ON THE GEOMETRY OF OUTER SPACE - PROBLEM SET 2

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(1) If $x \neq y \in \mathcal{X}_n$ show that there exist conjugacy classes $\alpha, \beta \in \mathcal{C}$ such that $\ell_x(\alpha) < \ell_y(\alpha)$ and $\ell_x(\beta) > \ell_y(\beta)$. Conclude that the functions $\ell_x, \ell_y \colon \mathcal{C} \to \mathbb{R}_+$ are distinct even projectively (i.e. they do not differ by a multiplicative constant). This implies that the map sending $\mathcal{X}_n$ to $P(\mathbb{R}_+^{\mathcal{C}})$ is injective.
(2) Compute the distance from x to y from problem (3) of problem set 1. Then compute the distance from y to x.
(3) Find a geodesic path from y to x from problem (3) problem set 1.
(4) Consider the map $\Phi \colon R_2 \to R_2$ defined by $a \to ab, b \to bab$.
  (a) Prove that this is a train track map.
  (b) Find the transition matrix of Φ and its largest eigenvalue λ.
  (c) Find the left eigenvector corresponding to this matrix and show that if x is the base rose with metric defined by this eigenvector then Φ as an automorphism of x stretches every edge by λ.
  (d) Determine the point $x\Phi$ (label the edges with the inverse marking). Compute $d(x, x\Phi)$?
  (e) Show that Φ^{-1} as a map on the rose is a train track map. To do this construct a graph whose vertices are the directed edges a, A, b, B (edges with reverse orientation are labeled by capital letters). Add a directed edge in this graph from e to e' if $D\Phi^{-1}(e) = e'$ (i.e. if the first edge in $\Phi^{-1}(e)$ is e'). Use this graph to determine all of the illegal turns, and check that edge images do not contain them.
  (f) Determine μ the eigen-value of Φ^{-1}. Determine the left eigenvector corresponding to it.
(5) Answer questions 4 for the the automorphism $\Phi \colon F_3 \to F_3$ defined by $\Phi(a) = b, \Phi(b) = c, \Phi(c) = ca$. Replace R_2 by R_3 wherever relevent.

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