Exclusion Processes and Pedestrians

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Transport team

Out-of-equilibrium

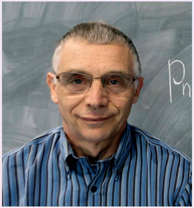
- ➥ Road Traffic
- ➥ Intracellular traffic
- ➥ Pedestrian traffic

- Experiments on pedestrian traffic (PEDIGREE Project)

  Data analysis

  Experiment-based models
- Today: Modeling based on exclusion processes

Work realized in collaboration
with Julien Cividini and Henk Hilhorst

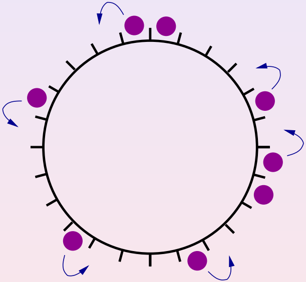


Plan

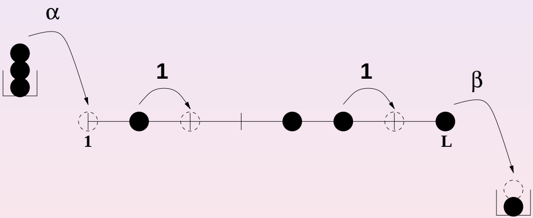
1. Introduction: exclusion processes
   - Update schemes
   - The frozen shuffle update
2. Crossing of two perpendicular pedestrian flows
   - Diagonal patterns and chevron effect
   - Mean-field approach
   - Wake and effective interaction
3. Domain-wall theory for deterministic TASEP with parallel update

Exclusion Processes

TASEP = Totally Asymmetric Simple Exclusion Process



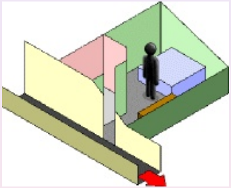
Periodic Boundary Conditions

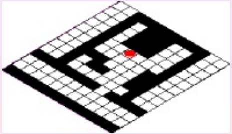


Open Boundary Conditions

Cellular automata models for pedestrians

- Floor field model
  [C. Burstedde et al, Physica A **295** (2001) 507-525]
- Ex: PEDGO Software





Cellular automaton = geometry + rules + update

Updates (discrete time)

- random sequential update
  - The unit time step is divided into microsteps $\delta t = 1/N$
  - One particle is randomly chosen and updated at each microtimestep
  - ➥ close to a continuous time
  - ➥ large fluctuations

- parallel update
  - All particles are updated in parallel
  - The state of a particle at time $t + 1$ is determined by the state of the system at time t
  - ➥ road traffic (Δt = 1 = reaction time)
  - ➥ conflicts are possible (crossing, lane changing...).

Updates

- ordered sequential update (forward, backward...)

➥ depends on the geometry of the system

- random shuffle update [Wölki et al (2006); Smith & Wilson (2007) J. Phys. A]
  - The order of the updates is chosen randomly at each time step
  - Each particle is updated once per time step according to this predefined order.

  ➥ proposed to model pedestrian traffic

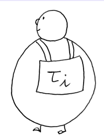
  ➥ no conflicts, bounded fluctuations

  ➥ but still, the same pedestrian can be updated twice in a row

- frozen shuffle update

Frozen shuffle update

- A phase $\tau_i \in [0, 1[$ is attached to each newly created pedestrian. It remains unchanged during the whole simulation.



- At each time step, pedestrians are updated in the order of increasing phases (time $t + \tau_i$).
  - τ_i = phase in the walking cycle, slight advance
  - pedestrians have the same speed, no large fluctuations
  - no conflicts

- Deterministic TASEP : all possible moves are accepted

➥ stochasticity comes only from the phases τ_i.

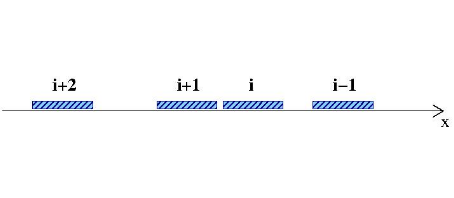
➥ analytical predictions

TASEP+PBC: [C. A-R, Cividini & Hilhorst, J. Stat. Mech. (2011) P07009]

TASEP+OBC: [C. A-R, Cividini & Hilhorst, J. Stat. Mech. (2011) P10013]

- Can also be applied to more realistic models.

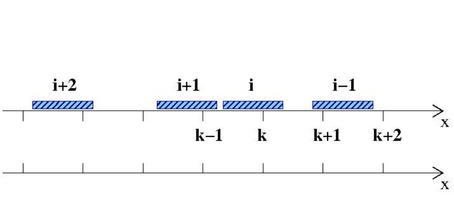
Mapping a free-flow state to a continuous model



- Continuous model: rods of length one sliding along the x-axis with velocity 1.

- add an underlying discrete network and take snapshots at integer times s.
  - ➡ Put a particle on site k if rod i overlaps position $x = k$.
  - ➡ A rod i jumps from site k to site $k + 1$ at time $s + \tau_i$.

