Conference: K-theory, $\mathbb{A}^1$-homotopy and quadratic forms

Organizers: Marco Schlichting and Heng Xie

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Tom Bachmann (University of Duisburg-Essen)

Title: Rational contractibility as an effectivity criterion

Abstract: In classical algebraic topology, ordinary cohomology theories h^* are distinguished among all cohomology theories by the dimension axiom: $h^*(pt) = 0$ unless $* = 0$. Moreover ordinary theories are classified by their *coefficient group* $h^0(pt)$.

Call a bigraded motivic cohomomology theory $h^{*,*}$ *ordinary* if $h^{*,0}(P) = 0$ for all $* \neq 0$, and also $h^{*,q}(\hat{\Delta}^\bullet_P) = 0$ for all $q > 0$ and all $*$. Here P ranges over all spectra of fields, and $\hat{\Delta}^\bullet_P$ denotes the semilocal cosimplicial scheme over P.

We show that ordinary motivic cohomology theories are classified by the homotopy invariant sheaf with generalised transfers $X \mapsto h^{0,0}(X)$. We also explain how Suslin's notion of rational contractibility shows that many motivic cohomology theories arising in practice are ordinary. As an application, we prove that two recent theories called"generalised motivic cohomology" are isomorphic, and hence identify the E_2 page of the Atiyah-Hirzebruch spectral sequence for hermitian K-theory.

Grzegorz Banaszak (Adam Mickiewicz University)

Title: Divisibility in K-groups and classical conjectures in Number Theory

Abstract: I will discuss divisibility and wild kernels in algebraic K-theory of global fields and present basic results concerning the divisible elements in K-groups. These results can be applied to investigate the imbedding obstructions in homology of GL's and the splitting obstructions in Quillen's localization sequence. I will discuss relation of divisible elements in K-groups of number fields with conjectures of Kummer-Vandiver, Iwasawa and Leopoldt. I will also present recent results, joint with Cristian Popescu, concerning Brummer-Stark conjecture and Galois equivariant Stickelberger splitting map in Quillen's localization sequence.

Federico Binda (University of Regensburg)

Title: Rigidity for relative 0-cycles.

Abstract: In this talk, we will present a relation between the classical Chow group of relative 0-cycles on a regular scheme $\mathcal{X}$, projective and flat over an excellent Henselian discrete valuation ring A with perfect residue field k, and the so-called cohomological Chow group of zero cycles of the special fiber. If k is algebraically closed and with finite coefficients (prime to the residue characteristic) these groups turn out to be isomorphic. This generalizes a previous argument due to Esnault-Kerz-Wittenberg to the case of regular models with arbitrary reduction. From this, one can re-prove in case of bad reduction that the étale cycle class map for relative 0-cycles with finite coefficients on $\mathcal{X}$ is an isomorphism, a result due to Saito and Sato in the case of semi-stable reduction. This is a joint work with Amalendu Krishna.

Oliver Braunling (University of Freiburg)

Title: K-theory of locally compact modules

Abstract: We discuss the K-theory of categories of modules over a number field, but additionally equipped with a locally compact topology. Although this is not an abelian category, such a category has a natural exact structure, and moreover Pontryagin duality makes it an exact category with duality (note that the dual of a discrete module will be a compact module, so this is very different from the standard duality on module categories). Generalizing results of Dustin Clausen, we explain how to compute the K-theory of such categories, and how it has links to number theory. In fact, Dustin Clausen originally began looking into such K-theory spectra in his work on lifting the Artin map of class field theory to a map of spectra.

Baptiste Calmès (Laboratoire de Mathématiques de Lens)

Title: Milnor-Witt motivic cohomology
Abstract: I'll give an overview of the theory of Milnor-Witt motives, as developed by Fasel, Déglise, Østvær and myself. It is a thickened version of Voevodsky's theory of motives, designed to be closer to the stable homotopy category of schemes defined by Morel and Voevodsky.

Emanuele Dotto (University of Bonn)

Title: The Hermitian K-theory of $\mathbb{Z}/2$-equivariant ring spectra.
Abstract: I will talk about joint work with C. Ogle, were we define the Hermitian and real K-theory of ring spectra "with genuine anti-involution", as well as a trace map to real topological Hochschild homology. The trace splits the restricted assembly map of the spherical group-ring, and this allows one to reformulate the Novikov conjecture in terms of the module structure of the "genuine L-theory spectrum" of the spherical group-ring.

