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Jw Camille Horbez

Boundary of relative FF graph & subgroup classif.

(0) Free Products

Start $G = G_1 * \dots * G_k * F_N$ If you only care of $G = F_N$, fine.

. $\mathcal{F} = \{G_1, \dots, G_k\}$ a ~~peripheral~~ part of structure

. g on $A < G$ is periph if conjugate in G_i

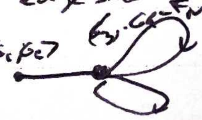
. $\mathrm{Out}(G, \mathcal{F}) = \{ \alpha \in \mathrm{Aut}(G) \mid$ ~~$\alpha(G_i)$ is conjugate~~ α preserves $[G_i] \} / \mathrm{Inn}(G)$

Main objects : ~~all~~ splittings ~~are~~ "relative to $\mathcal{F}$" $\overset{\text{Bass Serre}}{=}$ act° on trees in which G_i elliptic

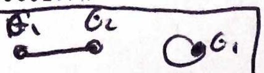
~~Def a (relative) free factor of $(G, \mathcal{F})$ is a factor in a free (relative) free splitting $G = A * B$ ... $G = A * B$ ...~~

Def. a free splitting = act° on a simplicial tree with trivial edge stab

eg. $\langle G_1, G_2 \rangle * \langle G_3 \dots G_k, F_N \rangle$ or $\langle G_1, G_2 \rangle$ Eg. $G = F_N * \mathbb{Z}$



. a proper free factor (relative) is a vertex gp of a free splitting + non peripheral & non trivial

Def Non sporadic if ∃ a proper free factor : G_1 G_2  G_1

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1bis

① Alternative
Generalises Ivanov for MCG
HM for $G = F_n$

Th 1 (assume $(G,\mathcal{F})$ non sporadic) [G Horbez]

$H \subset Out(G,\mathcal{F})$
[+ H fg if G is not a ~~...~~ toral rel hyp gp]

either:

(a) H v. preserves the conj class of a ~~...~~ proper free factor or a conj class [gdcotic]

(b) $H \ni h$ which is fully irreducible and atoroidal (no power preserves a proper free factor)

(b1) $H > F_2 =$ ~~...~~ st $\forall h \in F_2 - \{1\}$ is fully irred. and atoroidal

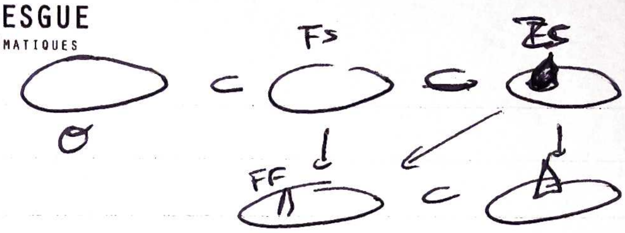
(b2) ~~H~~ ~~...~~ has a f idx subgp $H_0 \simeq \langle h \rangle \propto H'$
with . h fully irred (and atoroidal)
. H' contains no fully irred.

Th 1 bis (Uyanik for F_n, Φ)

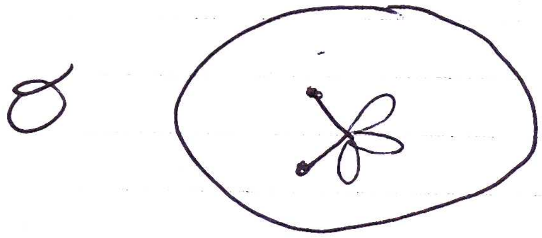
Comments (b1)(b2) ⇒ know sthg about H { act° on hyp spce / act° on R-tree ...

