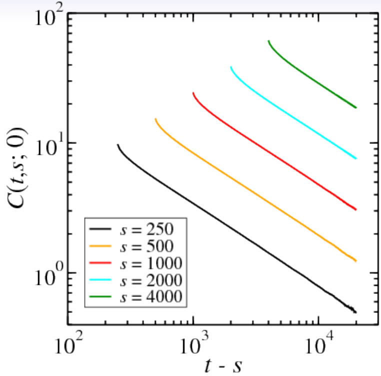
$$C(t,s;\mathbf{r}) = \left\langle \left(h(t,\mathbf{r}) - \langle \overline{h}(t) \rangle \right) \left(h(s,\mathbf{0}) - \langle \overline{h}(s) \rangle \right) \right\rangle = s^{-b} F_C\left(\frac{t}{s}, \frac{\mathbf{r}}{s^{1/z}} \right)$$

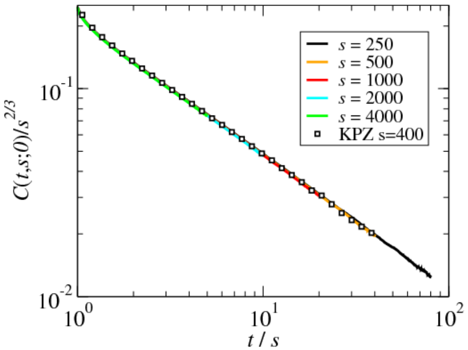
with ageing exponent : $\boxed{b = -2\beta}$

Kallabis & Krug 96

expect for $y = t/s \gg 1$: $F_C(y, \mathbf{0}) \sim y^{-\lambda_C/z}$ autocorrelation exponent

$1D$ relaxation dynamics, starting from an initially flat interface





observe all **3** properties of **ageing** : { slow dynamics; no TTI; dynamical scaling }

confirm **simple ageing** for the $1D$ KPZ universality class

pars pro toto

KALLABIS & KRUG 96 ; KRECH 97 ; BUSTINGORRY *et al.* 07-10 ; CHOU & PLEIMLING 10 ; D'AQUILA & TÄUBER 11/12 ; H.N.P. 12

extend Family-Viscek scaling to two-time responses :

analogue : TRM integrated response in magnetic systems

two-time integrated response :

* sample **A** with deposition rates $p_i = p \pm \epsilon_i$, up to time s,
* sample **B** with $p_i = p$ up to time s ;

then switch to common dynamics $p_i = p$ for all times $t > s$

$$\chi(t, s; \mathbf{r}) = \int_0^s \mathrm{d}u \, R(t, u; \mathbf{r}) = \frac{1}{L} \sum_{j=1}^{L} \left\langle \frac{h_{j+r}^{(\mathbf{A})}(t; s) - h_{j+r}^{(\mathbf{B})}(t)}{\epsilon_j} \right\rangle = s^{-a} F_\chi \left(\frac{t}{s}, \frac{|\mathbf{r}|^z}{s} \right)$$

with a : ageing exponent

expect for $y = t/s \gg 1$: $F_R(y, \mathbf{0}) \sim y^{-\lambda_R/z}$ autoresponse exponent

? Values of these exponents **?**

Effective action of the KPZ equation :

$$\mathcal{J}[\phi,\widetilde{\phi}] = \int \mathrm{d}t\mathrm{d}\mathbf{r}\ \left[\widetilde{\phi}\left(\partial_t\phi - \nu\nabla^2\phi - \frac{\mu}{2}(\nabla\phi)^2\right) - \nu T\widetilde{\phi}^{\,2}\right]$$

$\Longrightarrow$ Very special properties of KPZ in $d = 1$ spatial dimension !

