EXTREMAL COMBINATORICS
WARWICK, 18-22 SEPTEMBER 2017
BOOK OF ABSTRACTS

Isoperimetry in integer lattices

Ben Barber (University of Bristol)

The edge isoperimetric problem for a graph G is to determine, for each n, the minimum number of edges leaving any set of n vertices. Exact solutions are known only in special cases, for example when G is the usual integer lattice. The most natural open problem was to answer this question for the 'strong lattice', with edges between points at l_∞ distance 1. Whilst studying this question we in fact solved the edge isoperimetric problem asymptotically for every Cayley graph on $\mathbb{Z}^d$. I'll talk about how to go from the specification of a lattice to a corresponding near-optimal shape, for both this and the related vertex isoperimetric problem. Joint work with Joshua Erde.

Packing degenerate graphs

Julia Böttcher (LSE)

There has recently been much progress on (hyper)graph packing problems in Combinatorics, which became possible through the development of new techniques in the area. Highlights include the celebrated proof by Keevash of the existence of combinatorial designs whenever certain divisibility conditions are satisfied, or the proof of the so-called Tree Packing Conjecture for large bounded degree trees by Joos, Kim, Kühn and Osthus. Here, a family of graphs is said to pack into a given host graph if the members of the family can be embedded into the host-graph edge-disjointly.

Contributing to this body of research and improving on several other results, we can prove the following packing result for graphs of bounded degeneracy (but with potentially large maximum degree). A graph H has degeneracy at most D if there is an ordering of its vertices such that every vertex has at most D neighbours among the vertices preceding it in this order. For large n and arbitrarily small $\varepsilon > 0$ any family of graphs $G_1, \dots, G_m$, each of which has degeneracy bounded by a constant D, at most n vertices, and maximum degree $o(n/\log n)$, and whose number of edges sum to at most $(1-\varepsilon)\binom{n}{2}$ packs into the complete graph K_n on n vertices. For obtaining this result we use a rather natural randomised packing strategy and show that this preserves certain pseudorandomness properties.

In the talk I shall give an overview of the mentioned progress in the area, explain our result and its background, and give an outline of the methods that we use.

Joint work with Peter Allen, Jan Hladký, Diana Piguet.

Packing bounded degree trees into regular graphs

Padraig Condon (University of Birmingham)

A classical result of Komlós, Sárközy and Szemerédi states that every n-vertex graph G of minimum degree at least $(1/2+\varepsilon)n$ contains a copy of every bounded degree n-vertex tree T. We show that if one additionally assumes that G is (close to) regular then G has an approximate decomposition into edge-disjoint copies of T. More generally if T is a collection of bounded degree trees, each on at most n vertices, and if G is a $(1/2+\sigma)n$-regular n-vertex graph such that $e(T) \leq (1-o(1))e(G)$, then T packs into G. The special case when $G = K_n$ implies an approximate version (for bounded degree trees) of the tree packing conjecture of Gyárfás and Lehel (the latter also follows from several recent related results by various authors). Our proof involves Szemerédi's regularity lemma, results on Hamilton decompositions, random walks and a recent blow-up lemma for approximate decompositions due to Kim, Kühn, Osthus and Tyomkyn. This is

joint work with Jaehoon Kim, Daniela Kühn and Deryk Osthus.

Diameter-Ramsey Sets
Jan Corsten (LSE)

A finite set $A \subset \mathbb{R}^d$ is called *diameter-Ramsey* if for every $r \in \mathbb{N}$, there exists some $n \in \mathbb{N}$ and a finite set $B \subseteq \mathbb{R}^n$ with $\text{diam}(A) = \text{diam}(B)$ such that whenever B is coloured with r colours, there is a monochromatic set $A' \subset B$ which is congruent to A. We prove that sets of diameter 1 with circumradius larger than $1/\sqrt{2}$ are not diameter-Ramsey. In particular triangles with an angle larger than 135° are not diameter-Ramsey improving a result of P. Frankl, Pach, Reiher and Rödl. Furthermore we deduce that there are simplices which are almost regular but not diameter-Ramsey. This is joint work with Nóra Frankl.

Rainbow Matchings in Dirac Graphs
Matthew Coulson (University of Birmingham)

Let G be a balanced bipartite graph of order $2n$ and $\delta(G) \geq n/2$. Consider an edge colouring of G where each color appears at most μn times, for some $\mu > 0$. Then, we show G contains a rainbow perfect matching, i.e. a perfect matching in which each edge has a distinct colour. We use this result to obtain a lower bound on the number of k-factors in graphs of order n and minimum degree $\delta \geq n/2$ for $1 \leq k \leq \mu n$. Joint work with Guillem Perarnau.