(a) Suitable for induct° = A inv free fact → new ~~free~~ (split) ~~...~~ free prod structure

Reach sporadic case
$(G,\mathcal{F}) = A * B \qquad Out(G,\mathcal{F}) \simeq Aut(A) \times Aut(B)$
$(G,\mathcal{F}) = A * \qquad Out(G,\mathcal{F}) \simeq Aut(A) \ltimes A$



② Hyp spaces & bdaries

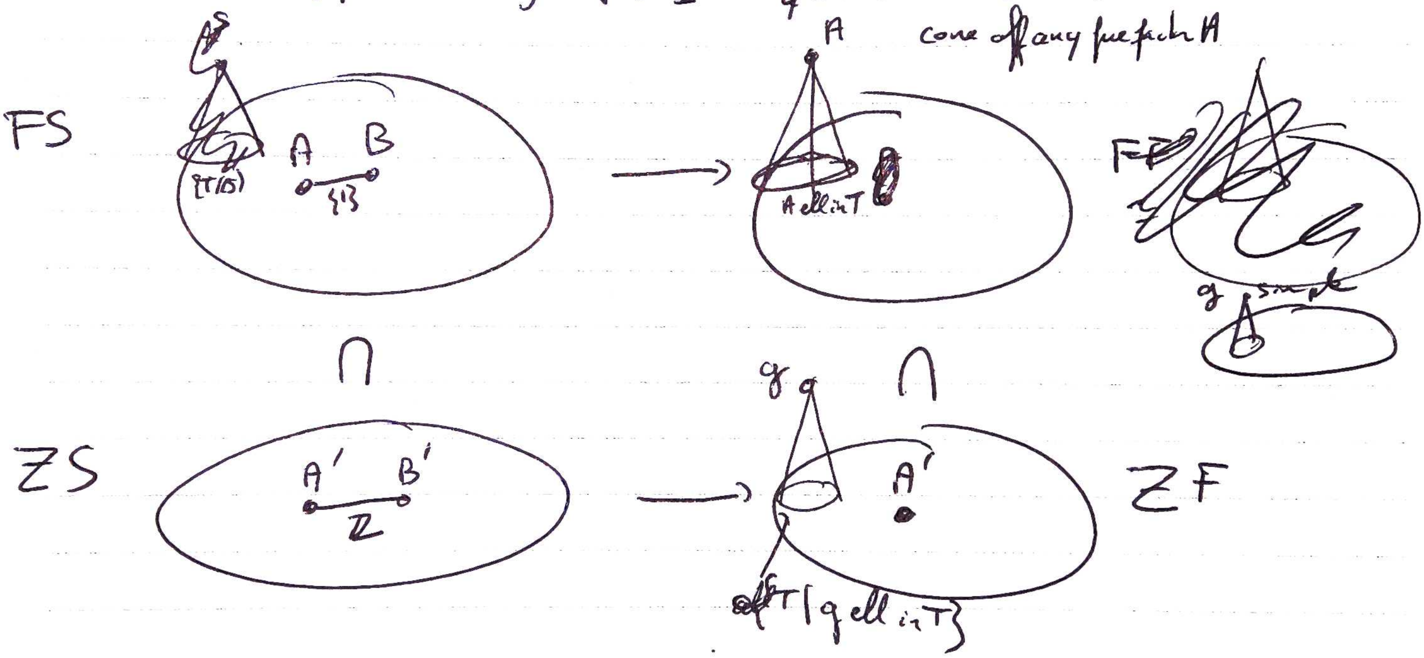


Outer space

O = metric trees + rel. free act° trivial edge stabs

∩ add missing simplices = ~~cone off over link of missing~~

cone off any free factor A



① O

② FS = add missing simplices

③ ZS = add all Z splittings = splittings over non-peripheral cyclic gps

④ FF = cone of A by {T | A ell in T}

= cone of g by {T | g ell in T}, g simple

⑤ ZF

⑥ Variants Intersect° graph by Mann / cosurface graph Dowdall-Taylor / Intersect° graph by Kapovich Lustig

[splitted tree + cone off g quadratic elt.] or

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Fact ⓐ all these spaces are hyp
Camille in this generality — Bestvina-Feighn (FF) + H Mosher
Handel Mosher (FS) + Hilion Horbez ~~[illegible]~~
B. Mann (ZS) Horbez
Mann (ZF) Nous
Kapovich-Rafi

BF
HM
Mann
Horbez
Horbez + G (FF instead of ZS) ZF
+ Kapovich-Rafi

ⓑ all related by coarsely alignment pres. maps

ⓒ ∄ map $ZS \longrightarrow FF$

- Coro (Dowdall-Taylor) $\underbrace{\partial_\infty ZF \subset \partial_\infty FF}_{\text{not closed}} \subset \partial_\infty ZS \subset \partial_\infty FS$
- $\overline{\mathcal{O}}$ = Very small trees: no tripod stab & arc stab are ~~max cyclic~~ root closed & non peripheral

