

# MASDOC summer school 'Topics in renormalisation group and regularity structures'

Monday 11 May - Friday 15 May 2015

Organiser: Stefan Adams

**David Brydges** (UBC Vancouver): *An introduction to the renormalisation group for probabilists*

Abstract: The  $\varphi^4$  model on the lattice  $\mathbb{Z}^d$  is a variation on the Ising model: instead of requiring, for every lattice site  $x$ , that the spin  $\varphi_x$  be plus or minus one,  $\varphi_x$  is real valued, but concentrated by a weight  $\exp[-g(\varphi_x^2 - 1)^2]$  near  $\pm 1$ . Similarly models where every spin  $\varphi_x$  has values in a sphere  $S^{n-1}$  have  $(\|\varphi\|^2 - 1)^2$  variants where  $\varphi$  has values in  $\mathbb{R}^n$ . There is even a precise mathematical definition of the  $n = 0$  component model which turns out to be a natural model for self-repelling walk. My colleagues Roland Bauerschmidt and Gordon Slade and I have recently used a rigorous version of the Wilson renormalisation group to prove that, on the four dimensional lattice, some of the critical exponents of these  $\|\varphi\|^4$  models have the precise logarithmic corrections predicted by theoretical physics. In these lectures I will explain the important ideas and some of the background in physics using the one component  $\varphi^4$  model on a special lattice called the hierarchical lattice as the initial example, and then passing to the models on  $\mathbb{Z}^d$ .

**Martin Hairer** (Warwick): *Regularity structures*

**Antti Kupiainen** (Helsinki): *Renormalization Group and Stochastic PDE's*

Abstract: We develop a Renormalization Group (RG) approach to the study of existence and uniqueness of solutions to stochastic partial differential equations driven by space-time white noise. The idea of the RG is to construct the solution of the SPDE scale by scale in space time. One solves the equation for smallest scales which leads to a "renormalized" equation for the remaining scales. Repeating this procedure one gets a sequence of effective equations on successive space time scales which can be viewed as a dynamical system in the "space of equations". In order to control this flow of equations one needs to renormalize some of the parameters in the original equation i.e. to add terms that diverge as a short scale regularization is removed. As examples we discuss the KPZ equation, the  $\phi_3^4$ -model and the Sine-Gordon model.

**Gordon Slade** (UBC Vancouver): *Recent applications of the renormalisation group to critical phenomena*

Abstract: We discuss recent results obtained using the renormalisation group method. The results include proofs of existence of logarithmic scaling factors in the critical behaviour of 4-dimensional multi-component spin models and weakly self-avoiding walk, and were obtained in collaborations with David Brydges, Roland Bauerschmidt, and Alexandre Tomberg. This provides a context for the course of David Brydges, which will explain some of the ideas in the renormalisation group method used to prove these results.

## Contributed talks

**Ajay Chandra** (Warwick): *A non-trivial scale invariant QFT with an anomalously scaling composite field*

Abstract: I will discuss methods and results for the construction and analysis of a hierarchical model which is based on an analog of the Wilson-Fisher fixed point studied earlier by Brydges, Mitter, and Scoppola. This is joint work with Abdelmalek Abdesselam and Gianluca Guadagni.

**Dirk Erhard** (Warwick): *On a scaling limit of the three-dimensional parabolic Anderson model with exclusion interaction*

Abstract: This talk is about the parabolic Anderson equation  $\partial u(x, t)/\partial t = \Delta u(x, t) + [\xi(x, t) - \rho]u(x, t)$ ,  $x \in \mathbb{Z}^d$ ,  $t \geq 0$ . Here,  $\Delta$  is the discrete Laplacian and the  $\xi$ -field is a stationary and ergodic dynamic random environment with mean  $\rho$  that drives the equation. The solution of the parabolic Anderson equation describes the evolution of a field of particles performing independent simple random walks with binary branching: particles jump at rate  $2d$ , split into two at rate  $\xi \vee 0$ , and die at rate  $-\xi \vee 0$ . I will focus on the case where  $\xi$  is given in terms of a simple symmetric exclusion process, i.e.,  $\xi$  can be described by a field of random walks that move independently from each other subject to the rule that no two random walks are allowed to occupy the same place at the same time. I will discuss the behaviour of the equation when time and space are suitably scaled. It turns out that in dimension two and three a renormalisation has to be carried out in order to see a non-trivial limit. This is joint work in progress with Martin Hairer.

**Cyril Labbe** (Warwick): *Singular stochastic PDEs on unbounded domains*

Abstract: So far, the SPDEs constructed with the theory of regularity structures have been considered on a torus. I will explain how one can adapt the theory to construct the solutions to some singular SPDEs on the whole space  $\mathbb{R}^d$  (PAM on  $\mathbb{R}^2$  and  $\mathbb{R}^3$ , SHE on  $\mathbb{R}$ ). Additionally, this construction allows one to start these SPDEs from a Dirac mass at the initial time. This is a joint work with Martin Hairer.