Block partitions in higher dimensions
Endre Csóka (Rényi Institute)

Given a sequence $S = (s_1, \ldots, s_m) \in [0,1]^m$, a block B of S is a subsequence $B = (s_i, s_{i+1}, \ldots, s_j)$. The size b of a block B is the sum of its elements. It is proved by Brny that for each positive integer n, there is a partition of S into n blocks $B_1, \ldots, B_n$ with $|b_i - b_j| \leq 1$ for every i, j. We consider a generalization of the problem in higher dimensions.

A part of the talk is joint work with Imre Bárány, Gyula Károlyi and Géza Tóth.

Testing subgraph-freeness in general graphs
Alberto Espuny Díaz (University of Birmingham)

Property testing of graph properties has been widely studied in two graph models, the bounded degree model and, especially, the dense graphs model. There is a third model, the general graphs model, in which problems have shown to have a more complex behaviour; this model is far less understood than the other two. The problem of testing subgraph-freeness in the general graphs model was studied by Alon, Kaufman, Krivelevich and Ron, who obtained several lower bounds on the complexity of any algorithm that tests triangle-freeness depending on the average degree of the graph. We extend some of these previous results beyond the triangle setting.

This is joint work with Felix Joos, Daniela Kühn, and Deryk Osthus.

A variant of the Kruskal-Katona theorem
Matthew Fitch (University of Warwick)

Let A be a family of sets of size r and let B be a family of sets of size $r-1$. Also let $|A| = x$ and $|B| = y$. For a set a in A, and a set b in B, say that a is above b if a contains b as a subset (and b is then below a). Note that because a is a set of size r, it contains at most r sets of size $r-1$. If for every set a in A, all r of the sets below it are in B, then the Kruskal-Katona theorem gives an upper bound on the size of x as a function of y (and respectively, a lower bound on y as a function of x). More specifically, this bound is found by letting A and B be initial segments in the colex ordering.

In this talk, we will look at a generalisation of this result, originally posed by Bollobás and Eccles. The set up is the same except that for every set a in A, we only require k of all the sets below it to be in B instead of all of them, where k is some integer smaller than r. We ask how large can we make x given a certain fixed y.

On the Möbius Function of A Pointed Graded Lattice
Samuel Asefa Fufa (Addis Ababa University)

In this paper, we use the pointed set partition lattice to introduce exponential pointed structures which are the pointed analog of exponential structures introduced by Stanley and new exponential structures from lattices. We show that this concept encompasses many examples introduced before. In particular, we introduce pointed decompositions of lattices and study their enumerative and geometric structure. We also show that exponential pointed structures satisfy pointed analogs of Stanley's compositional and exponential formulas. Joint work with Professor Melkamu Zeleke.

The extremal number for (a, b)-paths and other hypergraph trees
Zoltán Füredi (Rényi)

Let $\mathcal{F}$ be a (finite) class of k-uniform hypergraphs, and let $\mathrm{ex}(n, \mathcal{F})$ denote its Turan number, i.e., the maximum size of the $\mathcal{F}$-free, n-vertex, k-uniform hypergraphs. In other words, we consider maximal k-hypergraphs satisfying a *local constraint*. E.g., a Steiner system $S(n, k, t)$ is just a maximum k-hypergraph with no two sets intersecting in t or more elements.

In this lecture old and new *Turan type problems* are considered. To build a general theory we are especially interested around the Erdős-Ko-Rado range and try to find problems which are solvable with the known methods. The latest results are joint with T. Jiang, A. Kostochka, D. Mubayi, and J. Verstaëte.

The decomposition threshold of a given graph
Stefan Glock (University of Birmingham)

A fundamental theorem of Wilson states that, for every graph F, the edge set of every sufficiently large clique has a decomposition into copies of F (subject to some trivially necessary divisibility conditions). One of the main open problems in this area is to determine the minimum degree threshold which guarantees an F-decomposition in an incomplete host graph. We solve this problem for the case when F is bipartite and make significant progress towards the general case. This is joint work with Daniela Kühn, Allan Lo, Richard Montgomery and Deryk Osthus.

