

**2015-2016 Warwick EPSRC Symposium on  
Fluctuation-driven phenomena and large  
deviations**

**Random matrix theory and strongly correlated  
systems**

21 - 24 March 2016

Organisers: Oleg Zaboronski, Roger Tribe, Neil O'Connell, Yan Fyodorov, Gernot Akemann

**ABSTRACTS**

## Monday

Morning Session Chair: Gernot Akemann

Afternoon Session Chair: Mario Kieburg

### Arno Kuijlaars

*Propagation of singular behavior in UE and GUE sums*

I will discuss sums of Hermitian random matrices  $M + \sqrt{\tau}H$  where  $M$  is a matrix from a unitary ensemble of the form

$$\frac{1}{Z_n} e^{-n\text{Tr}V(M)}$$

and  $H$  is a scaled GUE matrix.

For very special choices of the potential  $V$  the eigenvalues of  $M$  has a density that vanishes at an interior point, or vanishes to higher order at an edge point. We show that the singular behavior persists for the eigenvalues of  $M + \sqrt{\tau}H$  for  $\tau$  up to a critical value  $\tau_{cr}$ .

In addition, the local scaling limits at the singular point continue to hold as well. This is joint work with Tom Claeys, Karl Liechty and Dong Wang

### Pierpaolo Vivo

*Universal fluctuation formulae for one-cut  $\beta$ -ensembles - with a combinatorial touch*

I discuss a recently obtained analytical formula for the covariance  $\text{Cov}(A,B)$  of two smooth linear statistics on the eigenvalues of a one-cut  $\beta$ -ensemble of random matrices. This allows to compute the generating function of the covariances of power traces for one-cut  $\beta$ -ensembles of random matrices in the limit of large matrix size. This formula depends only on the support of the spectral density, and is therefore universal for a large class of models. I briefly discuss the connection with the combinatorial problem of non-crossing pairings of 2 circles with 'k' and 'l' points on them, respectively.

In collaboration with Francesco Mezzadri and Fabio Deelan Cunden

[J. Phys. A: Math. Theor. 48, 315204 (2015); Phys. Rev. Lett. 113, 070202 (2014)]

### Khanh Duy Trinh

*On spectral measures of beta ensembles*

This talk concerns with the limiting behaviour of spectral measures of random Jacobi matrices of Gaussian, Wishart and MANOVA beta ensembles. We show that the spectral measures converge weakly to a limit distribution which is the semicircle distribution, Marchenko-Pastur distributions or the arcsine distribution, respectively. Regarding that convergence as the law of large number, a central limit theorem is then derived.

### Chris Joyner

*The probability distribution of spectral moments for the Gaussian beta-ensembles*

We derive the joint probability distribution of the first two spectral moments for the Gaussian beta-Ensembles random matrix ensembles in  $N$  dimensions for any  $N$ . This is achieved by making use of two complementary invariants of the domain where the spectral moments are defined. Our approach is significantly different from those employed previously to answer related questions and potentially offers new insights. We also discuss the problems faced when attempting to include higher spectral moments.

This is work together with U. Smilansky and T. Maciazek

## Tuesday

Morning Session Chair: Eugene Kanzieper (TBC)

Afternoon Session Chair: Uzy Smilansky

### Gregory Schehr

*Finite temperature free fermions and the Kardar-Parisi-Zhang equation at finite time*

I will consider a system of  $N$  one-dimensional free fermions confined by a harmonic well. At zero temperature ( $T=0$ ), it is well known that this system is intimately connected to random matrices belonging to the Gaussian Unitary Ensemble. In particular, the density of fermions has, for large  $N$ , a finite support and it is given by the Wigner semi-circular law. Besides, close to the edges of the support, the quantum fluctuations are described by the so-called Airy-Kernel (which plays an important role in random matrix theory). What happens at finite temperature  $T$ ? I will show that at finite but low temperature, the fluctuations close to the edge, are described by a generalization of the Airy kernel, which depends continuously on temperature. Remarkably, exactly the same kernel arises in the exact solution of the Kardar-Parisi-Zhang (KPZ) equation in 1+1 dimensions at finite time. I will also discuss recent results for fermions in higher dimensions.

### Nikos Zygoras

*Variants of geometric RSK correspondence and the multipoint distribution of the log-gamma polymer*

We extend Kirillov's geometric Robinson-Scheded-Knuth correspondence (gRSK) in two ways: one that replaces the input matrix by a general polygonal array and one which provides the geometric version of the polynuclear growth process (PNG). These variants of gRSK allow to write explicit integral formulae for the joint Laplace transform of point-to-point partition functions for the directed polymer model with log-gamma disorder. Under an assumption of convergence of certain series, we show that the joint law of the two point-to-point partition functions converges to the two point function of the Airy process. This is a joint work with Vu Lan Nguyen (Paris 7).