**Hao Shen** (Warwick): *Dynamical sine-Gordon equation*

Abstract: We will briefly review the sine-Gordon model in two space dimensions, especially the earlier work on this model using renormalisation group methods. We then discuss our recent result (joint with M. Hairer) on local well-posedness of the stochastic PDE counterpart of this model at high temperature.

**Weijun Xu** (Warwick): *Weak universality of  $\Phi_3^4$*

Abstract: The dynamical  $\Phi_3^4$  is widely considered as a universal model for magnetic fluctuations near critical temperature. In this talk, we will see that a large class of microscopic models with symmetric potential do rescale and converge to  $\Phi_3^4$ . The interesting feature is that the coefficient of the limiting equation is different from one's naive guess, and the renormalisation can be interpreted as a slight shift from critical point. On the other hand, if there is a weak asymmetry in the potential, then after re-centering, they just rescale to the heat equation with an additional linear term. Joint work with Martin Hairer.

## Programme

**Question session:** time for informal discussion with the lecturer on questions/examples/open problems.

### Monday, 11th May 2015; room MS.03

09:30 - 11:00	<b>Martin Hairer</b> REGULARITY STRUCTURES - I
11:00 - 11:30	Coffee - Common room
11:30 - 13:00	<b>Gordon Slade</b> RECENT APPLICATIONS OF THE RENORMALISATION GROUP TO CRITICAL PHENOMENA
13:00 - 15:00	Lunch - Common room
15:00 - 15:45	<b>Martin Hairer</b> REGULARITY STRUCTURES - II
15:45 - 16:15	Tea - Common room
16:15 - 17:00	<b>Dirk Erhard</b> ON A SCALING LIMIT OF THE THREE-DIMENSIONAL PARABOLIC ANDERSON MODEL WITH EXCLUSION INTERACTION
17:00 - 17:45	<b>Hao Shen</b> DYNAMICAL SINE-GORDON EQUATION
18:00 -	reception (wine & cheese with VC, followed by dinner in common room)

### Tuesday, 12th May 2015; room MS.03

09:30 - 11:00	<b>Antti Kupiainen</b> RENORMALIZATION GROUP AND STOCHASTIC PDE'S - I
11:00 - 11:30	Coffee - Common room
11:30 - 13:00	<b>Martin Hairer</b> REGULARITY STRUCTURES - III
13:00 - 15:00	Lunch - Common room
15:00 - 15:45	<b>David Brydges</b> AN INTRODUCTION TO THE RENORMALISATION GROUP FOR PROBABILISTS - I
15:45 - 16:15	Tea - Common room
16:15 - 17:00	<b>Question session with Brydges/Slade</b>
17:00 - 17:45	<b>Question session with Hairer</b>

**Wednesday, 13th May 2015; room MS.03**

09:30 - 11:00	<b>David Brydges</b> AN INTRODUCTION TO THE RENORMALISATION GROUP FOR PROBABILISTS - II
11:00 - 11:30	Coffee - Common room
11:30 - 13:00	<b>Antti Kupiainen</b> RENORMALIZATION GROUP AND STOCHASTIC PDE's - II
13:00 - 14:15	Lunch - Common room
14:15 - 15:00	<b>Question session with Kupiainen</b>
15:00 - 15:45	<b>Cyril Labbe</b> SINGULAR STOCHASTIC PDEs ON UNBOUNDED DOMAINS
15:45 - 16:00	Tea - Common room
16:00 - 17:00	<b>Chris Burdzy (U Washington) (probability seminar)</b> TWIN PEAKS
17:45 -	dinner common room

**Thursday, 14th May 2015; room B3.02**

09:30 - 11:00	<b>Martin Hairer</b> REGULARITY STRUCTURES - IV
11:00 - 11:30	Coffee - Common room
11:30 - 13:00	<b>David Brydges</b> AN INTRODUCTION TO THE RENORMALISATION GROUP FOR PROBABILISTS - III
13:00 - 15:00	Lunch - Common room
15:00 - 15:45	<b>Question session with Hairer</b>
15:45 - 16:15	Tea - Common room
16:15 - 17:00	<b>Weijun Xu</b> WEAK UNIVERSALITY OF $\Phi_3^4$
17:00 - 17:45	<b>Question session with Brydges/Slade</b>

**Friday, 15th May 2015; room MS.03**

09:30 - 11:00	<b>Martin Hairer</b> REGULARITY STRUCTURES - V
11:00 - 11:30	Coffee - Common room
11:30 - 13:00	<b>David Brydges</b> AN INTRODUCTION TO THE RENORMALISATION GROUP FOR PROBABILISTS - IV
13:00 - 14:15	Lunch - Common room
14:15 - 15:00	<b>Ajay Chandra</b> A NON-TRIVIAL SCALE INVARIANT QFT WITH AN ANOMALOUSLY SCALING COMPOSITE FIELD
15:00 - 15:45	<b>Question session with Brydges/Slade</b>
15:45 - 16:00	Tea - Common room
16:00 - 17:00	<b>Wendelin Werner (ETH) (colloquium)</b> AN ELEMENTARY APPROACH TO THE RENORMALIZATION FLOW
17:00 -	wine & cheese common room