Borel and measurable versions of the Lovász Local Lemma
Łukasz Grabowski (University of Lancaster)

Lovász Local Lemma is one of the most fundamental tools of combinatorics. It allows to find a colouring of a finite graph with desirable properties. I will describe a joint work with E. Csóka, A. Máthé, O. Pikhurko and K. Tyros in which we develop measurable and Borel versions of the local lemma for Borel graphs and graphings, i.e. infinite graphs which naturally arise e.g. from measure-preserving actions of countable groups.

On the number of edge-disjoint triangles in K_4-free graphs
Ervin Győri (Rényi Institute, Budapest)

Keszegh and I proved the quarter of a century old conjecture of Erdős that every K_4-free graph with n vertices and $t_2(n) + k$ edges contains k pairwise edge disjoint triangles. The half a century old history of the problem starting with another similar question of Erdős and some sketch of the proof will be presented too.

Independent sets in Hypergraphs and Ramsey properties of graphs and the integers
Robert Hancock (University of Birmingham)

Many important problems in combinatorics and other related areas can be phrased in the language of independent sets in hypergraphs. Recently Balogh, Morris and Samotij, and independently Saxton and Thomason developed very general container theorems for independent sets in hypergraphs; both of which have seen numerous applications to a wide range of problems. We use the container method to prove results that correspond to problems concerning tuples of disjoint independent sets in hypergraphs. We generalise the random Ramsey theorem of Rödl and Ruciński by providing a resilience analogue. This result also implies the random version of Turán's theorem due to Conlon and Gowers, and Schacht. We

prove a general subcase of the asymmetric random Ramsey conjecture of Kohayakawa and Kreuter. Both of these results in fact hold for uniform hypergraphs. We also strengthen the random Rado theorem of Friedgut, Rödl and Schacht by proving a resilience version of the result.

Stability and algorithms for independent transversals
Penny Haxell (University of Waterloo)

An *independent transversal* (IT) in a vertex-partitioned graph G is an independent set in G consisting of one vertex in each partition class. This is a very basic notion that comes up in many combinatorial problems. There are various criteria that guarantee the existence of an IT in a given graph G. For example, if each partition class has size at least $2\Delta(G)$ then G has an IT.

We consider *stability* questions for independent transversals, and show in particular that if each partition class of G has size close to $2\Delta(G)$ but G does not have an independent transversal, then it contains an induced subgraph with a very special structure.

The known proofs of the criteria mentioned above do not give efficient *algorithms* for actually finding an IT. Here we also discuss appropriate weakenings of such results that do have effective proofs.

Spanning trees in randomly perturbed graphs
Felix Joos (University of Birmingham)

In two seminal papers, Komlós, Sárközy and Szemerédi extended Dirac's theorem by showing that there is a universal (small) constant $c > 0$ such that for every $\epsilon > 0$, there is an N so that the following holds: Every graph G on at least $n > N$ vertices with minimum degree at least $(1/2 + \epsilon)n$ contains every tree on n vertices of maximum degree at most $cn/\log n$. Observe that the lower bound on the minimum degree of G as well as the upper bound on the maximum degree of the tree cannot be relaxed.

However, graphs certifying this are of very specific extremal nature. To illustrate this, Krivelevich, Kwan and Sudakov showed the following: For every $\alpha, \Delta > 0$, there is a C such that for every graph G of minimum degree at least αn and every tree T of maximum degree at most Δ, the graph $G \cup G(n, C/n)$ contains T asymptotically almost surely. We extend this result to the entire possible range of Δ and discover a surprising solution. This is joint work with Jaehoon Kim.

Forbidden vector-valued intersections
Peter Keevash (University of Oxford)

We solve a generalised form of a conjecture of Kalai motivated by attempts to improve the bounds for Borsuk's problem. The conjecture can be roughly understood as asking for an analogue of the Frankl-Rödl forbidden intersection theorem in which set intersections are vector-valued. We discover that the vector world is richer in surprising ways: in particular, Kalai's conjecture is false, but we prove a corrected statement that is essentially best possible, and applies to a considerably more general setting. Our methods include the use of maximum entropy measures, VC-dimension, Dependent Random Choice and a new correlation inequality for product measures. This is joint work with Eoin Long.

