

Integrals over unitary groups, maps on surfaces, and Euler characteristics

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This is joint work with Doron Puder (Tel Aviv University). For a positive integer r , fix a word w in the free group on r generators. Let G be any group. The word w gives a ‘word map’ from G^r to G : we simply replace the generators in w by the corresponding elements of G . We again call this map w . The push forward of Haar measure under w is called the w -measure on G . We are interested in the case $G = U(n)$, the compact Lie group of n -dimensional unitary matrices. A motivating question is: to what extent do the w -measures on $U(n)$ determine algebraic properties of the word w ?

For example, we have proved that one can detect the ‘stable commutator length’ of w from the w -measures on $U(n)$. Our main tool is a formula for the Fourier coefficients of w -measures; the coefficients are rational functions of the dimension n , for reasons coming from representation theory.

We can now explain all the Laurent coefficients of these rational functions in terms of Euler characteristics of certain mapping class groups. I’ll explain all this in my talk, which should be broadly accessible and of general interest. Time permitting, I’ll also invite the audience to consider some remaining open questions.