Exact critical exponents $\beta = 1/3$, $\zeta = 1/2$, $z = 3/2$, $\lambda_C = 1$ KPZ 86 ; Krech 97

related to precise symmetry properties :

A) tilt-invariance (Galilei-invariance) Forster, Nelson, Stephen 77

kept under renormalisation ! Medina, Hwa, Kardar, Zhang 89

$\Rightarrow$ exponent relation $\zeta + z = 2$ (holds for any dimension d)

B) time-reversal invariance Lvov, Lebedev, Paton, Procaccia 93
Frey, Täuber, Hwa 96

special property in $1D$, where also $\zeta = \frac{1}{2}$

Special KPZ symmetry in $1D$: let $v = \frac{\partial \phi}{\partial r}$, $\widetilde{\phi} = \frac{\partial}{\partial r}\left(\widetilde{p} + \frac{v}{2T}\right)$

$$\mathcal{J} = \int \mathrm{d}t\mathrm{d}r \left[\widetilde{p}\partial_t v - \frac{\nu}{4T}\left(\partial_r v\right)^2 - \frac{\mu}{2}v^2\partial_r\widetilde{p} + \nu T\left(\partial_r\widetilde{p}\right)^2\right]$$

is invariant under **time-reversal**

$$t \mapsto -t \ \ , \ \ v(t,r) \mapsto -v(-t,r) \ \ , \ \ \widetilde{p} \mapsto +\widetilde{p}(-t,r)$$

$\Rightarrow$ **fluctuation-dissipation relation** for $t \gg s$

$$\boxed{TR(t,s;r) = -\partial_r^2 C(t,s;r)}$$

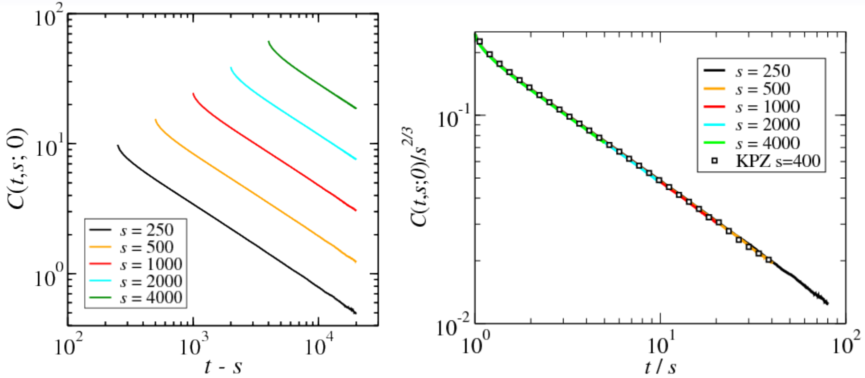
distinct from the equilibrium FDT $TR(t-s) = \partial_s C(t-s)$

Combination with ageing scaling, gives the ageing exponents :

$$\boxed{\lambda_R = \lambda_C = 1} \qquad \text{and} \qquad \boxed{1 + a = b + \frac{2}{z}}$$

Kallabis, Krug 96 mh, Noh, Pleimling '12

$1D$ relaxation dynamics, starting from an initially flat interface

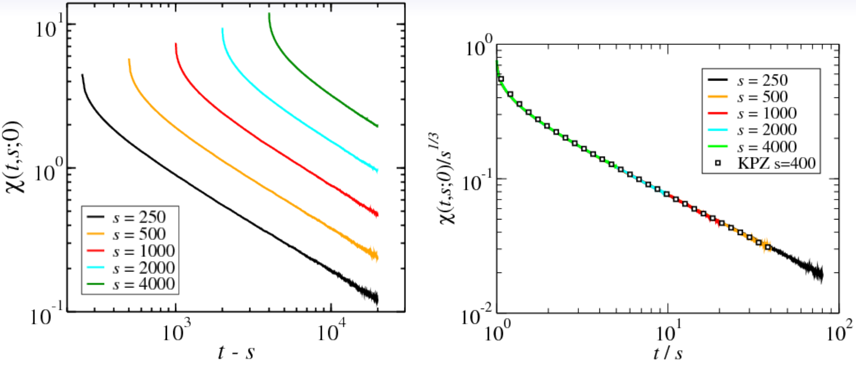


confirm simple ageing in the autocorrelator
confirm expected exponents $b = -2/3$, $\lambda_C/z = 2/3$

