New topologies for some spaces of n-homogeneous polynomials and applications on hypercyclicity of convolution operators

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(joint work with D. Pellegrino)

Relations Between Banach Space Theory and Geometric Measure Theory
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Hypercyclicity

- A mapping $f\colon X \longrightarrow X$, where X is a topological space, is *hypercyclic* if the set $\{x, f(x), f^2(x), \ldots\}$ is dense in X for some $x \in X$.
- The study of hypercyclic translation and differentiation operators on spaces of entire functions of one complex variable can be traced back to Birkhoff (1929) [3] and MacLane (1952) [8].
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Theorem

Let E' be separable and $(\mathcal{P}_{\Theta}(^{m}E))_{m=0}^{\infty}$ be a π_1-holomorphy type from E to $\mathbb{C}$. Then every convolution operator on $\mathcal{H}_{\Theta b}(E)$ which is not a scalar multiple of the identity is hypercyclic.

However, the spaces $\mathcal{P}_{\Theta}(^{m}E)$ need to be Banach spaces and thus $\mathcal{H}_{\Theta b}(E)$ becomes a Fréchet space. When the spaces $\mathcal{P}_{\Theta}(^{m}E)$ are quasi-Banach, the respective space $\mathcal{H}_{\Theta b}(E)$ is not Fréchet and then the arguments used to prove the result above, for instance the Hypercyclicity Criterion obtained independently by Kitai [7] and Gethner and Shapiro [5], do not work.

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$\mathcal{P}(^nE)$= Banach space of all continuous n-homogeneous polynomials from E to $\mathbb{C}$, $n \in \mathbb{N}$;

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Suppose that $(\mathcal{P}_\Delta(^nE), \|\cdot\|_\Delta)$ is a quasi-normed space of n-homogeneous polynomials defined on E such that the inclusion $\mathcal{P}_\Delta(^nE) \hookrightarrow \mathcal{P}(^nE)$ is continuous and let $C_{\Delta_n} > 0$ be such that $\|P\| \leq C_{\Delta_n} \|P\|_\Delta$, for all $P \in \mathcal{P}_\Delta(^nE)$. Suppose that $\mathcal{P}_f(^nE) \subset \mathcal{P}_\Delta(^nE)$ and the normed space $(\mathcal{P}_{\Delta'}(^nE'), \|\cdot\|_{\Delta'}) \subset \mathcal{P}(^nE')$ is such that the Borel transform

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We will show that the pair

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is a dual system. More precisely, we will prove that there exists a bilinear form $\langle \cdot ; \cdot \rangle$ on

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such that the following conditions hold:
$(S1)$ $\langle P; Q \rangle = 0$ for all $Q \in \mathcal{P}_{\Delta'}(^{n}E')$ implies $P = 0$.
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Let

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defined by

$$\langle P;Q\rangle = \mathcal{B}^{-1}(Q)(P).$$

It is clear that $\langle\cdot;\cdot\rangle$ is bilinear and it is not difficult to see that $(S1)$ and $(S2)$ hold.
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Now, let

$$U = \{P \in \mathcal{P}_{\Delta}\left({}^{n}E\right); \|P\|_{\Delta} \leq 1\}.$$

Using the Bipolar Theorem, we know that the bipolar of U, denoted by $U^{\circ\circ}$, coincides with the weak closure of the absolutely convex hull $\Gamma\left(U\right)$ of U. Consider the corresponding gauge

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$$U^{\circ}=\left\{ Q\in\mathcal{P}_{\Delta'}\left(^{n}E'\right);\left|<P,Q>\right|\leq1\text{ for all }P\in U\right\} .$$

It is easy to see that

$$U^{\circ}=\left\{ Q\in\mathcal{P}_{\Delta'}\left(^{n}E'\right);\left\Vert \mathcal{B}^{-1}\left(Q\right)\right\Vert \leq1\right\} ,$$

$$U^{\circ\circ}=\left\{ P\in\mathcal{P}_{\Delta}\left(^{n}E\right);\left|\mathcal{B}^{-1}\left(Q\right)\left(P\right)\right|\leq1,\text{ for all }\left\Vert \mathcal{B}^{-1}\left(Q\right)\right\Vert \leq1\right\}$$

Now, let

$$U = \{P \in \mathcal{P}_{\Delta}({}^{n}E); \|P\|_{\Delta} \leq 1\}.$$

Using the Bipolar Theorem, we know that the bipolar of U, denoted by $U^{\circ\circ}$, coincides with the weak closure of the absolutely convex hull $\Gamma(U)$ of U. Consider the corresponding gauge

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Hence

$$p_{U^{\circ\circ}}(P) = \inf\left\{\delta > 0; \left|\mathcal{B}^{-1}(Q)(P)\right| \leq \delta, \text{ for all } \left\|\mathcal{B}^{-1}(Q)\right\| \leq 1\right\}.$$

Using this equality it is easy to see that $p_{U^{\circ\circ}}$ is a norm on $\mathcal{P}_{\Delta}({}^{n}E)$.