Tree packing conjecture for bounded degree trees
Jaehoon Kim (University of Birmingham)

In 1976, Gyárfás and Lehel conjecture that if $T_1, \dots, T_n$ is a sequence of trees so that T_i has i vertices, then K_n has a decomposition into $T_1, \dots, T_n$. We prove this conjecture for large n when all trees $T_1, \dots, T_n$ have bounded maximum degree. This is joint work with Felix Joos, Daniela Kühn and Deryk Osthus.

Elusive problems in extremal graph theory
Daniel Král' (University of Warwick)

An empirical experience suggests that optimal solutions to extremal graph theory problems can be made asymptotically unique by introducing additional constraints. Lovasz conjectured that this phenomenon is true in general in the following sense: every finite feasible set of subgraph density constraints can

be extended further by a finite set of density constraints such that the resulting set is satisfied by an asymptotically unique graph. We will present a counterexample to this conjecture and discuss related results.

The talk is based on joint work with Andrzej Grzesik and László Miklós Lovász.

Chromatic index of random multigraphs
Gal Kronenberg (Tel Aviv university)

For a (multi)graph $G = (V, E)$, we denote by $\chi'(G)$ the minimum number of colors needed to color the edges of G properly. Clearly, $\Delta(G) \leq \chi'(G)$. Vizing proved that $\chi'(G) \leq \Delta(G) + \mu(G)$, where $\mu(G) = \max\{\mu(e) \mid e \in \binom{V}{2}\}$ is the maximum edge multiplicity of G.

Let $S \subseteq V$ and let $\rho(G) = \max\{\frac{e(G[S])}{\lfloor |S|/2 \rfloor} \mid S \subseteq V\}$. Since every color class forms a matching, we have that $\chi'(G) \geq \lceil \rho(G) \rceil$. In the '70s, Goldberg, and independently Seymour, conjectured that for any multigraph G, $\chi'(G) \in \{\Delta(G), \Delta(G) + 1, \lceil \rho(G) \rceil\}$. We show that their conjecture is true w.h.p. for random multigraphs.

The model $M(n, m)$ is the probability space consisting of all loopless multigraphs with n vertices and m edges, in which m pairs from $[n]$ are chosen independently at random with repetitions (that is, we permit edge repetitions in the standard random graph model $G(n, m)$). Our result states that, for a given $m := m(n)$, $M \sim M(n, m)$ typically satisfy $\chi'(G) = \max\{\Delta(G), \lceil \rho(G) \rceil\}$. In particular, we show that if n is even and $m := m(n)$, then $\chi'(M) = \Delta(M)$ for a typical $M \sim M(n, m)$. Furthermore, for a fixed $\varepsilon > 0$, if n is odd, then a typical $M \sim M(n, m)$ has $\chi'(M) = \Delta(M)$ for $m \leq (1 - \varepsilon)n^3 \log n$, and $\chi'(M) = \lceil \rho(M) \rceil$ for $m \geq (1 + \varepsilon)n^3 \log n$.

Joint work with Penny Haxell and Michael Krivelevich.

Hypergraph F-designs exist for arbitrary F
Daniela Kühn (University of Birmingham)

We show that given any r-uniform hypergraph F, the trivially necessary divisibility conditions are sufficient to guarantee a decomposition of any sufficiently large complete r-uniform hypergraph into edge-disjoint copies of F. The case when F is complete corresponds to the existence of block designs, a problem going back to the 19th century, which was recently settled by Keevash. In particular, our argument provides a new proof of this result, which employs purely probabilistic and combinatorial methods. We also obtain several further generalizations.

(Joint work with Stefan Glock, Allan Lo and Deryk Osthus.)

The minimum number of triangles in graphs of given order and size
Hong Liu (University of Warwick)

In 1941 Rademacher raised the question of determining the minimum number of triangles in a graph of given order and size. This problem has attracted much attention and, in a major breakthrough, was solved asymptotically by Razborov in 2008.

In this paper, we provide an exact solution for all large graphs whose edge density is bounded away from 1, confirming a conjecture of Lovász and Simonovits from 1975 in this range. Furthermore, we give a description of extremal graphs.

Joint work with Oleg Pikhurko and Katherine Staden.

Decomposing tournaments into paths
Allan Lo (University of Birmingham)

In this work we consider a generalisation of Kelly's conjecture which is due Alspach, Mason, and Pullman from 1976. Kelly's conjecture states that every regular tournament has an edge decomposition into Hamilton cycles, and this was proved by Kühn and Osthus for large tournaments. The conjecture of Alspach, Mason, and Pullman concerns general tournaments and asks for the minimum number of paths needed in an edge-decomposition of each tournament into paths. There is a natural

lower bound for this number in terms of the degree sequence of the tournament and they conjecture this bound is correct for tournaments of even order. Almost all cases of the conjecture are open and we prove many of them. This is joint work with Viresh Patel, Jozef Skokan and John Talbot.