N.B. : this confirmation is out of the stationary state

Kallabis & Krug 96 ; Krech 97 ; Bustingorry *et al.* 07-10 ; Chou & Pleimling 10 ; D'Aquila & Täuber 11/12 ; h.n.p. 12

relaxation of the integrated response, 1D

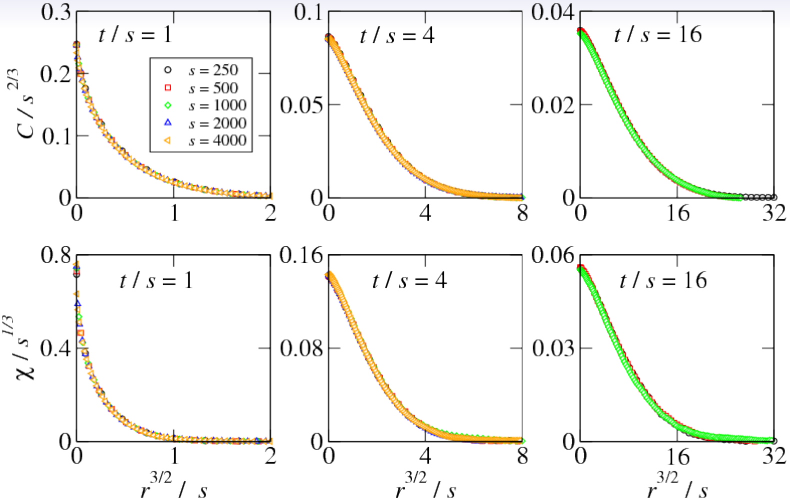


observe all **3** properties of **ageing** : $\left\{ \begin{array}{l} \text{slow dynamics} \\ \text{no TTI} \\ \text{dynamical scaling} \end{array} \right.$

exponents $a = -1/3$, $\lambda_R/z = 2/3$, as expected from FDR

N.B. : numerical tests for 2 models in KPZ class

Simple ageing is also seen in space-time observables



correlator $C(t,s;r) = s^{2/3} F_C\left(\frac{t}{s}, \frac{r^{3/2}}{s}\right)$

integrated response $\chi(t,s;r) = s^{1/3} F_\chi\left(\frac{t}{s}, \frac{r^{3/2}}{s}\right)$

} confirm $z = 3/2$

Values of some growth and ageing exponents in $1D$

model	z	a	b	$\lambda_R = \lambda_C$	β	ζ
KPZ	$3/2$	$-1/3$	$-2/3$	1	$1/3$	$1/2$
exp 1			$\approx -2/3^{\dagger}$	$\approx 1^{\dagger}$	0.336(11)	0.50(5)
exp 2	1.5(2)				0.32(4)	0.50(5)
EW	2	$-1/2$	$-1/2$	1	$1/4$	$1/2$
MH	4	$-3/4$	$-3/4$	1	$3/8$	$3/2$

liquid crystals — **Takeuchi, Sano, Sasamoto, Spohn 10/11/12**

cancer cells — **Huergo, Pasquale, Gonzalez, Bolzan, Arvia 12**

† scaling holds only for flat interface

Two-time space-time responses and correlators consistent with simple ageing for $1D$ KPZ

Similar results known for EW and MH universality classes

Roethlein, Baumann, Pleimling 06

3. Form of the scaling functions & LSI

Question : ? Are there model-independent results on the form of universal scaling functions ?

'Natural' starting point : try to draw analogies with conformal invariance at equilibrium

* Equilibrium critical phenomena : **scale-invariance**
* For sufficiently **local** interactions : extend to conformal invariance

space-dependent re-scaling (angles conserved) $\mathbf{r} \mapsto \mathbf{r}/b(\mathbf{r})$

BATEMAN & CUNNINGHAM 1909/10, POLYAKOV 70

In **two** dimensions : ∞ many conformal transformations ($w \mapsto \beta(w)$ complex analytic)

$\Rightarrow$ exact predictions for critical exponents, correlators, ... BPZ 84

Hidden assumptions :

1) extension scale-invariance $\longrightarrow$ conformal invariance **?**

formally : energy-momentum tensor symmetric & traceless Callan, Coleman, Jackiw '70

but counterexamples :

lattice animals	Miller & De Bell 93
hydrodynamics	Riva & Cardy 05
renormalised FT	Fortin, Grinstein, Stergiou 12

2) choice of so-called 'primary' scaling operators

not all physical models are unitary minimal CFTs $\longrightarrow$ SLE

3) how do primary operators transform ?

usual form

$$\phi'(w) = \beta'(w)^{\Delta}\phi(\beta(w))$$

alternative : logarithmic partner ψ Gurarie 93, Khorrami *et al.* 97,...

$$\psi'(w) = \beta'(w)^{\Delta}\left[\psi(\beta(w)) + \ln\beta'(w)\cdot\phi(\beta(w))\right]$$

Logarithmic conformal invariance has been found in, e.g.

