Metastability in the reversible inclusion process II

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joint work with Sander Dommers & Cristian Giardinà

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Outline

Inclusion process

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$$\mathcal{L}f(\eta) = \sum_{x,y \in S} r(x,y)\eta_x(d_N + \eta_y)\left(f(\eta^{x,y}) - f(\eta)\right) \qquad \text{generator}$$

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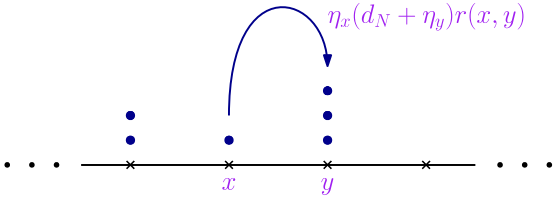
  - $r(x,y) \geq 0$ transition rates of a m-reversible RW on S
  - $d_N > 0$ constant tuning the rates of the underlying RW

Example:



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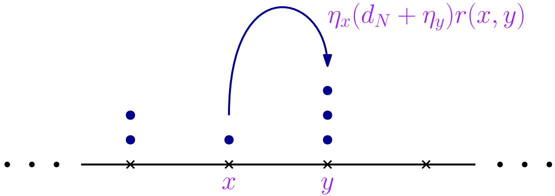

Remarkable facts:
Under suitable hypotheses, e.g. letting $N \to \infty$ and $d_N \longrightarrow 0$, the model displays condensation (*particles concentrate on single site*) and metastable behavior (*condensate may appears in different sites of S*)

[Grosskinsky, Redig, Vafayi 2011], [Chleboun 2012].

Metastable behavior

- **Symmetric IP** [Grosskinsky, Redig, Vafayi 2013]
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    $S^* = \arg\max\{m(x) : x \in S\}$
    $m(x)$ rev. measure for r s.t. $\max\{m(x) : x \in S\} = 1$

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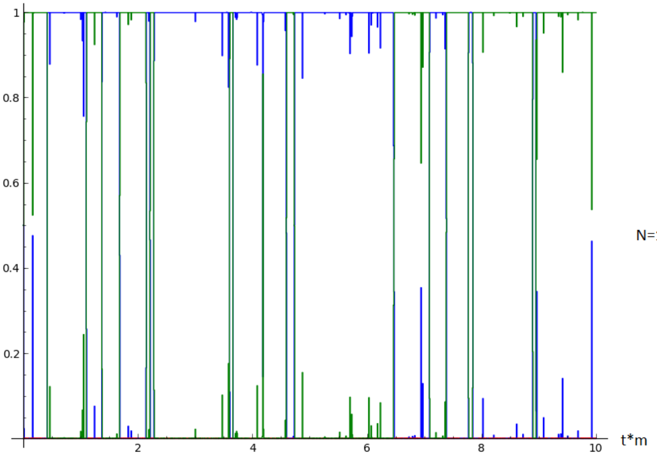
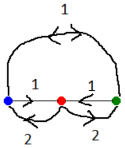
Main goal: Characterization of *further metastable timescales*, and *motion of the condensate between traps*.

Simulations and heuristics

First example





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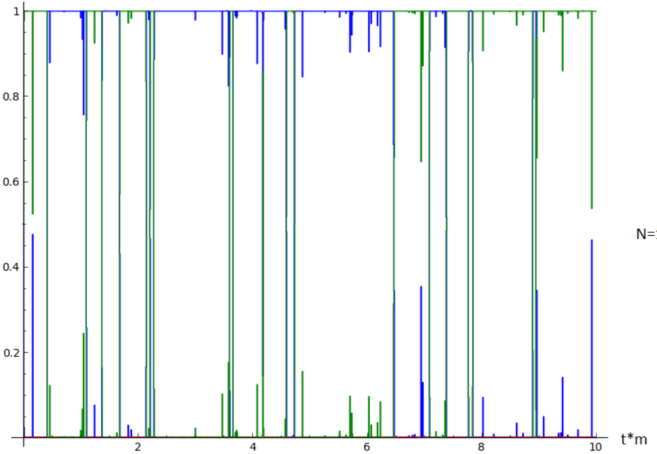

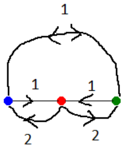

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On the timescale $1/d_N$, the condensate moves between sites **maximizing the measure** m.

