Scaling Cascades in Complex Systems

Non-Gaussian data assimilation via a localized hybrid ensemble transform filter

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Universität Potsdam/ University of Reading

Warwick, February 22nd 2016

Outline

Motivation

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Hybrid Scheme

- Ensemble Transform Particle Filter [Reich and Cotter, 2015]
- Hybrid Ensemble Transform Filter [Chustagulprom et al., 2015]
- Example: Single 1D Assimilation Step

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Non-spatially extended systems

- Likelihood splitting strategies
- Ensemble Inflation and Particle Rejuvenation
- Example: Lorenz 63 model

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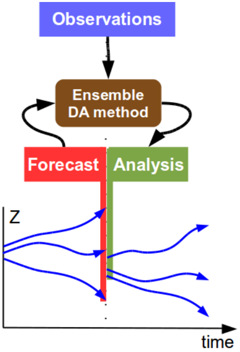
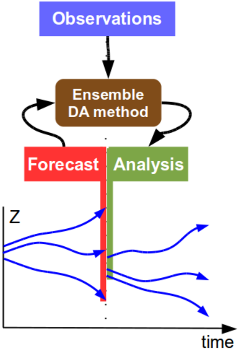
Spatially extended systems

- Localization
- Example: Lorenz 96 model

Sequential Data Assimilation Strategies

Scaling Cascades in Complex Systems





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  - + Robust
  - + Computationally affordable
  - − Inconsistent for non-Gaussian PDFs

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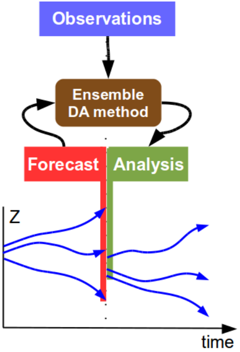

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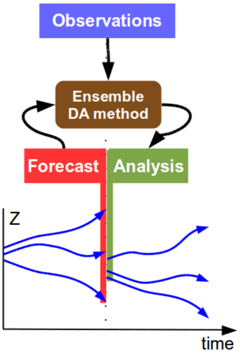

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- Hybrid schemes
  - \+ trade-off between accuracy and stability

Outline

Scaling Cascades in Complex Systems

Ensemble Transform Particle Filter [Reich and Cotter, 2015]

Scaling Cascades in Complex Systems

Importance weights: $$w_i = \frac{\exp\left(-\frac{1}{2}(Hz_i^{\mathrm{f}} - y_{\mathrm{obs}})^{\mathrm{T}} R^{-1} (Hz_i^{\mathrm{f}} - y_{\mathrm{obs}})\right)}{\sum_{j=1}^{M} \exp\left(-\frac{1}{2}(Hz_j^{\mathrm{f}} - y_{\mathrm{obs}})^{\mathrm{T}} R^{-1} (Hz_j^{\mathrm{f}} - y_{\mathrm{obs}})\right)} \quad (1)$$

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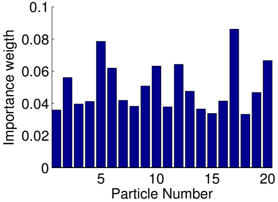

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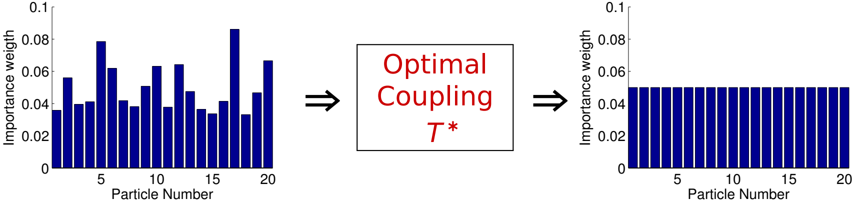

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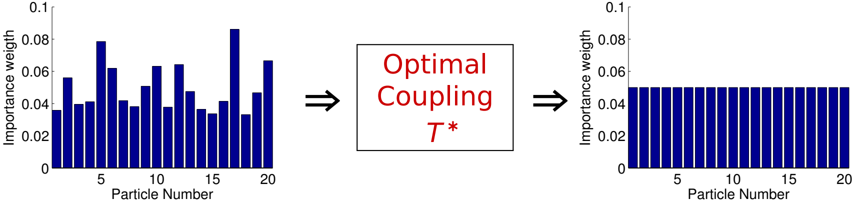

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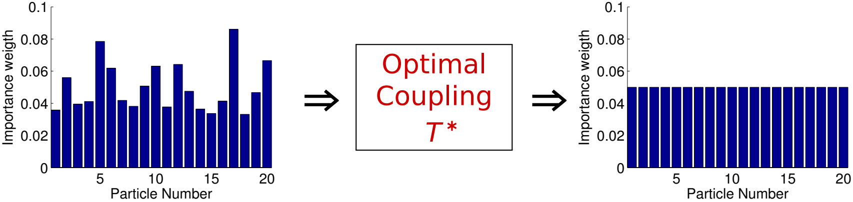