- critical $2D$ percolation Cardy 92, Watts 96, Mathieu & Ridout 07/08
- disordered systems Caux *et al.* 96
- sand-pile models Ruelle *et al.* 08-10

What about **time**-dependent critical phenomena **?** — Cardy 85

Characterised by **dynamical exponent** z : $t \mapsto tb^{-z}$, $\mathbf{r} \mapsto \mathbf{r}b^{-1}$

Can one extend to **local** dynamical scaling, with $z \neq 1$?

If $z = 2$, the **Schrödinger group** is an example : — Jacobi 1842, Lie 1881

$$t \mapsto \frac{\alpha t + \beta}{\gamma t + \delta} \;\; , \;\; \mathbf{r} \mapsto \frac{\mathcal{D}\mathbf{r} + \mathbf{v}t + \mathbf{a}}{\gamma t + \delta} \;\; ; \;\; \alpha\delta - \beta\gamma = 1$$

$\Rightarrow$ study **ageing** phenomena as paradigmatic example

essential : (i) **absence** of TTI & (ii) **Galilei**-invariance

Transformation $t \mapsto t'$ with $\beta(0) = 0$ and $\dot{\beta}(t') \geq 0$ and — MH *et. al.* 06

$$t = \beta(t') \;\; , \;\; \phi(t) = \left(\frac{\mathrm{d}\beta(t')}{\mathrm{d}t'} \right)^{-x/z} \left(\frac{\mathrm{d} \ln \beta(t')}{\mathrm{d}t'} \right)^{-2\xi/z} \phi'(t')$$

out of equilibrium, have **2 distinct** scaling dimensions, x and ξ.

mean-field for **magnets** : expect
$$\begin{cases} \xi = 0 \text{ in ordered phase } T < T_c \\ \xi \neq 0 \text{ at criticality } T = T_c \end{cases}$$

NB : if TTI (equilibrium criticality), then $\xi = 0$.

physical requirement :

co-variance of **response functions** under local scaling !

why : certain extended scaling symmetries **predict causality** for co-variant n-point functions ! MH & UNTERBERGER 03, MH 12

$\Rightarrow$ set of linear differential equations for $R(t,s)$ **most simple case** !

$$
\begin{aligned}
R(t,s) &= \left\langle \phi(t)\widetilde{\phi}(s) \right\rangle = s^{-1-a} f_R\left(\frac{t}{s}\right) \\
f_R(y) &= f_0 y^{1+a'-\lambda_R/z} (y-1)^{-1-a'} \underbrace{\Theta(y-1)}_{\text{causality}}
\end{aligned}
$$

$$
a = \frac{1}{z}(x+\widetilde{x}) - 1 \;,\; a'-a = \frac{2}{z}\left(\xi + \widetilde{\xi}\right) \;,\; \frac{\lambda_R}{z} = x + \xi
$$

magnetic example : $1D$ Glauber-Ising model at $T = T_c = 0$:

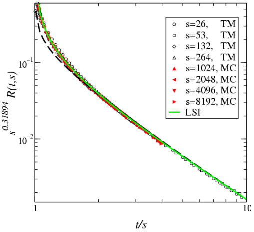
$$
a = 0 \;,\; a'-a = -\frac{1}{2} \;,\; \lambda_R = 1 \;,\; z = 2
$$

PICONE, MH 04
MH, ENSS, PLEIMLING 06

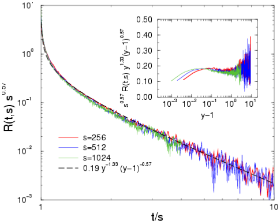
Particle models : comparison of $R(t,s)$ with LSI-prediction :

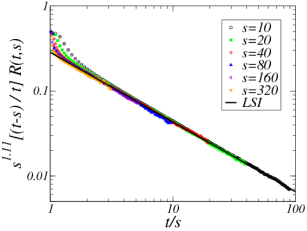
contact process (CP) nonequil. kinetic Ising (PC) voter Potts-3 (VP3)

$A \to 2A,\ A \to \emptyset$



$A \leftrightarrow 3A,\ 2A \to \emptyset$





CP : $a' - a \simeq 0.27$ PC : $a' - a \simeq 0.00(1)$ VP3 : $a' - a \simeq -0.1$

MH, ENSS, PLEIMLING 06
ENSS 06 ; HINRICHSEN 06

ÓDOR 06

CHATELAIN, TOMÉ, DE OLIVEIRA 11

? is this good general agreement already conclusive ?

