

LMS Meeting in Harmonic Analysis and PDE

Warwick, December 8th 2014

Pieter Blue - Edinburgh University

Decay for fields outside black holes

Abstract:

I will discuss energy and Morawetz (or integrated local decay) estimates for fields outside black holes, including the wave and Maxwell equation. These results use the Killing tensor, an unusual geometric object that exists in the Kerr spacetime.

Maria Carmen Reguera - Birmingham

Lower bounds for fractional Riesz transforms on general Cantor sets

Abstract:

In this talk we study estimates from below for the L^2 norm of the s -dimensional Riesz transform, with kernel $x/|x|^{s+1}$ for $s \in (d-1, d)$, of measures supported on very general Cantor sets in \mathbb{R}^d . The bounds obtained are written in terms of the densities of the cubes appearing in the construction of the Cantor sets. These estimates allow to establish an equivalence between the capacity γ_s associated with the s -dimensional Riesz kernel and the capacity $\dot{C}_{\frac{2}{3}(d-s), \frac{3}{2}}$ from non-linear potential theory associated to the Wolff potential for the so called uniformly disconnected compact sets.

The comparability of the two capacities was first understood by Mateu, Prat and Verdera, who proved $\gamma_s(E) \equiv \dot{C}_{\frac{2}{3}(d-s), \frac{3}{2}}(E)$ for any compact set E in the case $0 < s < 1$. The general case is still a big open problem in Geometric analysis. Our result is the latest contribution and it is based on previous work on the subject by Eiderman-Nazarov-Volberg, Mateu-Tolsa and Tolsa.

This is joint work with Xavier Tolsa.

James Robinson - Warwick

A characterisation of local existence for semilinear heat equations in Lebesgue spaces

Abstract:

We consider the nonlinear heat equation $u_t - \Delta u = f(u)$ with $u(0) = u_0$, with Dirichlet boundary conditions on a bounded domain $\Omega \subset \mathbb{R}^d$. We assume that $f: [0, \infty) \rightarrow [0, \infty)$ is continuous and non-decreasing. We show that if $q \in (1, \infty)$ then the equation has a local solution bounded in $L^q(\Omega)$ for all initial data in $L^q(\Omega)$ if and only if $\limsup_{s \rightarrow \infty} s^{-(1+2q/d)} f(s) < \infty$; and that if in addition $f(s)/s$

is non-decreasing then the equation has a local solution bounded in $L^1(\Omega)$ for all $u_0 \in L^1(\Omega)$ if and only if $\int_1^\infty s^{-(2+2/d)} f(s) \, ds < \infty$. Our proofs are also valid for the case $\Omega = \mathbb{R}^d$.

This is joint work with Robert Laister (University of the West of England), Mikolaj Sierzega (Warwick), and Alejandro Vidal-Lopez (Xi'an Jiaotong-Liverpool University)

Luis Vega - University of the Basque Country

The Talbot effect in a non-linear dynamics

Abstract:

In the first part of the talk I shall present a linear model based on the Schrodinger equation with constant coefficient and periodic boundary conditions that explains the so-called Talbot effect in optics. In the second part I will make a connection of this Talbot effect with turbulence through the Schrodinger map which is a geometric non-linear partial differential equation.