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which minimizes the cost function: $$J(\{t_{ij}\}) = \sum_{i,j=1}^{M} t_{ij} \| z_i^{\mathrm{f}} - z_j^{\mathrm{f}} \|^2 \quad (3)$$

Outline

Likelihood function splitting

$$\pi_Y(y_{\text{obs}}|z) \propto$$
$$\exp\left(-\ \frac{\alpha}{2}\ \ (Hz - y_{\text{obs}})^{\mathrm{T}} R^{-1} (Hz - y_{\text{obs}})\right) \times$$
$$\exp\left(-\frac{1-\alpha}{2} (Hz - y_{\text{obs}})^{\mathrm{T}} R^{-1} (Hz - y_{\text{obs}})\right)$$

with bridging parameter $\alpha \in [0, 1]$

Scaling Cascades in Complex Systems

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Related work

Ensemble Kalman Particle Filter (EKPF) [Frei and Künsch, 2013]: bridges the EnKF with perturbed observations and a SIR particle filter.

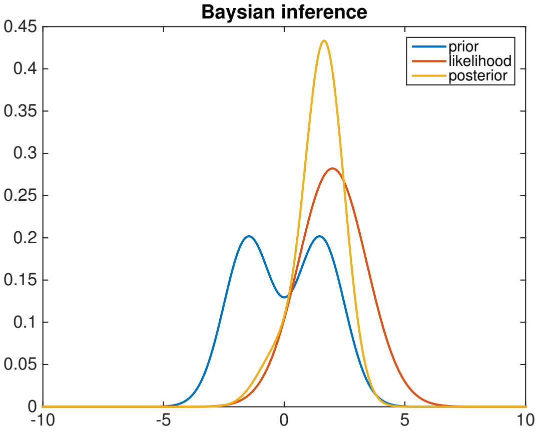
Outline

Bayesian Inference

for bimodal prior

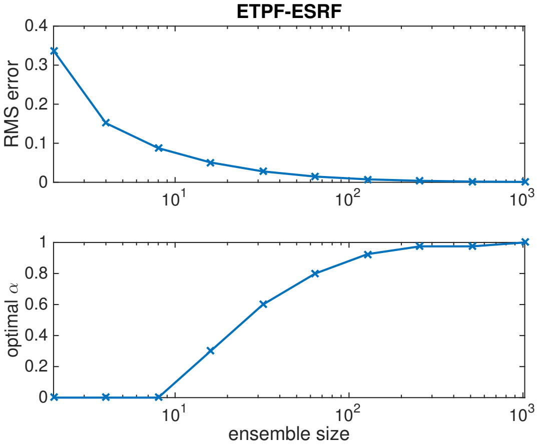
and

Gaussian likelihood



Example: Single 1D Assimilation Step

ETPF-ESRF
performance vs
ensemble size
for optimally chosen
bridging parameter
($\alpha = 0$: EnKF
$\alpha = 1$: ETPF)



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- **Adaptive**:
  keeping ETPF effective sample size

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constant

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Ensemble Inflation and Particle Rejuvenation

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Particle rejuvenation is applied to the analysis ensemble:

$$z_j^{\mathrm{a}} \rightarrow z_j^{\mathrm{a}} + \sum_{i=1}^{M} (z_i^{\mathrm{f}} - \bar{z}^{\mathrm{f}}) \frac{\beta \xi_{ij}}{\sqrt{M-1}} \tag{5}$$

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- β: Rejuvenation parameter
- ξ_{ij}'s: i.i.d. Gaussian random variables with mean zero and variance one
- $\sum_{j=1}^{M} \xi_{ij} = 0$ so as to preserve the ensemble mean

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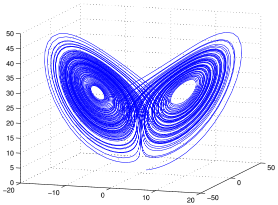
Scaling Cascades in Complex Systems

Example: Lorenz 63 model

$$\dot{x}_1 = 10(x_2 - x_1)$$
$$\dot{x}_2 = x_1(28 - x_3) - x_2$$
$$\dot{x}_3 = x_1 x_2 - 8/3 x_3$$