Observation : the **hidden assumption** $a = a'$, uncritically taken over from equilibrium, is often **invalid** out of equilibrium.
Observables **cannot** always be identified with scaling operators.

4. Logarithmic conformal & ageing invariance

generalise conformal invariance → doubletts $\Psi = \begin{pmatrix} \psi \\ \phi \end{pmatrix}$ ROZANSKY & SALEUR 92, GURARIE 93

generators : $\ell_n = -w^{n+1}\partial_w - (n+1)w^n \begin{pmatrix} \Delta & 1 \\ 0 & \Delta \end{pmatrix}$

two-point functions : have $\Delta_1 = \Delta_2$ GURARIE 93, RAHIMI TABAR *et al.* 97...

$$\begin{aligned} F &= \langle \phi_1(w_1)\phi_2(w_2)\rangle = 0 \\ G &= \langle \phi_1(w_1)\psi_2(w_2)\rangle = G_0|w|^{-2\Delta_1} \\ H &= \langle \psi_1(w_1)\psi_2(w_2)\rangle = (H_0 - 2G_0 \ln|w|)\,|w|^{-2\Delta_1} \\ &= w_2^{-2\Delta_1}(H_0 - 2G_0 \ln|y-1| - 2G_0 \ln|w_2|)\,|y-1|^{-2\Delta_1} \end{aligned}$$

with $w = w_1 - w_2$ and $y = w_1/w_2$.

Simultaneous log corrections to scaling **and** modified scaling function

Logarithmic conformal invariance has been found in, e.g.

- critical $2D$ percolation CARDY 92, WATTS 96, MATHIEU & RIDOUT 07/08
- disordered systems CAUX *et al.* 96
- sand-pile models RUELLE *et al.* 08-10

construct **logarithmic ageing-invariance** by the formal changes (generic case ; $x' = 0$ or $x' = 1$) :

$$x \mapsto \hat{x} = \begin{pmatrix} x & x' \\ 0 & x \end{pmatrix} \;,\; \xi \mapsto \hat{\xi} = \begin{pmatrix} \xi & \xi' \\ \mathbf{0} & \xi \end{pmatrix}$$

(**must** <u>show</u> : both dimension matrices $\hat{x}, \hat{\xi}$ are **simultaneously** Jordan !)

we find the co-variant two-point functions (with $y = t/s$) :

$$\begin{aligned}
\left\langle \phi(t)\widetilde{\phi}(s) \right\rangle &= s^{-(x+\widetilde{x})/2} f(y) \\
\left\langle \phi(t)\widetilde{\psi}(s) \right\rangle &= s^{-(x+\widetilde{x})/2} \left(g_{12}(y) + \ln s \cdot \gamma_{12}(y)\right) \\
\left\langle \psi(t)\widetilde{\phi}(s) \right\rangle &= s^{-(x+\widetilde{x})/2} \left(g_{21}(y) + \ln s \cdot \gamma_{21}(y)\right) \\
\left\langle \psi(t)\widetilde{\psi}(s) \right\rangle &= s^{-(x+\widetilde{x})/2} \left(h_0(y) + \ln s \cdot h_1(y) + \ln^2 s \cdot h_2(y)\right)
\end{aligned}$$

all scaling functions explicitly known

<u>**Question :**</u> **interesting models described by logarithmic LSI ?**

5. Numerical experiments

(A) Kardar-Parisi-Zhang (**KPZ**)

(B) directed percolation (**DP**)

(C) majority voter/Glauber models (**MV**) at $T = T_c$, triangular lattice

simple ageing of the correlators and responses, especially

$$C(t,s) = s^{-b} f_C\left(\frac{t}{s}\right) \quad , \quad R(t,s) = s^{-1-a} f_R\left(\frac{t}{s}\right)$$

$$f_C(y) \sim y^{-\lambda_C/z} \quad , \quad f_R(y) \sim y^{-\lambda_R/z} \qquad y \gg 1$$

values of the non-equilibrium exponents & scaling relations

KPZ in $1D$: $\lambda_C = \lambda_R = 1$, $1 + a = b + \frac{2}{z}$, $b = -2\beta = -\frac{2}{3}$, $z = \frac{3}{2}$

DP : $\lambda_C = \lambda_R = d + z + \frac{\beta}{\nu_\perp}$, $1 + a = b = \frac{2\beta}{\nu_\parallel}$

MV in $2D$: $\lambda_C = \lambda_R \simeq 0.732\, z$, $a = b = \frac{2\beta}{\nu_\parallel}$, $z \simeq 2.17$

what can be said on the form of the scaling function of the auto-response ?