Mapping a free-flow state to a continuous model



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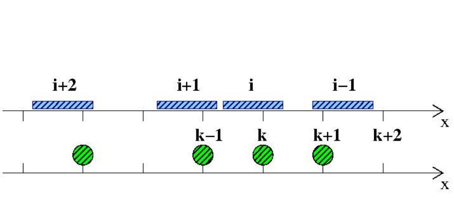

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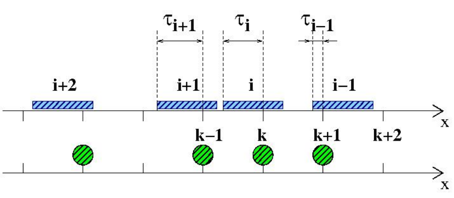

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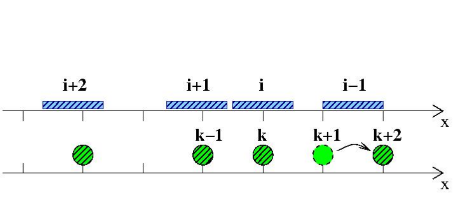

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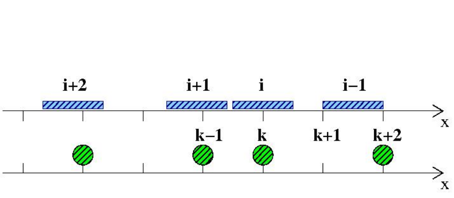

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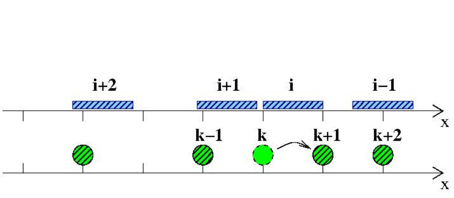

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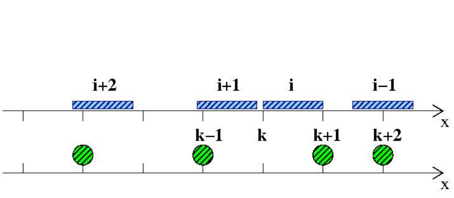

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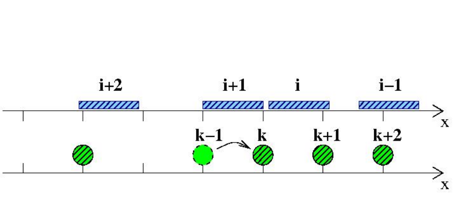

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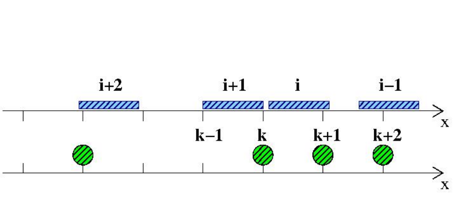

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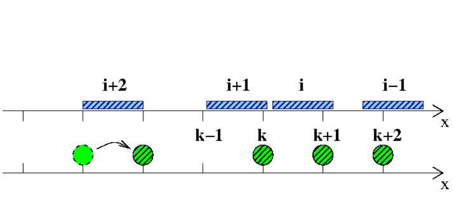

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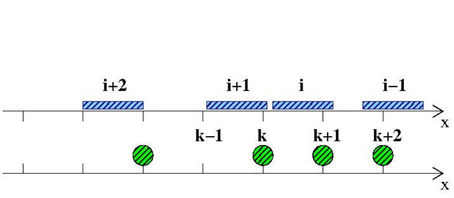

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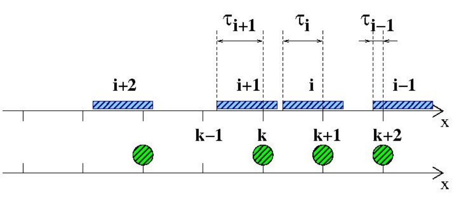

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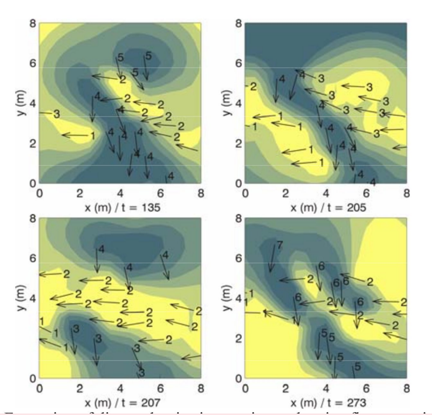
Intersection of two perpendicular pedestrian flows

Diagonal instability:

- observed in experiments

in [Hoogendoorn & Daamen, TGF'03 (Springer) 2005, pp. 121]



Intersection of two perpendicular pedestrian flows

Diagonal instability:

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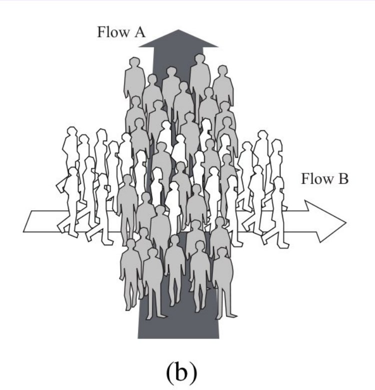