### Anna Maltsev

*Spectral properties of random fermionic models*

I will discuss our recent work on fermionic  $XY$  models with interactions that are taken to be i.i.d. Gaussian random variables. We focus on the ground state energy gap, density of states, and the local eigenvalue statistics in the case of all-to-all interactions. We also examine what happens as the number of interacting neighbors changes. This is based on a joint work with F. Mezzadri and F. Cunden.

### Maurice Duits

*Global fluctuations for non-colliding process*

In this talk I will discuss a recent approach for studying the global fluctuations for a class of non-colliding processes with determinantal correlations. The method is based on recurrence relations for the associated biorthogonal family of functions. The main results are Central Limit Theorems for multi-time linear statistics. The results show the universality of the Gaussian Free Field appearing in the global fluctuations for these models. Special attention will be given to Dyson's Brownian motion with one initial point and several endpoints. In that case, the underlying family is then given by multiple Hermite polynomials.

## **Maciej Nowak**

*Diffusion of characteristic polynomials and edge universality in non-hermitian random matrix models.*

We propose the mathematical framework for studying the evolution of non-hermitian random matrix ensembles, involving a new, hidden variable, which turns out to be crucial to unravel the dynamics of co-evolving eigenvalues and eigenvectors. We demonstrate this phenomenon on the example of complex Ornstein-Uhlenbeck process.

References:

- [1] Z. Burda, J. Grela, M. A. Nowak, W. Tarnowski and P. Warchoř, Phys. Rev. Lett. 114 (2014) 104102; Nucl. Phys. B897 (2015) 421.
- [2] J.-P. Blaizot, J. Grela, M. A. Nowak, W. Tarnowski and P. Warchoř, arXiv:1512.06599v2 [math-ph].
- [3] Y. Liu, M. A. Nowak and I. Zahed, arXiv:1602.02578v1 [hep-lat].

## **Ramis Movassagh**

*Eigenvalue Attraction*

Much work has been devoted to the understanding of the motion of eigenvalues in response to randomness. The folklore of random matrix analysis, especially in the case of Hermitian matrices, suggests that the eigenvalues of a perturbed matrix repel. We prove that the complex conjugate (c.c.) eigenvalues of a smoothly varying real matrix attract. We offer a dynamical perspective on the motion and interaction of the eigenvalues in the complex plane, derive their governing equations and discuss applications. C.c. pairs closest to the real axis, or those that are ill-conditioned, attract most strongly and can collide to become exactly real. We apply the results to the Hatano-Nelson model, random perturbations of a fixed matrix, real stochastic processes with zero-mean and independent intervals and discuss open problems.

## Wednesday

Morning Session Chair: Gregory Schehr

Afternoon Session Chair: Nina Snaith

### Igor Krasovsky

*Transition asymptotics for Toeplitz and Fredholm determinants*

We will discuss recent results on the asymptotic behaviour of certain Toeplitz and Fredholm determinants appearing in the theory of random matrices and related areas focusing on the situation where a transition between 2 different asymptotic regimes takes place. The talk is based on joint works with Th. Bothner, T. Claeys, P. Deift, and A. Its.

### Mihail Poplavskiy

*Asymptotic behaviour of the largest real eigenvalue for the real Ginibre ensemble*

We discuss the classical real Ginibre ensemble and the distribution of its biggest real eigenvalue. The distribution was recently obtained in the paper of B. Rider and C. Sinclair by using the classical method coming from C. Tracy and H. Widom papers. We present another elegant way, based on integrability of the model and skew-orthogonal polynomials technique, to obtain the distribution in a much shorter form. We also show how to simplify older answer to a new one by introducing Gaussian Brownian motion. Giving a pure probabilistic interpretation to the answer we are able to perform an asymptotic analysis of the distribution and find its tails. The talk is based on a joint project with R. Tribe and O. Zeitouni.

### Dmitry Savin

*Resonance width distribution beyond Porter-Thomas*

This talk discusses the distribution of resonance widths in quantum chaotic systems weakly coupled to the continuum via a finite number  $M$  of open channels. In contrast to the standard perturbative treatment of RMT, we do not a-priori assume the resonance widths being small compared to the mean level spacing. We show that to the leading order in weak coupling the perturbative  $\chi$ -square distribution of the resonance widths (in particular, the Porter-Thomas distribution at  $M=1$ ) should be corrected by a factor related to a certain average of the ratio of square roots of the characteristic polynomial ('spectral determinant') of the underlying RMT Hamiltonian. A simple single-channel expression is obtained that properly approximates the width distribution also at large resonance overlap, where the Porter-Thomas result is no longer applicable. [Based on a joint work with Yan Fyodorov, QMUL.]

### Nina Snaith

*Combining random matrix theory and number theory*

Many years have passed since the initial suggestion by Montgomery (1973) that in an appropriate asymptotic limit the zeros of the Riemann zeta function behave statistically like eigenvalues of random matrices, and the subsequent proposal of Katz and Sarnak (1999) that the same is true of families of more general L-functions. While this limiting behaviour is very informative, even more interesting are the intricacies involved in the approach to this limit, the understanding of which allows us to use random matrix theory in novel ways to shed light on major open questions in number theory.