N.B. : Galilei-invariance for KPZ is kept under renormalisation, unusual form

(A) **assumption :** $R(t,s) = \left\langle \psi(t)\widetilde{\psi}(s)\right\rangle$ $1D$ **KPZ equation/RSOS model**

good collapse $\Rightarrow$ **no** logarithmic corrections $\Rightarrow$ $\boxed{x' = \widetilde{x}' = 0}$

no logarithmic factors for $y \gg 1 \Rightarrow$ $\boxed{\xi' = 0}$

$\Rightarrow$ only $\widetilde{\xi}' = 1$ remains

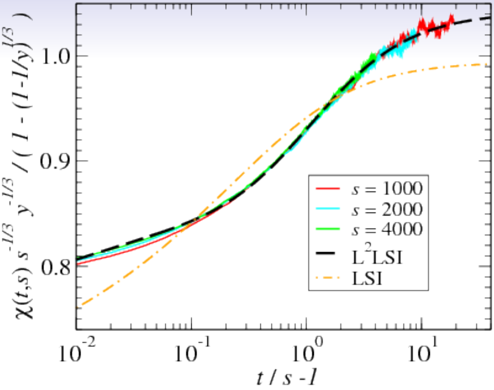
$$f_R(y) = y^{-\lambda_R/z}\left(1-\frac{1}{y}\right)^{-1-a'}\left[h_0 - g_0 \ln\left(1-\frac{1}{y}\right) - \frac{1}{2} f_0 \ln^2\left(1-\frac{1}{y}\right)\right]$$

use specific values of $1D$ KPZ class $\frac{\lambda_R}{z} - a = 1$

find integrated autoresponse $\chi(t,s) = \int_0^s \mathrm{d}u\, R(t,u) = s^{1/3} f_\chi(t/s)$

$$\begin{aligned} f_\chi(y) \;=\; & y^{1/3}\left\{A_0\left[1-\left(1-\frac{1}{y}\right)^{-a'}\right]\right. \\ & \left. +\left(1-\frac{1}{y}\right)^{-a'}\left[A_1 \ln\left(1-\frac{1}{y}\right) + A_2 \ln^2\left(1-\frac{1}{y}\right)\right]\right\} \end{aligned}$$

with free parameters A_0, A_1, A_2 and a'



non-log LSI with $a = a'$:
deviations $\approx 20\%$

non-log LSI with $a \neq a'$:
works up to $\approx 5\%$

log LSI : works **better than** $\approx 0.1\%$

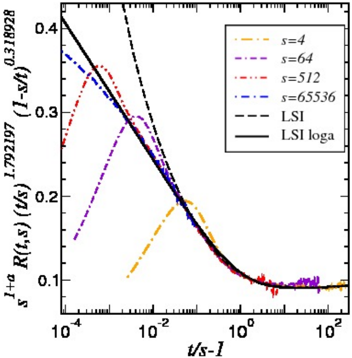
R	a'	A_0	A_1	A_2
$\langle\phi\widetilde{\phi}\rangle$ – LSI	-0.500	0.662	0	0
$\langle\phi\widetilde{\psi}\rangle$ – L^{1}LSI	-0.500	0.663	$-6 \cdot 10^{-4}$	0
$\langle\psi\widetilde{\psi}\rangle$ – L^{2}LSI	-0.8206	0.7187	0.2424	-0.09087

logarithmic LSI fits data at least down to $y \simeq 1.01$, with
$a' - a \approx -0.4873$ (can we make a conjecture ?)