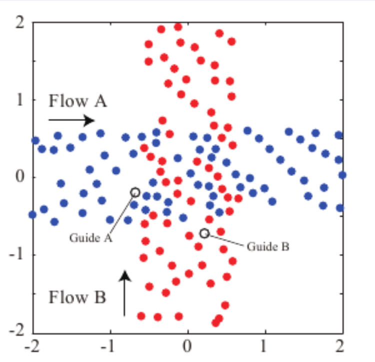

[Yamamoto & Okada, in 2011 IEEE Int. Conf. on Robotics and Automation (ICRA)]

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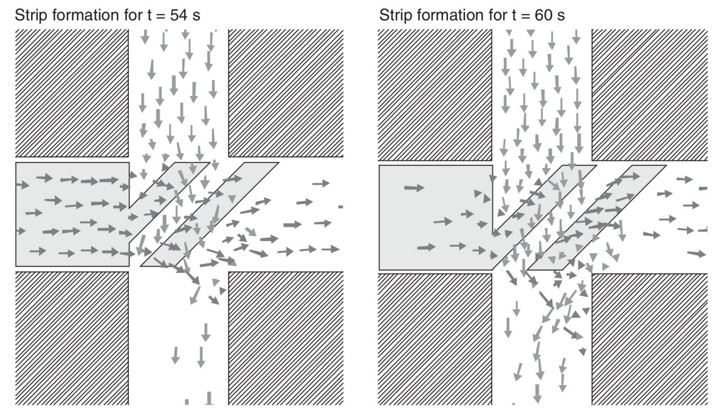

[Hoogendoorn & Bovy, Optim. Control Appl. Meth., 24 (2003) 153]

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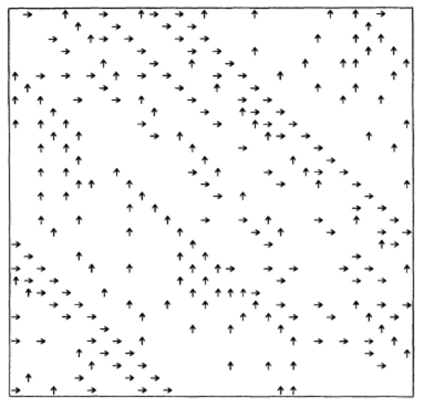
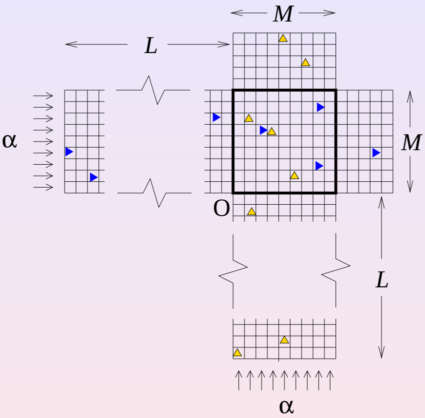


BML Model (city traffic)

PBC

[Biham, Middleton & Levine, PRA **46** (1992) R6124]

Intersection of two corridors



- $\mathcal{E}$ = Eastbound particles
- $\mathcal{N}$ = Northbound particles

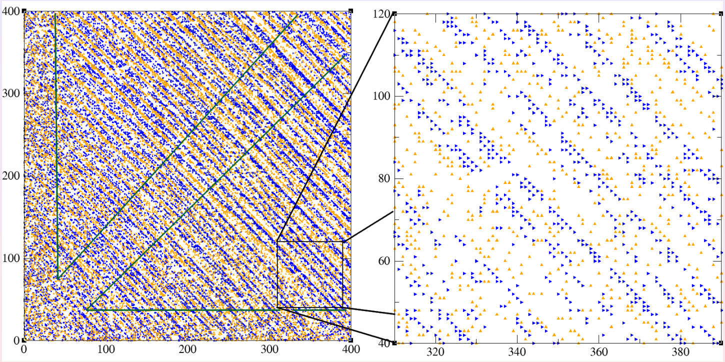
$n^{\mathcal{E}}(\mathbf{r})$, $n^{\mathcal{N}}(\mathbf{r})$ = boolean occupation variables

- As α increases: jamming transition
  [H. J. Hilhorst, C. A-R, J. Stat. Mech. (2012) P06009]

➥ Here we consider only the free flow phase.

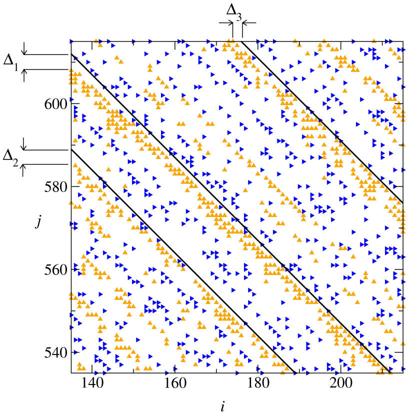
Observations

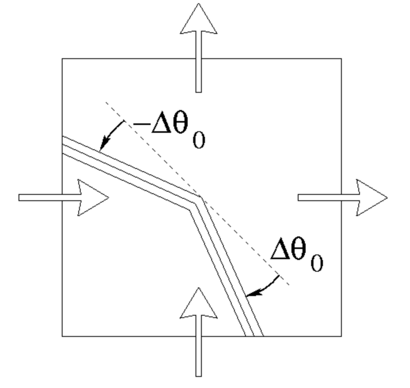
with frozen shuffle update $M = 400$



Number of encounters made by a particle: $g = \rho M$
= effective coupling constant governing pattern formation