**Nick Simm***Global fluctuations for the real component of the Ginibre orthogonal ensemble*

Let  $G$  be an  $N \times N$  real matrix of independent identically distributed standard Gaussians. The eigenvalues of  $G$  are known to form a two-component system of purely real and complex conjugated points. The real component of this system has interesting applications, from annihilating Brownian motions in probability theory to superconducting level crossings in quantum transport. I will discuss recent progress on our understanding of the real component, including a proof that linear statistics of the real eigenvalues fluctuate on a scale  $N^{1/4}$  and satisfy a central limit theorem as  $N$  goes to infinity.

**Elliot Paquette***The law of large numbers for the maximum of the log-characteristic polynomial associated to GUE*

We give a proof that the maximum of the centered log-characteristic polynomial of an  $N \times N$  GUE matrix is  $\log N + o(\log N)$  with high probability. This confirms the first term in a conjecture by Fyodorov and Simm. Moreover, we prove a general theorem about almost Gaussian fields that should be applicable to showing the law of large numbers for the log characteristic polynomial of random matrices.

This is joint work with Gaultier Lambert.

## Thursday

Morning session chair: Arno Kuijlaars

### Eugene Kanzieper

*Power spectrum analysis of long eigenlevel sequences in quantum chaology*

Fluctuations in quantum spectra are known to exhibit a high degree of universality which reflects the nature - regular or chaotic - of the underlying classical dynamics. Following Berry and Tabor (1977), statistics of level spacings in generic quantum systems with completely integrable classical dynamics is expected to mimic statistics of waiting times in a Poisson point process. For generic quantum systems with completely chaotic classical dynamics, Bohigas, Giannoni and Schmit (1984) conjectured that the level spacing distribution coincides with predictions of the Random Matrix Theory.

Recently, an alternative characterization of eigenvalue fluctuations was suggested by Relanõ et al. (2002). Interpreting long eigenlevel sequences as discrete-time random processes, these authors argued that the power spectrum of energy level fluctuations exhibits the  $1/\omega$  behavior for completely chaotic and  $1/\omega^2$  behavior for completely regular quantum systems.

In this talk, we present a rigorous theory of the power-spectrum and show that it can be expressed in terms of Painlevé VI function. We also outline the asymptotic (large- $N$ ), analysis of the resulting expression to confirm the small- $\omega$  behavior reported in various numerical experiments. Further work is required to analyze behavior of the power-spectrum in the domain  $\omega \sim O(N)$  and close to the Nyquist frequency. This is a joint work with Vladimir Osipov (Lund) and Roman Riser (Holon).

### Francesco Mezzadri

*Large deviations of radial statistics in the two-dimensional one-component plasma*

The two-dimensional one-component plasma is a ubiquitous model for several vortex systems. For special values of the coupling constant  $\beta q^2$  (where  $q$  is the particles charge and  $\beta$  the inverse temperature), the model also corresponds to the eigenvalues distribution of normal matrix models. Several features of the system are discussed in the limit of large number  $N$  of particles for generic values of the coupling constant. We show that the statistics of a class of radial observables produces a rich phase diagram, and their asymptotic behaviour in terms of large deviation functions is calculated explicitly, including next-to-leading orders in  $1/N$ . We demonstrate a split-off phenomenon associated to atypical fluctuations of the edge density profile. We also show explicitly that a failure of the fluid phase assumption of the plasma can break a genuine  $1/N$ -expansion of the free energy. Our findings are corroborated by numerical comparisons with exact finite- $N$  formulae valid for  $\beta q^2 = 2$ . This is work in collaboration with Fabio Deelan Cunden and Pierpaolo Vivo

### Uzy Smilansky

*Spectral statistics of the Unimodular Matrix Ensemble - A trace formula approach.*

The Unimodular Ensemble (UME) consists of complex Hermitian  $N \times N$  matrices with unimodular entries, with random phases which are uniformly distributed on the  $N(N-1)/2$  dimensional torus. Together with Hans Weidenmueller, we study some spectral statistics of the UME for large  $N$  and compute the leading and the first non-vanishing correction in a  $1/N$  expansion. Our purpose is to compare the trace-formula and the SUSY methods in addressing the same problem. In the present talk I shall mainly discuss the trace-formula approach.

**Mario Kieburg***An intriguing Relation between Eigenvalues & Singular Values of Bi-unitarily Invariant Random Matrices*

The relation between the eigenvalues and the singular values of an arbitrary complex square matrix is an old problem which was already studied by Schur, Weyl and others since the beginning of the last century. The only relations between the eigenvalues and singular values are given via inequalities for an arbitrary fixed matrix. This drastically changes when considering random matrices since we have an additional information due to the probability distribution. Very recently Holger Koesters and I could prove that there is a bijection mapping the joint probability distribution of the singular values to the one of the eigenvalues and vice versa for bi-unitarily invariant random matrix ensembles. I will sketch this proof in my talk and will outline the implications for spectral statistics.