Simulations and heuristics

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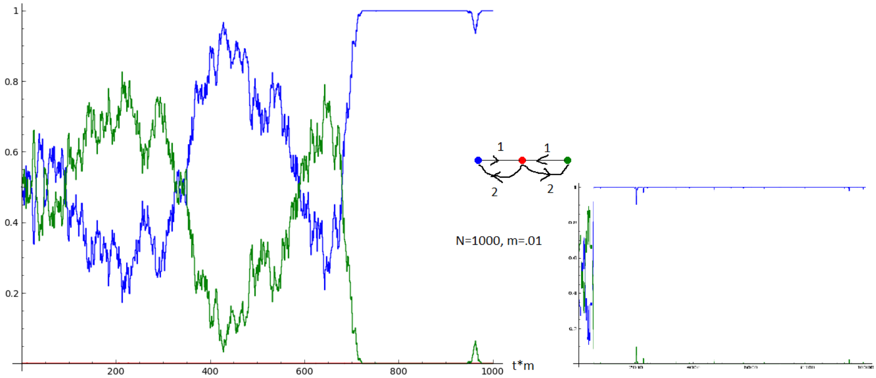

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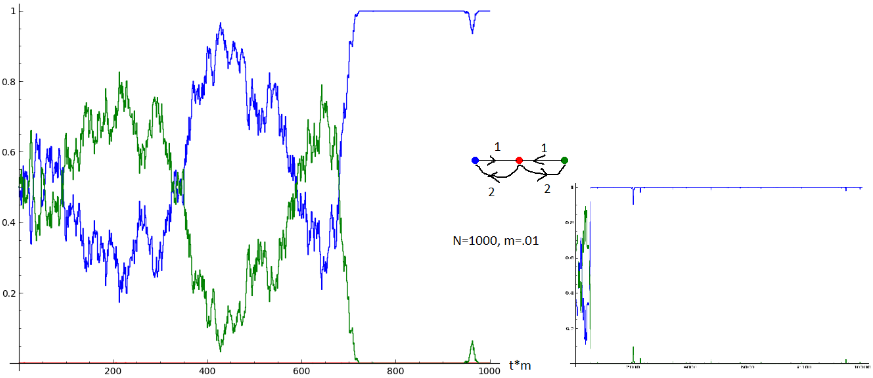

On the timescale $1/d_N$, condensation takes place *(though at a long scaled time)*, while once created, the condensate remains trapped for very long time on a vertex of S^*.

Metastable timescale(s)

Assume $\{r(x,y)\}_{x,y\in S^*}$ is reducible, and let $C_1,\ldots,C_m$, $m\geq 2$, the connected components of $(S^*, r_{|_{S^*}})$

$$S^* = \bigcup_{j=1}^{m} C_j \ , \ \ C_i \cup C_j = \emptyset, \text{ for } i \neq j$$

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- Define a new set of metastable sets $\mathcal{E}_1,\ldots,\mathcal{E}_m$:

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- Verify the hypotheses H_0, H_1 and H_2 of [Beltrán, Landim, 2010] $\longrightarrow$ **compute capacities** $\text{Cap}_N(\mathcal{E}_i,\mathcal{E}_j)$.

Capacity versus Metastability

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Its definition comes from correspondence btw reversible dynamics and electrical networks. If $A, B \subset \Omega$, and μ reversible measure

$$\mathrm{Cap}(A, B) := \sum_{\eta \in A} \mu(\eta) \mathbb{P}_\eta[\tau_B < \tau_A^+]$$

Advantages:

I Fact. If A e B are metastable sets, the mean metastable time btw A e B is (**roughly**) $\sim \mu(A)/\text{Cap(A,B)}$.

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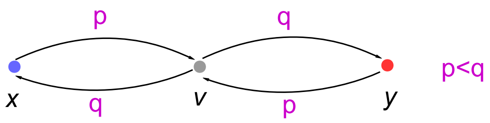
On the other hand, unless very simple systems, the precise computation of capacities may be complicated.
$\longrightarrow$ we start from a **simple IP dynamics** to understand **traps**.