Specifying a positive threshold function via extremal points

Vadim Lozin (Mathematics Institute, University of Warwick, UK)

An extremal point of a positive threshold Boolean function f is either a maximal zero or a minimal one. It is known that if f depends on all its variables, then the set of its extremal points completely specifies f within the universe of threshold functions. However, in some cases, f can be specified by a smaller set. The minimum number of points in such a set is the specification number of f. Hu showed in [Sze-Tsen Hu. *Threshold Logic*. University of California Press, Berkeley, 1965.] that the specification number of a threshold function of n variables is at least $n+1$. Anthony, Brightwell, Shawe-Taylor proved in [M. Anthony, G. Brightwell, J. Shawe-Taylor. On specifying Boolean functions by labelled examples. *Discrete Applied Mathematics*, 61(1):1-25, 1995] that this bound is attained for nested functions and conjectured that for all other threshold functions the specification number is strictly greater than $n+1$. In the present paper, we resolve this conjecture negatively by exhibiting threshold Boolean functions of n variables, which are non-nested and for which the specification number is $n+1$. On the other hand, we show that the set of extremal points satisfies the statement of the conjecture, i.e. a positive threshold Boolean function depending on all its n variables has $n+1$ extremal points if and only if it is nested. To prove this, we reveal an underlying structure of the set of extremal points.

Joint work with I. Razgon, V. Zamaraev, E. Zamaraeva, N. Zolotykh

On subgraphs of C_{2k}-free graphs and a question of Kühn and Osthus

Abhishek Methuku (Central European University)

Kühn and Osthus showed that every bipartite C_{2k}-free graph G contains a C_4-free subgraph with at least $1/(k-1)$ fraction of the edges of G. We will present a new and short proof of this result. Kühn and Osthus also showed that a C_{2k}-free graph which is obtained by pasting together C_4's has average degree at most $16k$ and asked whether there exists a number $d = d(k)$ such that every C_{2k}-free graph which is obtained by pasting together C_{2l}'s has average degree at most d if $k > l \geq 3$ are given integers. We answer this question negatively.

Let c denote the largest constant such that every C_6-free graph G contains a bipartite C_4-free subgraph having c fraction of edges of G. Győri *et al.* showed that $3/8 \leq c \leq 2/5$. We prove that $c = 3/8$. More generally, we show that for any $\varepsilon > 0$, and any integer $k \geq 2$, there is a C_{2k}-free graph G_1 which does not contain a bipartite subgraph of girth greater than $2k$ with more than $\left(1 - \frac{1}{2^{2k-2}}\right) \frac{2}{2k-1}(1+\varepsilon)$ fraction of the edges of G_1, and a C_{2k}-free graph G_2 which does not contain a bipartite and C_4-free subgraph with more than $\left(1 - \frac{1}{2^{k-1}}\right) \frac{1}{k-1}(1+\varepsilon)$ fraction of the edges of G_2.

Our proof uses statements similar to the following one, which we prove using probabilistic ideas, generalizing a theorem of Erdős: For any $\varepsilon > 0$, and any integers a, b, $k \geq 2$, there exists an a-uniform hypergraph H of girth greater than k which does not contain any b-colorable subhypergraph with more than $\left(1 - \frac{1}{b^{a-1}}\right)(1+\varepsilon)$ fraction of the hyperedges of H. We also prove further generalizations of this theorem.

This is joint work with Dániel Grósz and Casey Tompkins.

Triangle-tilings in graphs without large independent sets

Richard Mycroft (University of Birmingham)

We study the minimum degree necessary to guarantee the existence of perfect and almost-perfect triangle-tilings in an n-vertex graph G with sublinear independence number. In this setting, we show that if $\delta(G) \geq n/3 + o(n)$

then G has a triangle-tiling covering all but at most four vertices; this minimum degree condition is best-possible up to the $o(n)$ error term. Also, for every $r \geq 5$, we asymptotically determine the minimum degree threshold for a perfect triangle-tiling under the additional assumptions that G is K_r-free and n is divisible by 3. This is joint work with József Balogh, Andrew McDowell and Theodore Molla.