Grigory Garkusha (Swansea University)

Title: Reconstructing rational stable motivic homotopy theory
Abstract: Using a recent computation of the rational minus part of $\mathrm{SH}(k)$ by Ananyevskiy-Levine-Panin, a theorem of Cisinski-Déglise and a version of the Röndigs-Østvær theorem, rational stable motivic homotopy theory over an infinite perfect field of characteristic different from 2 is recovered from finite Milnor-Witt correspondences in the sense of Calmès-Fasel.

Jens Hornbostel (University of Wuppertal)

Title: Chow-Witt groups of classifying spaces
Abstract: We compute the Chow-Witt rings of the classifying spaces for the symplectic and special linear groups. In the structural description we give, contributions from real and complex realization are clearly visible. The computations for the symplectic groups show that Chow-Witt groups are a symplectically oriented ring cohomology theory. Using our computations for special linear groups, we also discuss the question when an oriented vector bundle of odd rank splits off a trivial summand. This is joint work with Matthias Wendt.

Kevin Hutchinson (University College Dublin)

Title: The module structure on the homology of SL_2 of rings.
Abstract: For a commutative ring R the integral homology groups of $\mathrm{SL}_2(R)$ are naturally modules over the group ring $\mathbb{Z}[R^\times/(R^\times)^2]$. We will discuss the nature and uses of this structure in the calculation of low-dimensional homology in the cases when R is a ring with valuation, a global field or a ring of S-integers in a global field and will illustrate with examples. We will discuss connections with K-theory, homology stability and Milnor-Witt K-theory.

Jeremy Jacobson (University of Emory)

Title: On the signature of a quadratic form, real cohomology, and the powers of the fundamental ideal.
Abstract: The question of whether or not -1 is a sum of squares in a given field is intimately connected to the Galois cohomology of the field and the powers of the fundamental ideal in the Witt ring of quadratic forms over the field. The connection is via the classic notion of the signature of a quadratic form. In this talk, we discuss 'globalizations' of these connections to algebraic varieties, in particular, our recent work relating the powers of the fundamental ideal in the Witt ring of a scheme to real cohomology.

Max Karoubi (Paris Diderot University)

Title: The Witt group of real algebraic varieties
Abstract: The topology of real algebraic varieties V has some history going back to the 19th century. In the 70's, a connection was made between this topology and the purely algebraic Witt group $W(V)$ defined via quadratic forms on algebraic vector bundles over V. In this talk we improve these results, mainly due to Mahé and Brumfiel, by introducing a new topological invariant $WR(X)$, where X is the space of complex points of V, provided with the involution defined via complex conjugation. There is a map $W(V) \to WR(X)$ which is an isomorphism modulo bounded 2-primary torsion. There is also another map $WR(X) \to KO(Y)$, where Y is the space of real points, KO denoting the usual KO-theory, with the same property. Precise bounds are given about these 2-primary torsions in terms of the dimension of V. This is joint work with M. Schlichting and C. Weibel.

Bernhard Koeck (University of Southampton)

Title: Operations on higher K-groups revisited
Abstract: A couple of years ago, Grayson surprised the mathematical community with an algebraic description of higher algebraic K-groups in terms of generators and relations. After reviewing that description we show how to implement exterior power operations in this new context. We also purely algebraically prove that these operations satisfy the axioms of a special lambda ring. This is joint work with Tom Harris and Lenny Taelman.

Markus Land (University of Regensburg)

Title: K- and L-Theory of C^*-algebras
Abstract: In this talk I want to explain a spectrum level comparison between topological K-theory and algebraic L-theory for C^*-algebras.

I will focus on explaining how one can obtain a natural map from connective topological K-theory to algebraic L-Theory and why no such map can exists for periodic K-theory. I will then explain that this map induces an equivalence on periodic theories after inverting 2.