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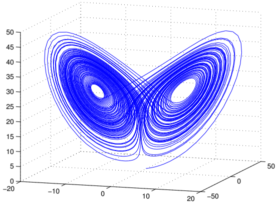
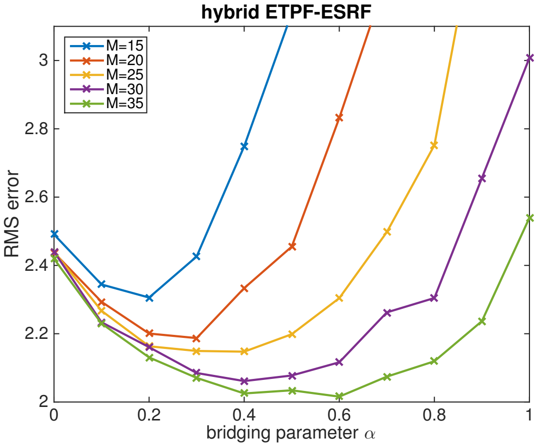


Perfect model DA experiments:

- Implicit midpoint method with time step $\Delta t = 0.01$.
- x_1 observed every 12 time-steps with error variance $R = 8$
- Particle rejuvenation $\beta = 0.2$
- 100,000 assimilation cycles
- OTP solved using FastEMD algorithm

Example: Lorenz 63 model

Skill dependence on bridging parameter for different ensemble sizes using fixed likelihood splitting



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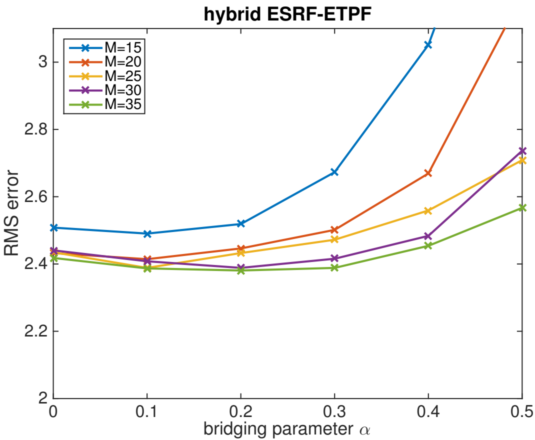
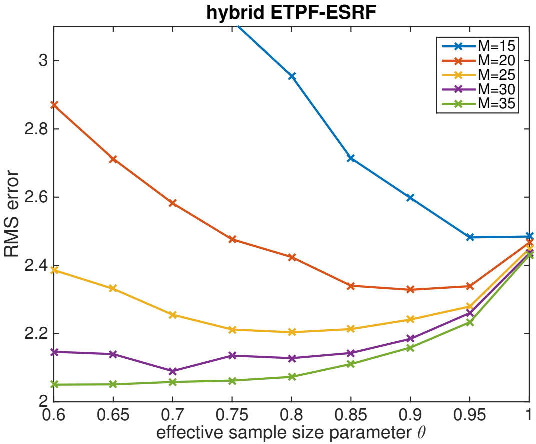
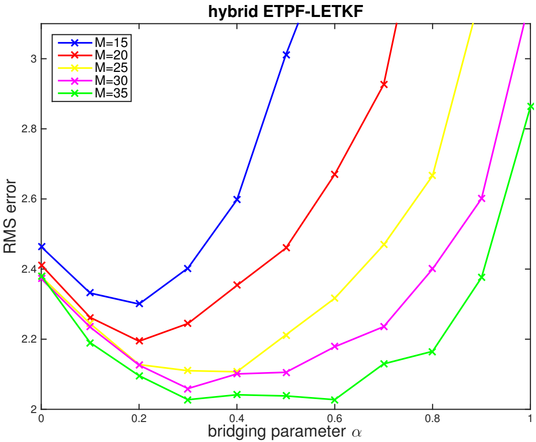


Example: Lorenz 63 model

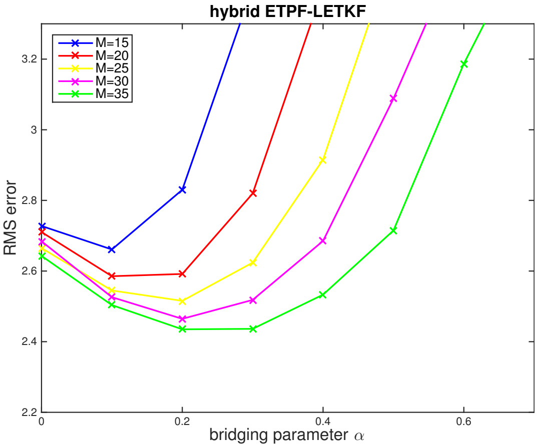
Skill dependence on effective ETPF sample size for different ensemble sizes using adaptive likelihood splitting



Skill dependence on bridging parameter for different ensemble sizes using fixed likelihood splitting and wrong parameter values (10.2, 28.2, 2.5)



Skill dependence on bridging parameter for different ensemble sizes using fixed likelihood splitting and wrong parameter values (10.3, 28.4, 2.9)



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R-Localization

Localized measurement error covariance elements:

$$r_{qq}^{LOC}(x_k) = \frac{r_{qq}}{\rho\left(\frac{\|x_k - x_q\|}{R_{\text{loc}}}\right)},$$

with ρ a compactly supported tempering function, e.g., Gaspari-Cohn function.