Unavoidable trees in tournaments
Tássio Naia (University of Birmingham)

An oriented tree T on n vertices is unavoidable if every tournament on n vertices contains a copy of T. We obtained a sufficient condition for T to be unavoidable, and use this to prove that almost all labeled oriented trees are unavoidable, verifying a conjecture of Bender and Wormald. We additionally proved that every tournament on $n+o(n)$ vertices contains a copy of every oriented tree T on n vertices with polylogarithmic maximum degree, improving a result of Kühn, Mycroft and Osthus. This is joint work with Richard Mycroft.

A characterization of testable hypergraph properties
Deryk Osthus (University of Birmingham)

We provide a combinatorial characterization of all testable properties of k-graphs (i.e. k-uniform hypergraphs). Here, a k-graph property $\mathbf{P}$ is testable if there is a randomized algorithm which makes a bounded number of edge queries and distinguishes with probability $2/3$ between k-graphs that satisfy $\mathbf{P}$ and those that are far from satisfying $\mathbf{P}$. For the 2-graph case, such a combinatorial characterization was obtained by Alon, Fischer, Newman and Shapira. Our results for the k-graph setting are in contrast to those of Austin and Tao, who showed that for the somewhat stronger concept of local repairability, the testability results for graphs do not extend to the 3-graph setting.

Joint work with Felix Joos, Jaehoon Kim, and Daniela Kühn.

On the independence numbers and the chromatic numbers of random subgraphs
Andrei M. Raigorodskii (Moscow Institute of Physics and Technology; Moscow State University)

The classical theory of random graphs gives us a series of deep results concerning the independence numbers and the chromatic numbers of random subgraphs of the complete graphs.

Recently many new results have been obtained for the same characteristics of random subgraphs of Kneser's graphs and their important generalizations.

In our talk, we will expose some of the recent advances in the above-mentioned directions.

Covering and tiling hypergraphs with tight cycles
Nicolás Sanhueza-Matamala (University of Birmingham)

Given $3 \leq k \leq s$, we say that a k-uniform hypergraph C_s^k is a tight cycle on s vertices if there is a cyclic ordering of the vertices of C_s^k such that every k consecutive vertices under this ordering form an edge. We prove that if $k \geq 3$ and $s \geq 2k^2$, then every k-uniform hypergraph on n vertices with minimum codegree at least $(1/2 + o(1))n$ has the property that every vertex is covered by a copy of C_s^k. Our result is asymptotically best possible for infinitely many pairs of s and k, e.g. when s and k are coprime.

A perfect C_s^k-tiling is a spanning collection of vertex-disjoint copies of C_s^k. When s is divisible by k, the problem of determining the minimum codegree that guarantees a perfect C_s^k-tiling was solved by a result of Mycroft. We prove that if $k \geq 3$ and $s \geq 5k^2$ is not divisible by k and s divides n, then every k-uniform hypergraph on n vertices with minimum codegree at least $(1/2+1/(2s)+o(1))n$ has a perfect C_s^k-tiling. Again our result is asymptotically best possible for infinitely many pairs of s and k, e.g. when s and k are coprime with k even.

Joint work with Jie Han (São Paulo) and Allan Lo (Birmingham),

Homomorphism threshold for graphs
Mathias Schacht (University of Hamburg)

The interplay of minimum degree and 'structural properties' of large graphs with a given

forbidden subgraph is a central topic in extremal graph theory. For a given graph F we define the homomorphism threshold as the infimum α such that every n-vertex F-free graph G with minimum degree $> \alpha n$ has a homomorphic image H of bounded size (independent of n), which is F-free as well. Without the restriction of H being F-free we recover the definition of the chromatic threshold, which was determined for every graph F by Allen et al. The homomorphism threshold is less understood and we present recent joint work with O. Ebsen on the homomorphism threshold for odd cycles.

Proof of Komlós's conjecture on Hamiltonian subsets

Maryam Sharifzadeh (University of Warwick)

Komlós conjectured in 1981 that among all graphs with minimum degree at least d, the complete graph K_{d+1} minimises the number of Hamiltonian subsets, where a subset of vertices is Hamiltonian if it contains a spanning cycle. We prove this conjecture when d is sufficiently large. In fact we prove a stronger result: for large d, any graph G with average degree at least d contains almost twice as many Hamiltonian subsets as K_{d+1}, unless G is isomorphic to K_{d+1} or a certain other graph which we specify.