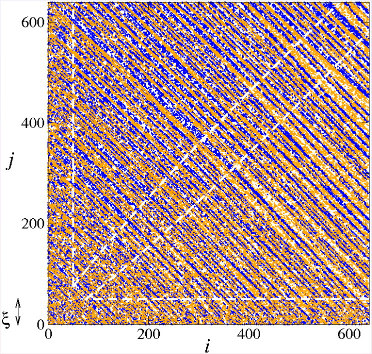
Observations



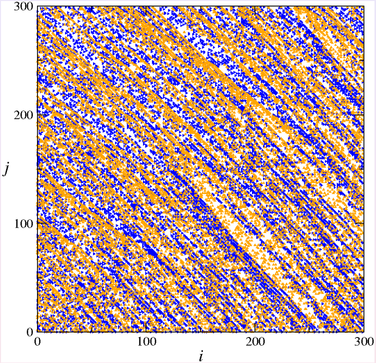


➥ Tilt $\Delta\theta$

Observations



- frozen shuffle update
- $M = 640$



- alternating parallel update
- $M = 300$

Observations

PBC: no tilt

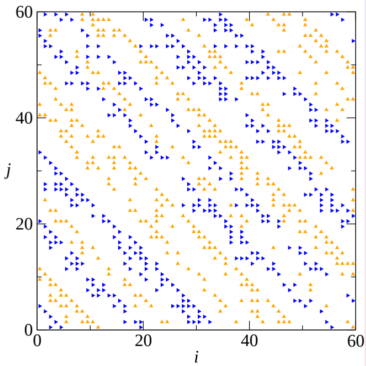
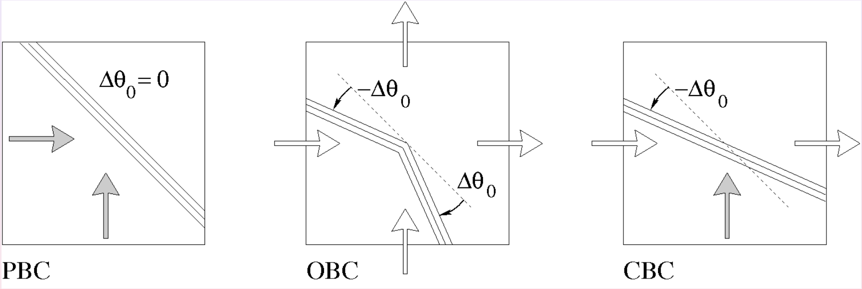


Figure from Chloé Barré

Observations

Summary: pattern depends on the boundary conditions



Mean field equations

We postulate some mean-field equations:

$$\begin{aligned}\rho^{\mathcal{E}}_{t+1}(\mathbf{r}) &= [1-\rho^{\mathcal{N}}_t(\mathbf{r})]\rho^{\mathcal{E}}_t(\mathbf{r}-\boldsymbol{e}_x)+\rho^{\mathcal{N}}_t(\mathbf{r}+\boldsymbol{e}_x)\rho^{\mathcal{E}}_t(\mathbf{r}) \\ \rho^{\mathcal{N}}_{t+1}(\mathbf{r}) &= [1-\rho^{\mathcal{E}}_t(\mathbf{r})]\rho^{\mathcal{N}}_t(\mathbf{r}-\boldsymbol{e}_y)+\rho^{\mathcal{E}}_t(\mathbf{r}+\boldsymbol{e}_y)\rho^{\mathcal{N}}_t(\mathbf{r})\end{aligned}$$

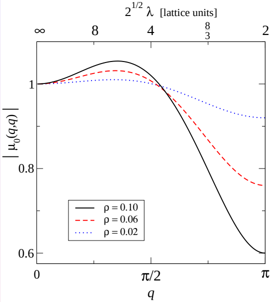
- pair correlations $\langle n^{\mathcal{E}} n^{\mathcal{N}} \rangle$ have been factorized
- interaction terms $\langle n^X n^X \rangle$ between same-type particles have been neglected (low density)

Simulations: same patterns as for the particle model

Mean field equations

- PBC
  - Linear stability analysis
    $\rho_t^{\mathcal{E},\mathcal{N}}(\mathbf{r}) = \overline{\rho} + \delta\rho_t^{\mathcal{E},\mathcal{N}}(\mathbf{r})$

➥ Most unstable mode traveling in the $(1, 1)$ direction with wavelength



$$\lambda_{\max} = 2\pi/|\mathbf{q}|_{\max} = \sqrt{2}\pi/\arccos[(1-2\overline{\rho})/(2-2\overline{\rho})]$$
$$= 3\sqrt{2}[1-(\sqrt{3}/\pi)\overline{\rho}] + \mathcal{O}(\overline{\rho}^2),$$

Mean field equations

- OBC
  - Linear stability analysis:

➥ In preparation [Cividini & Hilhorst]
➥ no sign of the chevron effect

Chevron effect = non linear effect

Chevron effect

- $v^{\mathcal{E}}(\mathbf{r})$ average eastward velocity (in the stationary state)
- $v^{\mathcal{N}}(\mathbf{r})$ average northward velocity