(B) assumption : $R(t,s) = \left\langle \psi(t)\widetilde{\psi}(s) \right\rangle$ **1D critical contact process**

good collapse $\Rightarrow$ **no** logarithmic corrections $\Rightarrow$ $\boxed{x' = \widetilde{x}' = 0}$

$$h_R(y) = \left(1 - \frac{1}{y}\right)^{a-a'} \left[h_0 - g_{12,0}\widetilde{\xi}' \ln(1-1/y) - g_{21,0}\xi' \ln(y-1) - \frac{1}{2} f_0 \widetilde{\xi}'^2 \ln^2(1-1/y) + \frac{1}{2} f_0 \xi'^2 \ln^2(y-1) \right]$$



find empirically :
very small amplitude of
$\ln^2$-terms

$\Rightarrow$ $\boxed{f_0 = 0}$

require both $\xi \neq 0$, $\widetilde{\xi}' \neq 0$

BUT : logarithmic factor for $y \gg 1$ **?**

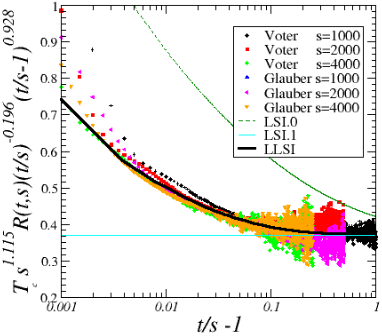
logar. LSI fit data, at least down to $y \simeq 1.002$; with $a' - a \simeq -0.002$.

(C) <u>**assumption :**</u> $R(t,s) = \left\langle \psi(t)\widetilde{\psi}(s) \right\rangle$ **$2D$ majority voter/Glauber model**

(triangular lattice)

good collapse $\Rightarrow$ **no** logarithmic corrections $\Rightarrow$ $\boxed{x' = \widetilde{x}' = 0}$

$$h_R(y) \;=\; \left(1-\frac{1}{y}\right)^{a-a'} \left[h_0 - g_{12,0} \ln(1-1/y) - \frac{1}{2} f_0 \ln^2(1-1/y) \right]$$



no logarithmic terms for $y \gg 1$

$\Rightarrow$ $\boxed{\xi' = 0}$

can normalise $\widetilde{\xi}' = 1$

F. Sastre (2013) *preliminary*

logar. LSI fit data, at least down to $y \simeq 1.005$.

6. Outlook : growth on semi-infinite substrates

properties of growing interfaces near to a boundary ?

$\rightarrow$ crystal dislocations, face boundaries . . .

Experiments : Family-Vicsek scaling not always sufficient

FERREIRA *et. al.* 11
RAMASCO *et al.* 00, 06
YIM & JONES 09, . . .

$\rightarrow$ **distinct** global and **local** interface fluctuations

- **anomalous scaling**, growth exponent β larger than expected
- **grainy interface morphology**, facetting

! analyse simple models on a **semi**-infinite substrate !

frame co-moving with average interface deep in the bulk

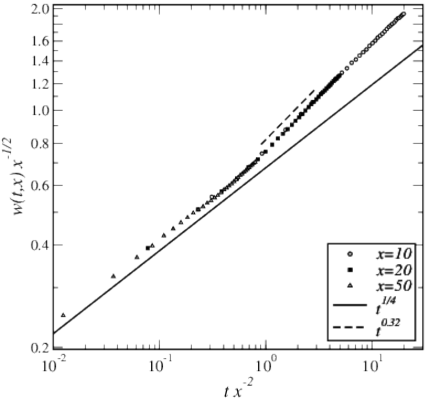
characterise interface by

- height profile $\langle h(t, \mathbf{r}) \rangle$
- width profile $w(t, \mathbf{r}) = \left\langle [h(t, \mathbf{r}) - \langle h(t, \mathbf{r}) \rangle]^2 \right\rangle^{1/2}$

$h \rightarrow 0$ as $|\mathbf{r}| \rightarrow \infty$

specialise to $d = 1$ space dimensions ; boundary at $x = 0$, bulk $x \to \infty$

cross-over for the phenomenological growth exponent β near to boundary



bulk behaviour $w \sim t^{\beta}$

'surface behaviour' $w_1 \sim t^{\beta_1}$?

cross-over, if causal interaction with boundary

experimentally observed, e.g. for semiconductor films

Nascimento, Ferreira, Ferreira 11

EW-class Allegra, Fortin, mh 13

values of growth exponents (bulk & surface) :