This is joint work with Jaehoon Kim, Hong Liu and Katherine Staden.

Exploring the projective normgraph

Tibor Szabó (FU Berlin)

The projective normgraphs $NG(t, q)$ provide tight constructions for the Turán number of complete bipartite graphs $K_{t,s}$ with $s > t!$. In this talk we discuss their automorphism group and explore their small subgraphs. In particular we count their 3-degenerate subgraphs and prove that $NG(4, q)$ does contain (many) $K_{4,6}$. Some of these result also extend the work of Alon and Shikhelman on generalized Turán numbers. We also give a new, more elementary proof for the $K_{4,7}$-freeness of $NG(4, q)$. The talk represents joint work with Tomas Bayer, Tamás Mészáros, and Lajos Rónyai.

Tilings in graphs and directed graphs

Andrew Treglown (University of Birmingham)

Given two graphs G and H, a perfect H-tiling in G is a collection of vertex-disjoint copies of H in G which together cover all the vertices in G. (Perfect H-tilings are also referred to as H-factors or perfect H-packings.) Over recent decades a wealth of research has been undertaken on the topic in the setting of graphs, directed graphs, hypergraphs and random graphs. For example, the celebrated Hajnal-Szemerédi theorem determines the minimum degree threshold that ensures a graph G contains a perfect K_r-tiling. In this talk I will survey some recent joint work including:

(i) a directed version of the Hajnal–Szemerédi theorem;
(ii) a result on perfect H-tilings in randomly pertubed dense graphs;
(iii) algorithmic aspects of the problem.

This includes joint work with Andrzej Czygrinow, Louis DeBiasio and Theo Molla; József Balogh and Adam Zsolt Wagner, and; Jie Han.

Strong forms of stability

Konstantinos Tyros (Koç University)

The simplest case of Turán's problem asks for the maximal edge density in a triangle free graph. By Mantel's theorem we have that the balanced bipartite graph is extremal. Independently, by Erdős and Simonovits, it was proven that every triangle free graph with edge density close to the maximal one is close in edit distance to the balanced bipartite graph. This phenomenon is called stability. Since then, several problems of similar type have been considered. In general, given a hereditary class $\mathcal{G}$ of graphs, we consider the problem of determining the maximal value of the density of copies of some graph H (or a linear combination of such densities) among the members of $\mathcal{G}$. We introduce stronger forms of stability and we provide sufficient conditions for them. In this way, we reprove a list of known results and we obtain some new ones. This is a joint work with Oleg Pikhurko and Jakub Sliačan.

Preferential attachment with weighted choice

Mark Yarrow (University of Sheffield)

We use preferential attachment with replacement to select r vertices from G. V_{n+1} attaches to each of these r vertices with probabilities $[P(V_1), P(V_2), ..., P(V_r)]$ where the locations x_i of each of V_i satisfy $x_1 \leq x_2 \leq ... \leq x_r$. Using the theory of stochastic approximation we have found a complete set of stable fixed points proving results on when condensation can occur. When we choose $r = 3$ and the vector to be $[0, 0, 1]$ we have the same model previously studied by Freeman and Jordan (2017).

Supersaturation of even cycles in linear hypergraphs

Liana Yepremyan (University of Oxford)

A classic result of Bondy and Simonovits and independently, of Erdős from 1970s says that the maximum number of edges in an n-vertex graph not containing C_{2k}, the cycle of length $2k$, is $O(n^{1+1/k})$. In an unpublished work, Simonovits obtained the corresponding supersaturation result, that is, there is a constant c (depending on k) such that for all large n, every n-vertex graph G with more than $cn^{1+1/k}$ edges contains $\Omega((e/n)^{2k})$ many C_{2k}'s, this bound being achieved by a random graph with the same edge density. In a recent work on the number of C_{2k}-free graphs, Morris and Saxton also obtained a 'balanced' version of the supersaturation of C_{2k}'s.

In this talk, we extends Simonovits' result to linear cycles in linear hypergraphs. Our result is self-contained and includes the $r = 2$ case and hence is a direct extension of Simonovits' result. In comparison to Morris' and Saxton's proof for $r = 2$, this proof is simpler; it relies on some ideas from earlier work of Faudree and Simonovits as well as new reduction theorems that we develop. We believe that the reduction theorems are of independent interest and can be useful for other supersaturation problems. This is joint work with Tao Jiang.