We denote the completion of the space $(\mathcal{P}_{\Delta}({}^{n}E), p_{U^{\circ\circ}})$ by

$$\left(\mathcal{P}_{\widetilde{\Delta}}({}^{n}E), \|\cdot\|_{\widetilde{\Delta}}\right).$$

So the restriction of $\|\cdot\|_{\widetilde{\Delta}}$ to $\mathcal{P}_{\Delta}({}^{n}E)$ is $p_{U^{\circ\circ}}$.

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Theorem

The linear mapping

$$\widetilde{\mathcal{B}}\colon \left[\mathcal{P}_{\widetilde{\Delta}}\left({}^{n}E\right)\right]' \longrightarrow \mathcal{P}_{\Delta'}\left({}^{n}E'\right)$$
$$\widetilde{\mathcal{B}}\left(T\right)\left(\varphi\right) = T\left(\varphi^{n}\right)$$

is a topological isomorphism.

Remark

When $\mathcal{P}_{\Delta}\left({}^{n}E\right)$ is a Banach space then we have $\|\cdot\|_{\widetilde{\Delta}} = \|\cdot\|_{\Delta}$ and $\mathcal{P}_{\widetilde{\Delta}}\left({}^{n}E\right) = \mathcal{P}_{\Delta}\left({}^{n}E\right)$.
In fact, in this case, U is the closed unit ball in $\mathcal{P}_{\Delta}\left({}^{n}E\right)$, hence balanced and convex and $\Gamma(U) = U$. By using the Bipolar Theorem we have $U^{\circ\circ} = \overline{\Gamma(U)}^{w} = \overline{U}^{w} = U$, and the last equality follows from Banach-Mazur Theorem. Hence

$$\begin{aligned} p_{U^{\circ\circ}}\left(P\right) &= \inf\left\{\delta > 0; P \in \delta U^{\circ\circ}\right\} = \inf\left\{\delta > 0; P \in \delta U\right\} \\ &= \inf\left\{\delta > 0; \|P\|_{\Delta} \leq \delta\right\} = \|P\|_{\Delta}. \end{aligned}$$

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Definition

A *holomorphy type* Θ from E to $\mathbb{C}$ is a sequence of Banach spaces $(\mathcal{P}_\Theta(^nE))_{n=0}^\infty$, the norm on each of them being denoted by $\|\cdot\|_\Theta$, such that the following conditions hold true:
(1) Each $\mathcal{P}_\Theta(^nE)$ is a linear subspace of $\mathcal{P}(^nE)$.
(2) $\mathcal{P}_\Theta(^0E)$ coincides with $\mathcal{P}(^0E) = \mathbb{C}$ as a normed vector space.
(3) There is a real number $\sigma \geq 1$ for which the following is true: given any $k \in \mathbb{N}_0$, $n \in \mathbb{N}_0$, $k \leq n$, $a \in E$ and $P \in \mathcal{P}_\Theta(^nE)$, we have

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Definition

Let $(\mathcal{P}_\Delta({}^nE), \|\cdot\|_\Delta)$ be quasi-normed spaces for all $n \in \mathbb{N}_0$ with $\mathcal{P}_\Delta({}^0E) = \mathbb{C}$. A sequence $(\mathcal{P}_\Delta({}^nE))_{n=0}^{\infty}$ is *stable for derivatives* if

(1) $\hat{d}^k P(x) \in \mathcal{P}_\Delta({}^kE)$ for each $n \in \mathbb{N}_0$, $P \in \mathcal{P}_\Delta({}^nE)$, $k = 0, 1, \ldots, n$ and $x \in E$.

(2) For each $n \in \mathbb{N}_0$, $k = 0, 1, \ldots, n$, there is a constant $C_{n,k} \geq 0$ such that

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Let $(\mathcal{P}_{\Delta}({}^{n}E))_{n=0}^{\infty}$ *be a sequence stable for derivatives. If* $P \in \mathcal{P}_{\widetilde{\Delta}}({}^{n}E)$, $k = 0, 1, \ldots, n$ *and* $x \in E$, *then*

$$\hat{d}{}^{k} P(x) \in \mathcal{P}_{\widetilde{\Delta}}\left({}^{k}E\right)$$

and

$$\left\|\hat{d}^{k} P(x)\right\|_{\widetilde{\Delta}} \leq C_{n,k} \|P\|_{\widetilde{\Delta}} \|x\|^{n-k},$$

where $C_{n,k}$ *is the constant of Definition 4.*

Corollary

If $(\mathcal{P}_{\Delta}({}^{n}E))_{n=0}^{\infty}$ *is stable for derivatives for* $C_{n,k} \leq \frac{n!}{(n-k)!}$, *then* $\left(\mathcal{P}_{\widetilde{\Delta}}({}^{n}E)\right)_{n=0}^{\infty}$ *is a holomorphy type.*