If time permits, I will outline how this comparison can be used to obtain a commutative diagram in which both the Baum-Connes assembly map and the Farrell-Jones assembly map (for L-theory) appear. This allows to prolong injectivity results from the operator algebraic side to the L-theoretic side.
All of this is joint work with Thomas Nikolaus.

Alexander D. Rahm (University of Luxembourg)

Title: Computing torsion in the cohomology of arithmetic groups
Abstract: This talk describes works involving a technique called Torsion Subcomplex Reduction (TSR), which was developed by the speaker for computing torsion in the cohomology of discrete groups acting on suitable cell complexes. TSR enables one to skip machine computations on cell complexes, and to access directly the reduced torsion subcomplexes, which yields results on the cohomology of matrix groups in terms of formulas. TSR has already yielded general formulas for the cohomology of the tetrahedral Coxeter groups as well as, at odd torsion, of SL_2 groups over arbitrary number rings (in joint work of M. Wendt and the speaker). The latter formulas have allowed Wendt and the speaker to refine the Quillen conjecture. Furthermore, progress has been made to adapt TSR to Bredon homology computations. In particular for the Bianchi groups, yielding their equivariant K-homology, and, by the Baum-Connes assembly map, the K-theory of their reduced $C^\star$-algebras, which would be very hard to compute directly. As a side application, TSR has allowed the speaker to provide dimension formulas for the Chen-Ruan orbifold cohomology of the complexified Bianchi orbifolds, and to prove (jointly with F. Perroni) Ruan's crepant resolution conjecture for all complexified Bianchi orbifolds.

Oscar Randal-Williams (University of Cambridge)

Title: E_∞-cell structures and general linear groups
Abstract: The nerve of the groupoid of finitely-generated projective R-modules forms an E_∞-algebra $X(R)$, even before group-completing it to form the algebraic K-theory space of R. There is a notion of attaching an E_∞-cell to an E_∞-algebra, and one may ask how $X(R)$ may be constructed from the trivial E_∞-algebra by iterated cell attachments. (After group-completion, attaching an E_∞-cell corresponds to attaching a spectrum cell, so the spectrum homology of $K(R)$ gives a lower bound for the necessary E_∞-cells.) I will explain some ongoing work with S. Galatius and A. Kupers in which we use a theory of "E_∞ cellular homology" to produce cellular models for such E_∞-algebras $X(R)$ with constraints (for certain rings R) on the dimensions of the cells which arise. Such constraints give immediate information about the ordinary homology of $X(R)$, and hence about the group homology of general linear groups over R (stability, secondary stability, ...).

Husney Parvez Sarwar (University of Warwick)

Title: K-theory of monoid algebras and questions of Gubeladze.
Abstract: We will begin with some questions of Gubeladze on K-theory of monoid algebras. The question is whether or not the homotopy invariance property of K-theory extends to the monoid extension. We will present some results in this direction. This is a joint work with Amalendu Krishna.

David Sprehn (University of Copenhagen)

Title: Stable homology of classical groups
Abstract: Quillen computed the homology of the general linear groups over a finite field of characteristic p, with

coefficients away from characteristic p. He wasn't able to compute the p-torsion. However, by comparing the groups over various field extensions, he gave a beautiful argument showing that, in the stable limit, the p-torsion always vanishes. For p odd, Nathalie Wahl and I were able to prove the analogous result for the other classical groups (symplectic, orthogonal, and unitary groups).

Alexander Vishik (University of Nottingham)

Title: Motives of affine quadrics
Abstract: Affine quadrics may be considered as non-split spheres. It appears that their motives behave much better than that of projective quadrics. In particular, the motive of an affine quadric $A_q = q = 1$ determines the respective quadratic form q. Moreover, by the result of Bachmann, the reduced motive of A_q is invertible. This permits to embed quadratic forms (or, which is the same, the (0)[0]-stable homotopy group of spheres) into Pic($DM(k)$). I will describe the subgroup in Pic($DM(k)$) generated by such objects. The key tool here is the functor of Bachmann.

Charles Weibel (Rutgers University)

Title: The Witt group of real surfaces
Abstract: We compare the algebraic Witt group $W(V)$ of quadratic forms for a 2-dimensional algebraic variety V over R with its topological counterpart, $WR(V_C)$, which is based on symmetric forms on Real vector bundles. They are isomorphic if $p_g = 0$. This is joint work with Karoubi.