~~Relatio~~ **Th** [BR (Reynolds), G Horbez] $\partial_\infty FF \simeq \{\text{arational trees}\}/\sim$

The inclus° $\mathcal{O} \longrightarrow FF$ extends continuously to an homeo ~~[illegible]~~ $\{\text{arational trees}\}/\sim \longrightarrow$ ~~[illegible]~~ $\partial_\infty FF$ & ~~[illegible]~~

Def [Reynolds]. $T \in \partial\mathcal{O}$ arational if
- $T \notin \mathcal{O}$
- $\forall A$ free factor, $A \curvearrowright T$ is rel. free & discrete

Def $T \sim T'$ if ∃ bij° pres alignment.

Th [Dowdall Taylor in free SP, GH] The inclus° $\mathcal{O} \longrightarrow ZF$ extends cont. to an homeo $\{\text{rel. free arational trees}\}/\sim \longrightarrow \partial_\infty ZF$ or Lovett ~~[illegible]~~

Comments : → quite rigid : T arational, $T/\!/T' \Rightarrow T' \sim T$

Example → T arational non free $\Rightarrow$ 

eg of free arational: $f = F_n \curvearrowright \mathbb{R}_n$ irrep, + attractive tree
...

Witness : T not arat^{nal} ⇒ ∃ canonical finite collect° that shows it (minimal lt rel free)

6 (4)

(3) Utilis° du bnd : Alternative de Gromov:

$\mathrm{I} \to$, $F_2 \Rightarrow$ OK (1)

unbded orbits + fix pt at ∞ $\Longrightarrow$ Study stab of tree [GL] $\Rightarrow$ (2)

Remains to prove: bded orbits $\Longrightarrow$ periodic FF

(3) Utilis° du bnd. RW arg^t de Camille

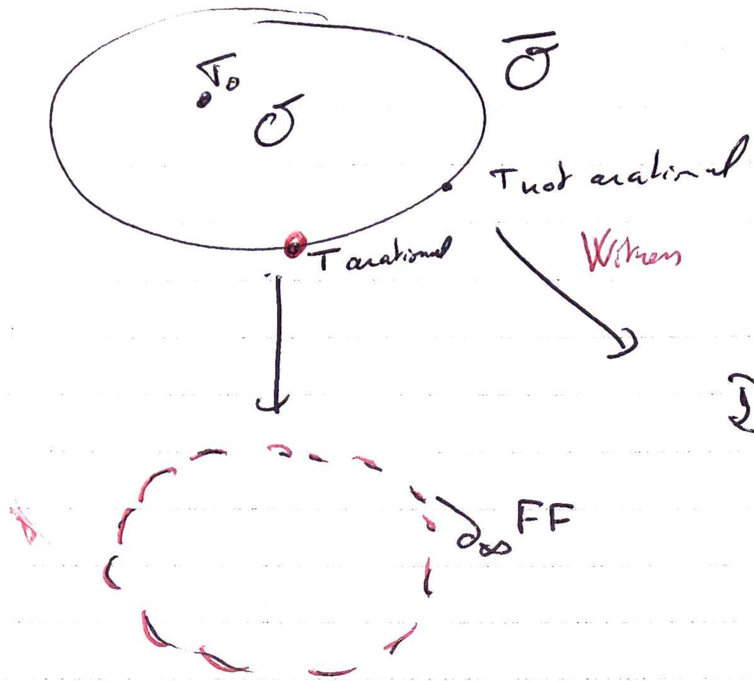
Propagation H

Cpactness $\to$ ν harmonic on $\overline{\mathcal{O}}$

(start at random along ν and move at random along μ, distribution is still given by ν)

$\mu * \nu = \nu$

limits of $\mu^n \delta_o$



$\mathcal{D}$ = {finite coll° of free factors}

Bded orbits $\Longrightarrow$ ν({arational trees}) = 0

$\Longrightarrow$ ν harmonic on $\mathcal{D}$

$\Longrightarrow \exists$ finite orbit on $\mathcal{D}$