Analysis of a 3- sites IP

Consider the IP defined through the underlying RW on $S = \{v, x, y\}$ with transition rates s.t.

$$\begin{cases} r(y,x) = r(x,y) = 0 \\ m(x) = m(y) = 1 > m(v) \end{cases}$$

$\Longrightarrow \eta^{N,x}, \eta^{N,y}$ are disconnected components of $(S^*, r_{|S^*})$



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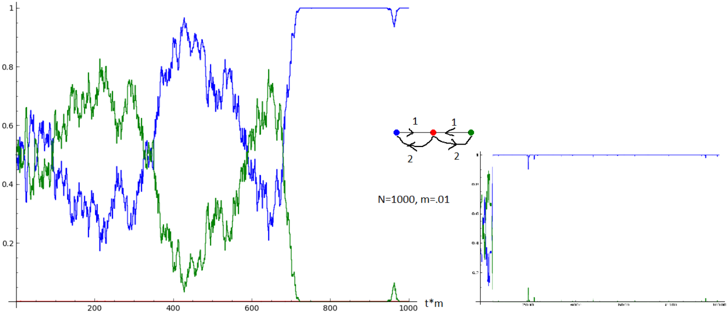

Capacities for the 3-sites IP

Proposition 1.

In the above notation and for $d_N \log N \to 0$,

$$\lim_{N\to\infty} \frac{N}{d_N^2} \cdot \mathrm{Cap}_N(\eta^{N,x}, \eta^{N,y}) = \left(\frac{1}{r(v,x)} + \frac{1}{r(v,y)}\right)^{-1} \cdot \frac{m(v)}{1-m(v)}$$

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Following [Beltrán, Landim 2010], hypothesis H_0 is verified:

$$\lim_{N\to\infty} \frac{N}{d_N^2} p_N(\eta^{N,x}, \eta^{N,y}) = \left(\frac{1}{r(v,x)} + \frac{1}{r(v,y)} \right)^{-1} \cdot \frac{m(v)}{1-m(v)} =: p^{(2)}(x,y)$$

Dynamics of the condensate in the 3-sites IP

As a consequence (hypotheses H_1 and H_2 are easily verified), for

$$X_N(t) = \sum_{z \in S^*} z \mathbb{1}_{\{\eta_z(t) = N\}}$$

Proposition 2.

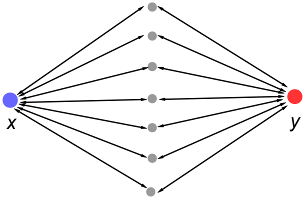
Let $d_N \log N \to 0$ as $N \to \infty$, and $\eta_z(0) = N$ for some $z \in S^*$. Then

$$X_N(t \cdot N/d_N^2) \text{ converges weakly to } x(t) \qquad \text{as } N \to \infty$$

where $x(t)$ is a Markov process on S^* with symmetric rates $p^{(2)}(x, y)$.

Easy extension I

Consider the IP defined through the following underlying RW



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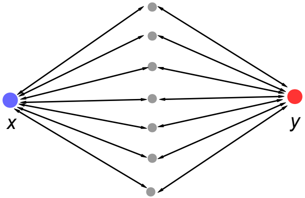

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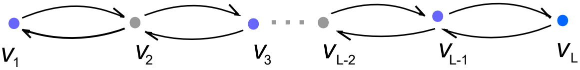

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For $S^* = \bigcup_{j \in J} C_j$, C_j connected component of $(S^*, R_{|_{S^*}})$. For $I \subsetneq J$

$$S_1^* = \bigcup_{i \in I} C_i \,, \quad S_2^* = \bigcup_{i \in J \setminus I} C_i \quad \text{and} \quad \mathcal{E}_1 = \bigcup_{x \in S_1^*} \eta^{N,x} \,, \quad \mathcal{E}_2 = \bigcup_{y \in S_2^*} \eta^{N,y}$$