Heng Xie (University of Warwick)

Title: Witt groups of real algebraic spheres
Abstract: In the first part of this talk, we will construct a long exact sequence connecting Witt groups $W^i(Q_d)$ of a quadric to Witt groups of its even part clifford algebra with the canonical involution. In the second part, we will use this long exact sequence to discuss an application to Witt groups of real affine algebraic spheres.

Abstract of short talks:

Anwar Alameddin (University of Liverpool)

Title: Motivic Measures through Waldhausen K-theories
Abstract: The Grothendieck ring of varieties encodes essential information about them, including their stable birational classes, ℓ-adic characteristic, Hodge characteristic, etc. In this talk, I will explain how Waldhausen K-theories give rise to motivic measures through the envelope topology and how that can be applied to extend the (modified) Grothendieck ring of varieties to a ring spectrum. Then, I will discuss recent work regarding the Euler-Poincareé characteristics induced by motivic spaces with the cdh-topology.

Andrei Druzhinin (Chebyshev Laboratory, St Peterborough)

Title: Cancellation theorem for Grothendieck-Witt-correspondences and Witt-correspondences
Abstract: The cancellation theorem for Grothendieck-Witt-correspondences and Witt-correspondences between smooth varieties over an infinite prefect field k, $char k \neq 2$, is proved. The isomorphism

$$\mathrm{Hom}_{\mathbf{DM}^{GW}_{\mathrm{eff}}}(A^\bullet, B^\bullet) \simeq \mathrm{Hom}_{\mathbf{DM}^{GW}_{\mathrm{eff}}}(A^\bullet(1), B^\bullet(1))$$

for any pair of objects $A^\bullet, B^\bullet \in \mathbf{DM}^{GW}_{eff}$ in the category of effective GW-motives is obtained.
The result yields that the canonical functor $\Sigma^\infty_{\mathbb{G}_m^{\wedge 1}} : \mathbf{DM}^{GW}_{\mathrm{eff}}(k) \to \mathbf{DM}^{GW}(k)$ is fully faithful, where $\mathbf{DM}^{GW}(k)$ is the category of non-effective GW-motives, and the natural isomorphism

$$\mathrm{Hom}_{\mathbf{DM}^{GW}}(M^{GW}(X), \Sigma^\infty_{\mathbb{G}_m^{\wedge 1}}(A^\bullet)[i]) = \mathbb{H}^i_{nis}(X, A^\bullet),$$

where $A^\bullet$ is a motivic complex of sheaves with GW-transfers (in particular, a homotopy invariant sheave with GW-transfers), and $M^{GW}(X)$ denotes the GW-motive of a smooth variety X; and similarly for Witt-motives.

Christian Dahlhausen (University of Regensburg)

Title: Continuous K-theory and cohomology of analytic spaces
Abstract: Morrow proposed a definition of continuous K-theory which associates to every Tate ring A (e.g. an affinoid algebra) a K-theory spectrum $\mathrm{K}^{\mathrm{cont}}(A)$. Besides the algebraic structure, it also reflects the topology of

A. In this short talk, we are interested in the special case that A is an affinoid algebra of dimension d over a complete discretely valued field k. I will sketch how one can relate the group $\mathrm{K}_{-d}^{\mathrm{cont}}(A)$ with the cohomology group $\mathrm{H}^d(\mathcal{M}(A);\mathbb{Z})$ of the associated Berkovich spectrum $\mathcal{M}(A)$ using Raynaud-Gruson's *platification par éclatement*. This is part of my ongoing PhD project advised by M. Kerz and G. Tamme.

Jonas Irgens Kylling (University of Oslo)

Title: Algebraic cobordism of number fields.
Abstract: I will report on work in progress on calculations of the motivic homotopy groups of MGL (the algebraic cobordism spectrum) over number fields. We use the motivic slice spectral sequence. The main step is to calculate over the reals where the differentials are determined by C_2-equivariant Betti realization and comparison with Hill-Hopkins-Ravenel. This is related to previous work by Hu-Kriz, Rognes-Weibel, Hill, Yagita and Ormsby-Østvær.