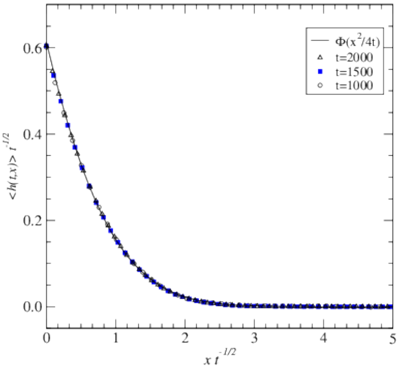
$\beta = 0.25$ $\beta_{1,\text{eff}} \simeq 0.32$ Edwards-Wilkinson class

$\beta \simeq 0.32$ $\beta_{1,\text{eff}} \simeq 0.35$ Kardar-Parisi-Zhang class need explicit boundary interactions in Langevin equation $h_1(t) = \partial_x h(t,x)|_{x=0}$

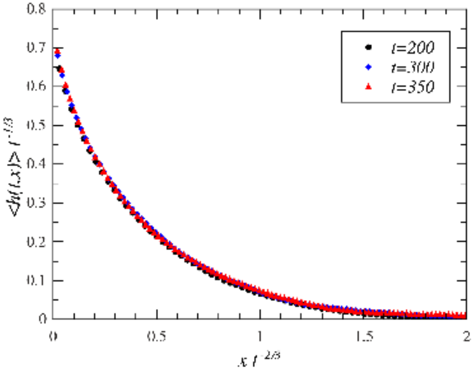
$$\left(\partial_t - \nu\partial_x^2\right) h(t,x) - \frac{\mu}{2}\left(\partial_x h(t,x)\right)^2 + \eta(t,x) = \nu\left(\kappa_1 + \kappa_2 h_1(t)\right)\delta(x)$$

height profile $\langle h(t,x)\rangle = t^{1/\gamma}\Phi\left(xt^{-1/z}\right)$, $\gamma = \frac{z}{z-1} = \frac{\zeta}{\zeta-\beta}$

EW & exact solution, $h(t,0) \sim \sqrt{t}$ self-consistently



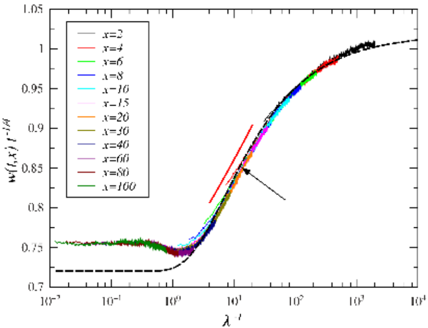
KPZ

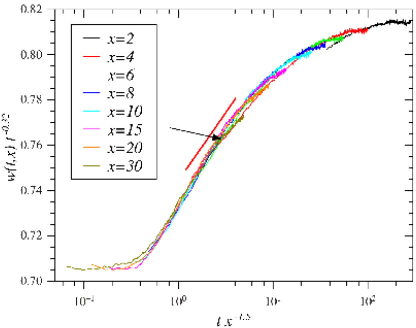


Scaling of the width profile :

EW & exact solution $\lambda^{-1} = 4tx^{-2}$ KPZ

bulk boundary





same growth scaling exponents in the bulk and near to the boundary

large **intermediate scaling regime** with effective exponent (**slopes**)

agreement with RG for non-disordered, local interactions LOPÉZ, CASTRO, GALLEGO 05

? ageing behaviour near to a boundary ?

7. Conclusions

- physical ageing occurs naturally in many **irreversible** systems relaxing towards **non**-equilibrium stationary states
  considered here : absorbing phase transitions & surface growth
- scaling phenomenology analogous to simple magnets
- **but** finer differences in relationships between non-equilibrium exponents
- a major difference w/ equilibrium : intrinsic **absence** of time-translation-invariance $\Rightarrow$ **2** scaling dimensions
- shape of scaling functions : **logarithmic** local scale-invariance ?
  performed numerical experiments on auto-response function :
  (i) $1D$ KPZ equation (ii) $1D$ critical directed percolation
  (iii) $2D$ majority voter/Glauber models
- **surprises** in scaling near a boundary : height/width profiles

studies of the ageing properties, via two-time observables, might become a **new tool**, also for the analysis of complex systems !