Hypothesis: Moving stripes are mutually impenetrable

➥ Possible only if

$$\tan\theta(\mathbf{r}) = \frac{v^{\mathcal{N}}(\mathbf{r})}{v^{\mathcal{E}}(\mathbf{r})}$$

Chevron effect

<u>Particle model:</u>

• *Definition of velocity*

$$v^{\mathcal{E},\mathcal{N}}(\mathbf{r}) = \frac{J^{\mathcal{E},\mathcal{N}}(\mathbf{r})}{\langle n^{\mathcal{E},\mathcal{N}}(\mathbf{r})\rangle}$$

where $J^{\mathcal{E},\mathcal{N}}(\mathbf{r})$ = stationary current

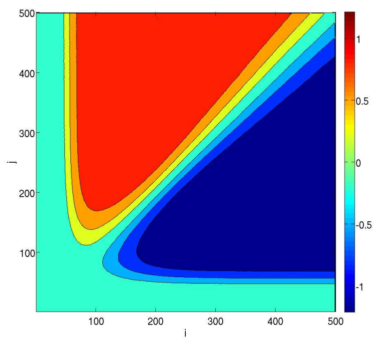
• If $J^{\mathcal{E},\mathcal{N}}(\mathbf{r}) = J$, then

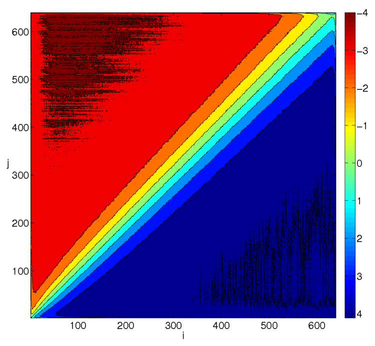
$$\tan\theta(\mathbf{r}) = \frac{\langle n^{\mathcal{E}}(\mathbf{r})\rangle}{\langle n^{\mathcal{N}}(\mathbf{r})\rangle}$$

• Setting $\theta = \frac{\pi}{4} + \Delta\theta$ and expanding yields

$$\Delta\theta(\mathbf{r}) \simeq \frac{\langle n^{\mathcal{E}}(\mathbf{r})\rangle - \langle n^{\mathcal{N}}(\mathbf{r})\rangle}{2\langle n^{\mathcal{N}}(\mathbf{r})\rangle}$$

Chevron effect



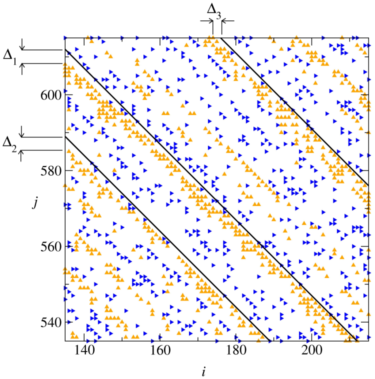


- Mean-field equations
- alternating parallel update

Is the system able to sustain modes with tilted stripes?

Chevron effect

In the upper triangle:

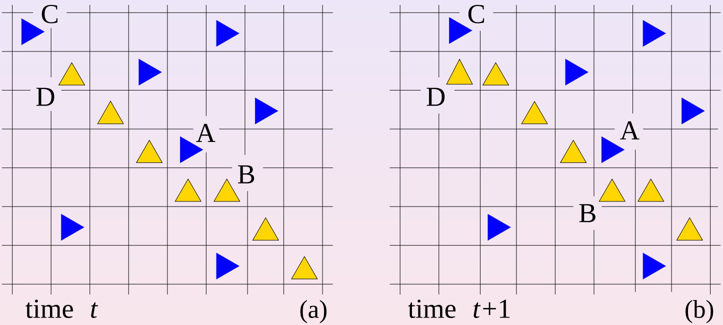


Asymmetry:

- $\mathcal{E}$ stripes (orange) are dense and narrow
- $\mathcal{N}$ stripes (blue) are sparse and wide

Chevron effect: Identifying a tilted mode

Idealized tilted mode: (alternating parallel update)
Expected near the entrance of $\mathcal{E}$ particles



figures taken before the hopping of $\mathcal{E}$ particles

➥ Structure of the stripe is preserved

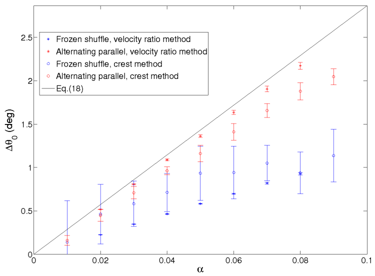
Chevron effect: Identifying a tilted mode

$$\tan\theta = 1-\rho_{\text{kink}} = 1-\rho^{\mathcal{E}} = 1-J^{\mathcal{E}}$$

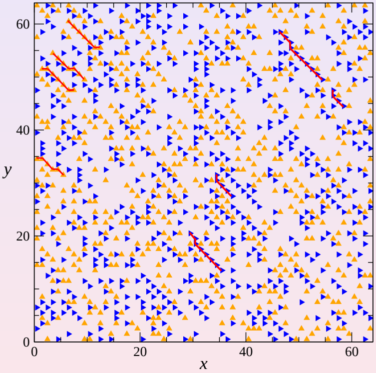
- To lowest order:

$$\Delta\theta(\mathbf{r}) = \frac{\alpha}{2}\left(\frac{180}{\pi}\right)^{\circ}$$

- Should give an upper bound



In direct simulations:

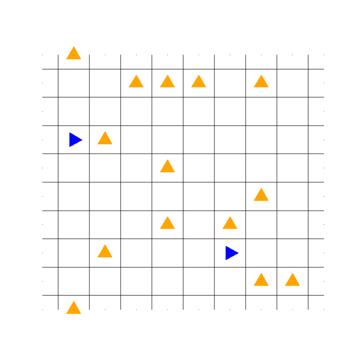


alternating parallel update
with $\alpha = 0.15$
and $M = 64$

Effective interactions

From which microscopic mechanism does the (tilted) diagonal pattern emerge?