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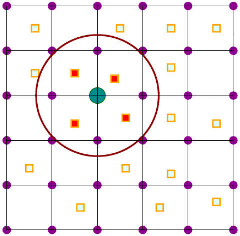

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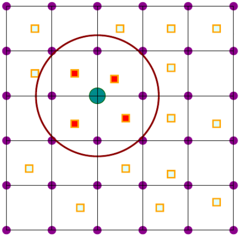

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- Local updates for each gridpoint
- Directly applicable to Kalman Filters
- Directly applicable to ETPF [Cheng and Reich, 2015] (local weights and transport cost)

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Hybrid Scheme
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- Hybrid Ensemble Transform Filter [Chustagulprom et al., 2015]
- Example: Single 1D Assimilation Step

Non-spatially extended systems
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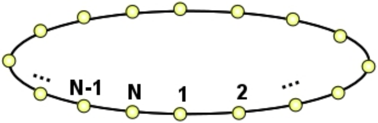
Conclusions and Prospect

Example: Lorenz 96 model

$$\dot{x}_j = (x_{j+1} - x_{j-2})x_{j-1} - x_j + F,$$
$$x_j = x_{j+N}$$

where $F = 8$ and $N = 40$.



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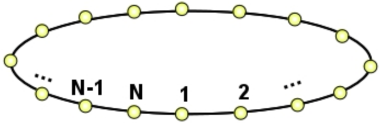
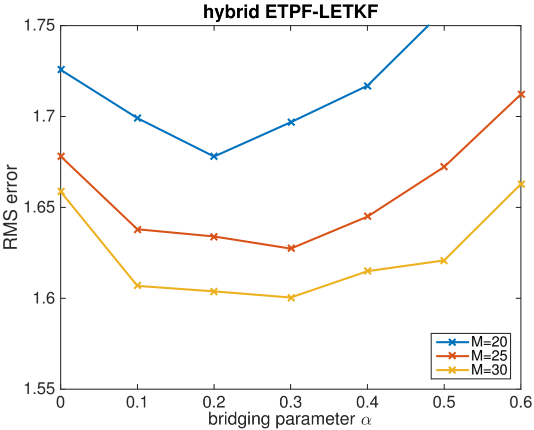


Perfect model DA experiments:

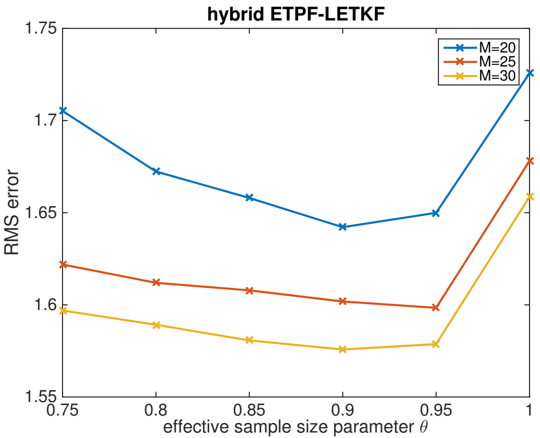
- Implicit midpoint method with time step $\Delta t = 0.005$.
- Odd variables observed every 22 time-steps
- Particle rejuvenation $\beta = 0.2$
- Localisation radius is $R_{\text{loc}} = 4$
- 50,000 assimilation cycles
- OTP solved using FastEMD algorithm

Example: Lorenz 96 model

Skill dependence on bridging parameter for different ensemble sizes using fixed likelihood splitting



Skill dependence on effective ETPF sample size for different ensemble sizes using adaptive likelihood splitting



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  Other adaptation criteria must be tested
- Proposed hybrid filter can be combined with R-localization.

Ongoing Research and Prospect

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- Impact of systematic model errors on hybrid scheme
- Hybrid ensemble transform smoother

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Thanks!

Scaling Cascades in Complex Systems

Mathematisches Forschungsinstitut Oberwolfach

Member of the Leibniz Association

Data Assimilation: The Mathematics of Connecting Dynamical Systems to Data

Organizers: Jana de Wiljes, Potsdam
Sebastian Reich, Potsdam and Reading
Andrew Stuart, Warwick

Date (ID): 15 – 21 May 2016 (1620a)

Deadline: 13 March 2016

Recent Advances on the Global Nonlinear Stability

Mathematical Theory of Evolutionary Fluid-Flow Structure Interactions

Organizers: Barbara Kaltenbacher, Klagenfurt
Igor Kukavica, Los Angeles
Irena Lasiecka, Memphis
Roberto Triggiani, Memphis

Date (ID): 20 – 26 November 2016 (1647b)

Deadline: 18 September 2016