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Definition

Let $(\mathcal{P}_\Theta(^mE))_{m=0}^\infty$ be a holomorphy type from E to $\mathbb{C}$. A given $f \in \mathcal{H}(E)$ is said to be of Θ-*holomorphy type of bounded type* if

$(i)\frac{1}{m!}\hat{d}^m f(0) \in \mathcal{P}_\Theta(^mE)$, for all $m \in \mathbb{N}_0$,

(ii) $\lim_{m\to\infty}\left(\frac{1}{m!}\|\hat{d}^m f(0)\|_\Theta\right)^{\frac{1}{m}} = 0$.

The vector subspace of $\mathcal{H}(E)$ of all such f is denoted by $\mathcal{H}_{\Theta b}(E)$ and becomes a Fréchet space with the topology τ_Θ generated by the family of seminorms

$$f \in \mathcal{H}_{\Theta b}(E;F) \mapsto \|f\|_{\Theta,\rho} = \sum_{m=0}^{\infty} \frac{\rho^m}{m!}\|\hat{d}^m f(0)\|_\Theta,$$

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for all $\phi \in E'$, $b \in \mathbb{C}$ and $m \in \mathbb{N}$;

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A holomorphy type $(\mathcal{P}_\Theta({}^mE))_{m=0}^\infty$ from E to $\mathbb{C}$ is said to be a π_1-*holomorphy type* if the following conditions hold:
(i)Polynomials of finite type belong to $(\mathcal{P}_\Theta({}^mE))_{m=0}^\infty$ and there exists $K > 0$ such that

$$\|\phi^m \cdot b\|_\Theta \leq K^m \|\phi\|^m \cdot |b|$$

for all $\phi \in E'$, $b \in \mathbb{C}$ and $m \in \mathbb{N}$;
(ii)For each $m \in \mathbb{N}_0$, $\mathcal{P}_f({}^mE)$ is dense in $(\mathcal{P}_\Theta({}^mE), \|\cdot\|_\Theta)$.

Example

F., Matos and Pellegrino introduced the class $\mathcal{P}_{N,((r,q);(s,p))}\left({}^{m}E\right)$ of all Lorentz $((r,q);(s,p))$-nuclear n-homogeneous polynomials on E and proved that if E' has the bounded approximation property, then the Borel transform establishes an topological isomorphism between $\left[\mathcal{P}_{N,((r,q);(s,p))}({}^{n}E)\right]'$ and $\mathcal{P}_{as((r',q');(s',p'))}\left({}^{m}E'\right)$, where $\mathcal{P}_{as((r',q');(s',p'))}\left({}^{m}E'\right)$ denotes the space of all absolutely Lorentz $((r',q');(s',p'))$-summing n-homogeneous polynomials on E'.

Using our technique we may consider the space $\mathcal{P}_{\widetilde{N},((r,q);(s,p))}\left({}^{m}E\right)$ and its dual (via Borel transform) is also $\mathcal{P}_{(s',m(r';q'))}\left({}^{m}E'\right)$.

It is not difficult to prove that the sequence $\left(\mathcal{P}_{\widetilde{N},((r,q);(s,p))}\left({}^{m}E\right)\right)_{m=0}^{\infty}$ is a π_1-holomorphy type.

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It is not difficult to prove that the sequence $\left(\mathcal{P}_{\widetilde{N},((r,q);(s,p))}\left({}^m E\right)\right)_{m=0}^{\infty}$ is a π_1-holomorphy type.

Theorem

Let E' be separable and $(\mathcal{P}_{\Theta}(^{m}E))_{m=0}^{\infty}$ be a π_1-holomorphy type from E to $\mathbb{C}$. Then every convolution operator on $\mathcal{H}_{\Theta b}(E)$ which is not a scalar multiple of the identity is hypercyclic.

Corollary

If E' is separable, then every convolution operator on $\mathcal{H}_{\widetilde{N}b,((r,q);(s,p))}(E)$ which is not a scalar multiple of the identity is hypercyclic.

Theorem

Let E' be separable and $(\mathcal{P}_\Theta({}^m E))_{m=0}^\infty$ be a π_1-holomorphy type from E to $\mathbb{C}$. Then every convolution operator on $\mathcal{H}_{\Theta b}(E)$ which is not a scalar multiple of the identity is hypercyclic.

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Thank you very much!!!