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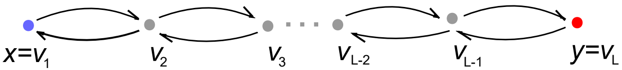
Then, for $d_N \log N \to 0$,

$$\lim_{N \to \infty} \frac{N}{d_N^2} \cdot \mathrm{Cap}_N(\mathcal{E}_1, \mathcal{E}_2) = \sum_{\substack{x \in S_1^* \\ y \in S_2^*}} \sum_{v \in S \setminus S^*} \left(\frac{1}{r(v,x)} + \frac{1}{r(v,y)} \right)^{-1} \cdot \frac{m(v)}{1 - m(v)}$$

Analysis of a IP on on $\{1, 2, \ldots, L\}$, $L \geq 4$

Let $S = \{x = v_1, v_2, \ldots, v_L = y\}$ with $L \geq 4$ and consider the IP defined through the following RW



with transition rates s.t. $S^* = \{x, y\}$

Capacities for the IP on $\{1, 2, \ldots, L\}$

Proposition 3.

In the above notation and for $d_N \log N \to 0$*,*

$$\lim_{N\to\infty} \frac{N^2}{d_N^{L-1}} \cdot \mathrm{Cap}_N(\eta^{N,x}, \eta^{N,y}) \geq c > 0$$

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Upper bound on these capacities is still under investigation, but rough (not-matching) estimates show

$$\mathrm{Cap}_N(\eta^{N,x}, \eta^{N,y}) \leq d_N^3/N^2 \ll d_N^2/N \longrightarrow \text{third timescale}$$

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Are there further metastable timescales associated to length of minimal path btw connected components? (to investigate)

Comment on the results

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We conjecture the existence of many (at least three) metastable timescales:

Consider the IP on a finite set S with reversible rates $r(x, y)$.

Let $\left\{C_1^{(k)}, \ldots C_{m(k)}^{(k)}\right\}$, for $k = 1, 2$, be partitions of S^*, of cardinality $0 \leq m(k) \leq |S^*|$, such that

- $C_j^{(1)}$'s are the connected components of $(S^*, r_{|_{S^*}})$
- $C_j^{(2)}$' are s.t $x \in S^*$ if $d(x, S^* \setminus \{x\}) \leq 2$ (d graph distance on (S, r)).

Let $T_1 = N/d_N^2$ and $T_2 = N^2/d_N^3$, and

$$\mathcal{E}_j^{(k)} = \bigcup_{x \in C_j^{(k)}} \eta^{N,i}\,, \quad \forall j\,,\ k = 1, 2$$

(metastable sets at timescale T_k)

$$X_N^{(k)}(t) = \sum_{j=1}^{m^{(k)}} j \mathbb{1}_{\left\{\eta(t) \in \mathcal{E}_j^{(k)}\right\}}\,,\ k = 1, 2$$

(processes projected on metastable sets)

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Following the techniques of [Beltrán, Landim, 2010-2014],

Conjecture 1.

Let $d_N \log N \to 0$ as $N \to \infty$, and $\eta_z(0) = N$ for some $z \in S^*$. Then, for $k = 1, 2$

$$X_N^{(k)}(t \cdot T_k) \text{ converges weakly to } x^{(k)}(t) \qquad \text{as } N \to \infty$$

where $x^{(k)}(t)$ is a Markov process on $\{1, \dots, m(k)\}$ with $x^{(k)}(0) = \sum_{j=1}^{m(k)} j \mathbb{1}_{\{\eta(0) \in \mathcal{E}_j^{(k)}\}}$.

Upper and lower bound on capacities

Recall that by the Dirichlet principle

$$\mathrm{Cap}_N(A,B) = \inf_{f:f_{|A}=1, f_{|B}=0} \{D_N(f)\}$$

where the Dirichlet form of the IP is

$$D_N(f) = \frac{1}{2}\sum_{\eta} \mu_N(\eta) \sum_{x,y\in S} R(\eta,\eta^{x,y})\,(f(\eta^{x,y}) - f(\eta))^2$$

with

$$\mu_N(\eta) = \frac{1}{Z_N}\prod_{x\in S} m(x)^{\eta_x} w_N(\eta_x)$$

$$w_N(k) = \frac{\Gamma(k+d_N)}{k!\Gamma(d_N)}$$

$$R(\eta,\eta^{x,y}) = r(x,y)\eta_x(\eta_y + d_N)$$

(see computation at the blackboard)

Future projects

Formation of the condensate in the TASIP
(joint with S. Dommers, S. Grosskinsky)

Consider the IP on $S = \mathbb{Z}/L\mathbb{Z}$ in the totally asymmetric case, with $r(x, x-1) = 0 \ \forall x \in S$ (jumps to the right).