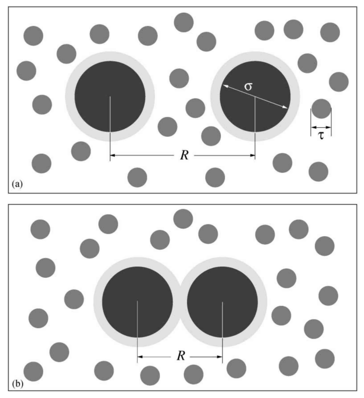
➥ effective interaction between two $\mathcal{E}$ particles crossing a flow of $\mathcal{N}$ particles



Effective interactions

Environment-mediated interactions:

Most well known : depletion forces



[C. Likos, Physics Reports **348** (2001) 267]

Effective interactions

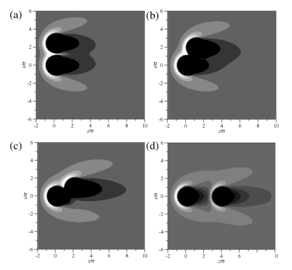
Environment-mediated interactions: extensively studied

- in equilibrium soft matter
  [C. Likos, Effective interactions in soft condensed matter physics, Physics Reports **348** (2001) 267]

Effective interactions

Environment-mediated interactions: extensively studied

- in equilibrium soft matter
- and more recently in out-of-equilibrium systems



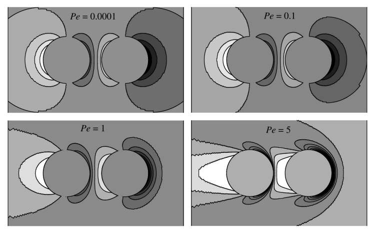
[Dzubiella, Löwen & Likos, Depletion forces in nonequilibrium, PRL **91** (2003) 1]

Approx: perturbation of the density field due to the two large particles
= superposition of the perturbation due to each large particle separately.

Effective interactions

Environment-mediated interactions: extensively studied

- in equilibrium soft matter
- and more recently in out-of-equilibrium systems



[Khair & Brady, On the motion of two particles translating with equal velocities through a colloidal dispersion, Proc. R. Soc. A **463** (2007) 223]

No superposition approximation but

- interactions in the bath are neglected
- probes are taken aligned

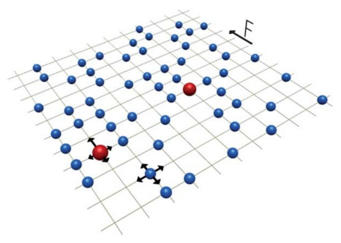
Focus: prediction of effective forces

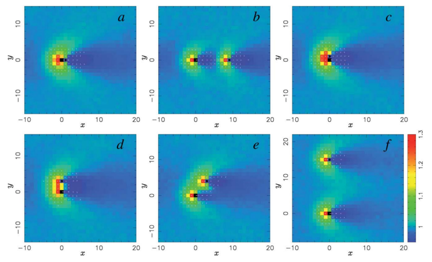
Effective interactions

- Forces are not relevant for pedestrians
  In our case: interaction comes from the dynamical rules

Effective interactions

- Discrete model

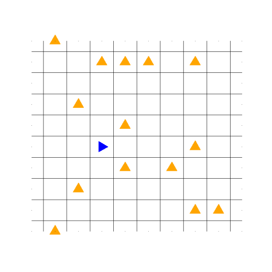




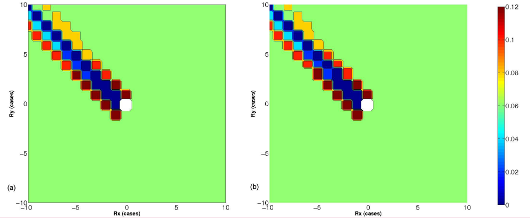
[Mejía-Monasterio & Oshanin, Bias- and bath-mediated pairing of particles driven through a quiescent medium, The royal soc. of chem. **7** (2011) 993]

Numerical study ➪ existence of an attractive interaction between the intruders resulting in a statistical pairing

Ensemble averaged wake



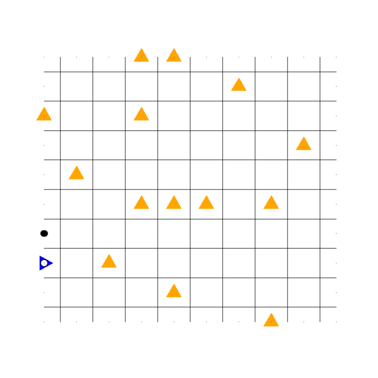
Frozen shuffle update



Theory   Simulation

Wake of a single $\mathcal{E}$ particle

Microscopic structure of the wake:



Central part of the wake : the shadow

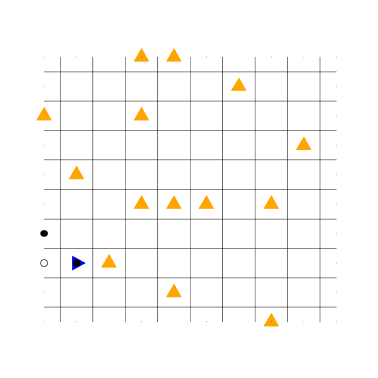
Construction:

- Before move: white dot
- After move: black dot

Again, at low density, *tan*$\theta \simeq 1 - \rho^{\mathcal{N}}$

Wake of a single $\mathcal{E}$ particle

Microscopic structure of the wake:



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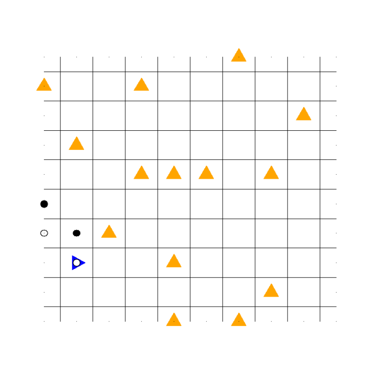

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Wake of a single $\mathcal{E}$ particle

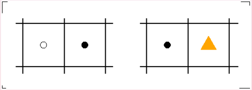
Microscopic structure of the wake:

Central part of the wake : the shadow

Construction:

- Before move: white dot
- After move: black dot

Two types of rows:

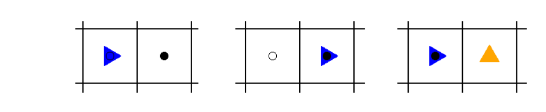


Again, at low density, $tan\theta \simeq 1 - \rho^{\mathcal{N}}$

Two $\mathcal{E}$ particles in a flow of $\mathcal{N}$ particles

Frozen shuffle update:

Let us put a second $\mathcal{E}$ particle (phase τ_0) in the shadow of the first one (phase 0) :



D_W D_B K_τ

$$P_S(t) = P_W(t) + P_B(t) + \int_0^1 d\tau \; p_\tau(t)$$

Two $\mathcal{E}$ particles in a flow of $\mathcal{N}$ particles

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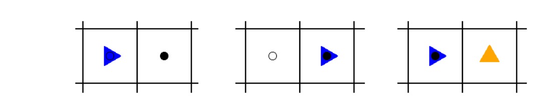


D_W D_B K_τ

Low density limit :

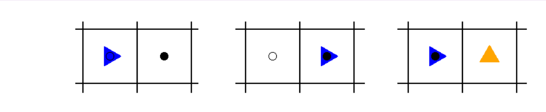
$$\begin{cases} P^{fs}_W(t+1) &= (1-\rho^{fs})P^{fs}_W(t) + \rho^{fs}(1-\tau_0)P^{fs}_B(t) + P^{fs}_{>\tau_0}(t) \\ P^{fs}_B(t+1) &= (1-2\rho^{fs}+\rho^{fs}\tau_0)P^{fs}_B(t) + P^{fs}_{<\tau_0}(t) \\ P^{fs}_{<\tau_0}(t+1) &= \rho^{fs}\tau_0 P^{fs}_W(t) + \rho\tau_0 P^{fs}_B(t) \\ P^{fs}_{>\tau_0}(t+1) &= \rho^{fs}(1-\tau_0)P^{fs}_W(t) \end{cases}$$

where $P^{fs}_{<\tau_0}(t) \equiv \int_{\tau=0}^{\tau_0} p_\tau(t)d\tau$ and $P^{fs}_{>\tau_0}(t) \equiv \int_{\tau=\tau_0}^{1} p_\tau(t)d\tau$.

Two $\mathcal{E}$ particles in a flow of $\mathcal{N}$ particles

Frozen shuffle update:

Let us put a second $\mathcal{E}$ particle (phase τ_0) in the shadow of the first one (phase 0) :



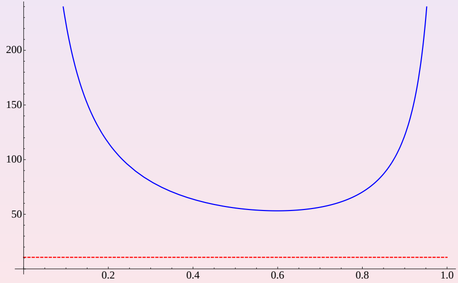
D_W D_B K_τ

$$P_S^{\mathrm{fs}}(t+1) = P_S^{\mathrm{fs}}(t) - \rho^{\mathrm{fs}}(1-\tau_0)P_B^{\mathrm{fs}}(t)$$

Two $\mathcal{E}$ particles in a flow of $\mathcal{N}$ particles

Diagonalization of the transfer matrix
$\rightarrow$ time evolution of $P_S^{\mathrm{fs}}(t)$ (linear combination of exponentials).