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Main goal: Provide rigourous arguments to compute the condensation time.

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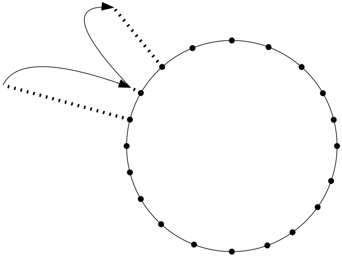

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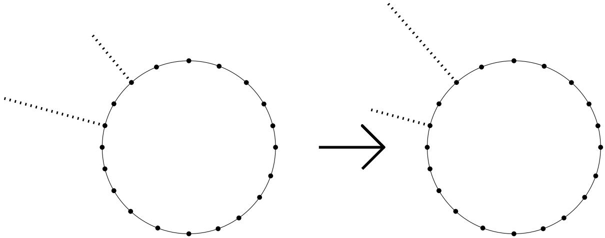
II step *The largest condensate loses $O(m_1 - m_2)$ particles before they are absorbed by the other condensate. At the hand, the two condensates had roughly exchanged mass.*



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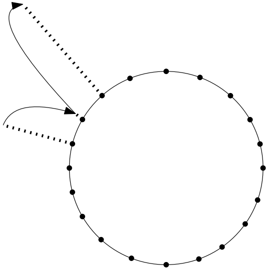
This can be represented by a cyclic mechanism of 4 main steps:
III step *As in the first step, the smallest condensate (now on the left) loses one particle which is absorbed by the other condensate. Difference $m_2 - m_1$ increases.*



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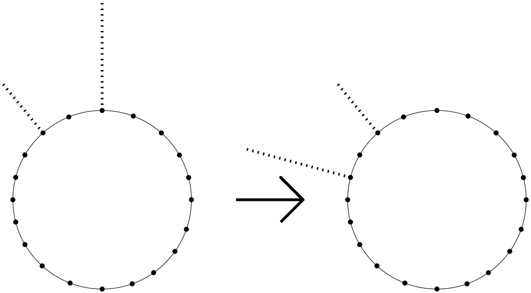
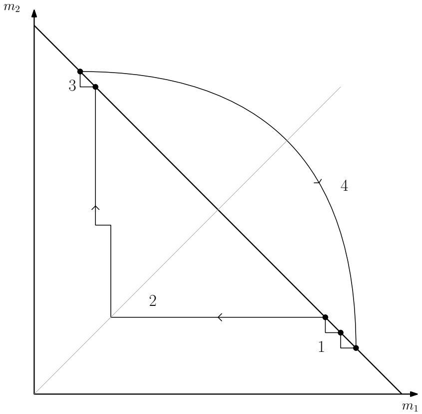
By simulations, the slower dynamical step along nucleation is the union of two half-condensates, of size $m_1 > m_2$.
This can be represented by a cyclic mechanism of 4 main steps:
IV step *The largest condensate (on the right) moves faster until reaches the configuration at step I .*



Reduction to a random walk

$$\begin{cases} m_1 = \#\text{particles of condensate on the left} \\ m_2 = \#\text{particles of condensate on the right} \end{cases} \quad \text{with}$$

$m_1 + m_2 = N$.



Conclusions and open problems

We studied the reversible IP on a finite set in the limit $N \to \infty$ and for $d_N \to 0$ with $d_N \log N \to 0$ by martingale approach:

- We derive the dynamics of the condensate at timescale $\sim 1/d_N$;
- We prove the existence of a longer metastable timescale $\sim N/d_N^2$ and derive dynamics of the condensate in simple IP processes (1D RW);
- We conjecture the existence of longer metastable timescales (at least one) $\sim N^2/d_N^a$ with $a = a(r) \geq 3$ and possibly dependent on the shortest length of paths between components of $(S^*, r_{|S^*})$.

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Thank you for your attention!