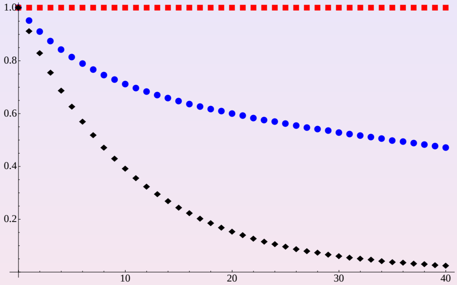
Longest characteristic decay time as a function of τ_0



red: uncorrelated case

Two $\mathcal{E}$ particles in a flow of $\mathcal{N}$ particles

Probability that the second particle stays in the shadow during the first t timesteps



- blue: frozen shufle update
- red: alternating parallel update
- black: uncorrelated

Alternating parallel update: overlap of shadows ➪ several particles can be localized in the shadow of the first particle.

Conclusion

- Exemple of an effective interaction that can be solved analytically.
- Calculations can be extended to other update schemes, provided the free flow phase is deterministically shifted forward with velocity 1 at each time step.
- The angle of the wake = angle of the long-lived global mode identified before.
- In the full problem, angle may be different and depend on the update, though the order of magnitude should be the same.

[J. Cividini and C. A-R, *Wake-mediated interaction between driven particles crossing a perpendicular flow*, J. Stat. Mech. (2013) P07015]

Domain wall picture

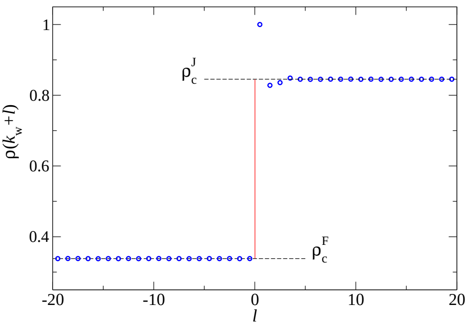
[Kolomeisky, Schütz, Kolomeisky and Straley, J. Phys. A: Math. Gen. **31** (1998) 6911]

- phenomenological picture
- can be more easily extended to non stationary states, variants of the ASEP
- physical understanding

Domain wall picture

For deterministic updates for which free flow has velocity 1:

- microscopic definition of the wall
- wall position = position of the leftmost particle that has ever been blocked



$\alpha = \beta = 0.4$
$L = 3000$
frozen shuffle update

Domain wall picture

Usual domain wall theory not appropriate for this case.
➪ Extension of the domain wall theory to the case of deterministic parallel update

- There are correlations between successive steps of the domain wall

➥ Memory effect

➥ Two coupled master equations (+BC)

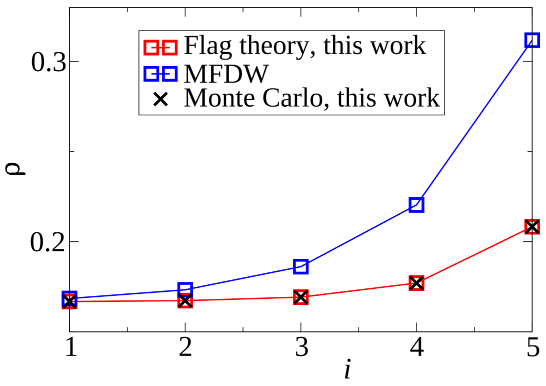
- Continuum limit of this exactly soluble model

➥ Fokker-Planck equation for the position of the wall

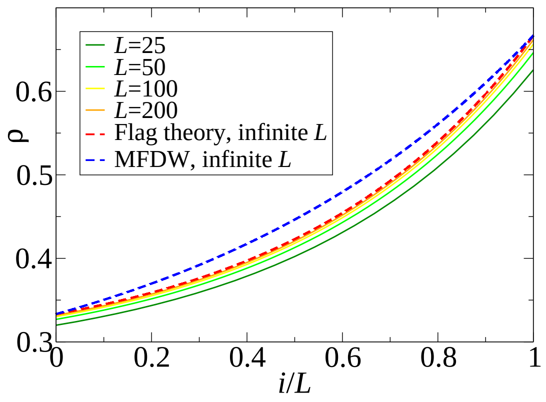
  ➪ with a diffusion constant different from the one that would be obtained by the usual DW theory

  ➪ in agreement with Monte-Carlo simulations.

Domain wall picture



Domain wall picture



Conclusion

For more details:
http://www.th.u-psud.fr/page_perso/Appert/

Thank-you

Experimental study of pedestrians

PEDIGREE Project 2009-2011 (LPT, IMT, CRCA, Bunraku)

- Experiments on pedestrian traffic
  - Ring [Moussaid et al, PLoS Computational Biology **8** (2012) 1002442]
  - 1D circle [Jelic et al, PRE **85** (2012) 036111]

- Models
- Continuous model for bidirectional crowd motion [C. A.-R., P. Degond, S. Motsch, NHM **6** (2011) 351]
- Following model [S. Lemercier et al, Eurographics (2012)]

And more generally, transport...

Road Traffic

- Kinetic model for a bi-directional road [C. A.-R., H. Hilhorst, G. Schehr, J. Stat. Mech. (2010)]
- Response of a multi-lane highway to a local perturbation [C. A.-R., J. Du Boisberranger, Transp. Res. C (2013)]

Intracellular Traffic

- **Intracellular transport** (collab. Sarrebrücken, L. Santen, M. Ebbinghaus, I. Weber, S. Klein)