BANK OF ENGLAND

The Dynamics of Money
University of Warwick
4th February 2019

Contagion in Financial Networks

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Clearing & Conservation of Money

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Locally conserved quantity

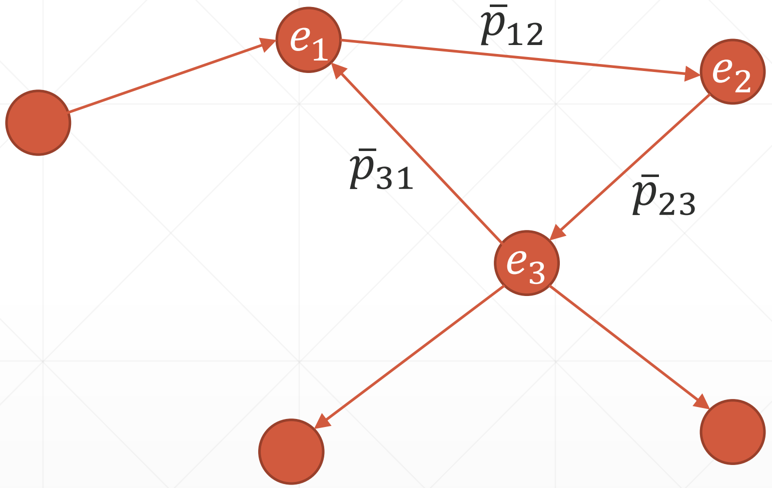
$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

- Fluid dynamics
- Heat
- Electromagnetism
- Quantum mechanics
- General relativity

Clearing payments

- Financial institutions with some cash buffers: e_i

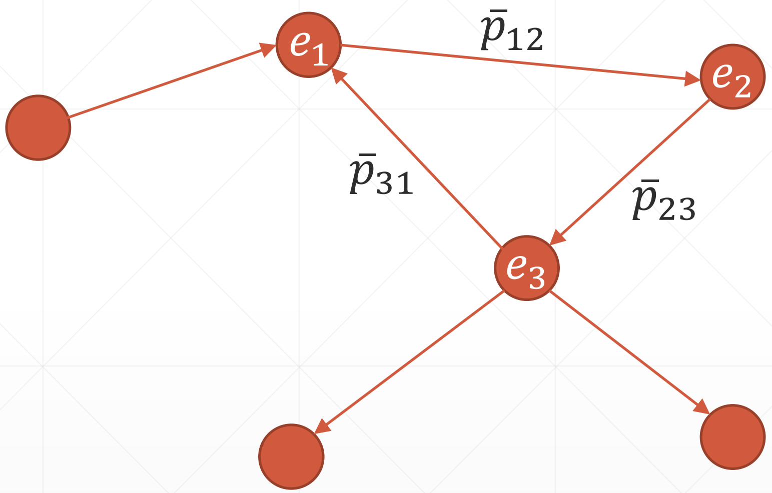
- And with bilateral obligations between them: $\bar{p}_{ij}$



- How do we compute payments between institutions?

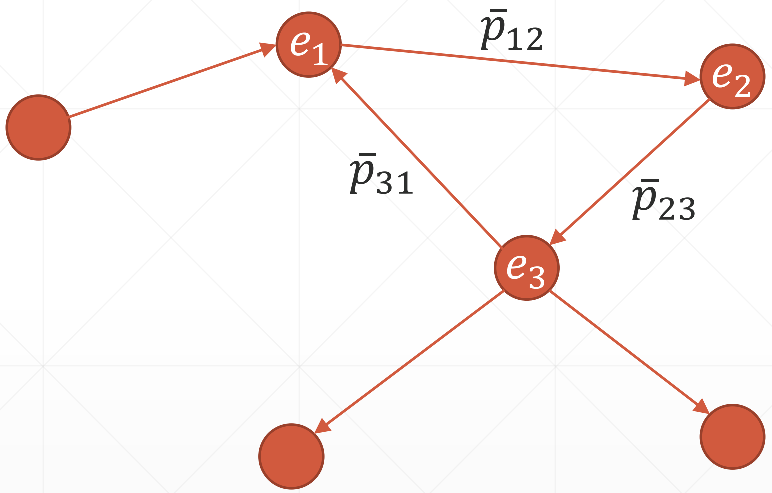
Eisenberg and Noe (2001)

- Main assumptions:
  - Limited liability
  - Absolute priority
- Institutions pay:
  - All their obligations, if they can
  - As much as they can pro rata, otherwise
- Key results:
  - There exist greatest and least clearing payments
  - In many cases clearing payments are unique
  - Clearing payments can be easily computed



Local conservation of money

- Visentin et al (2016): Money is locally conserved in Eisenberg and Noe
- Local conservation implies global conservation
- No amplification!
- Can we break it?
  - Rogers and Veraart (2013): Bankruptcy costs destroy money
  - Barucca et al (2016): Ex-ante valuation



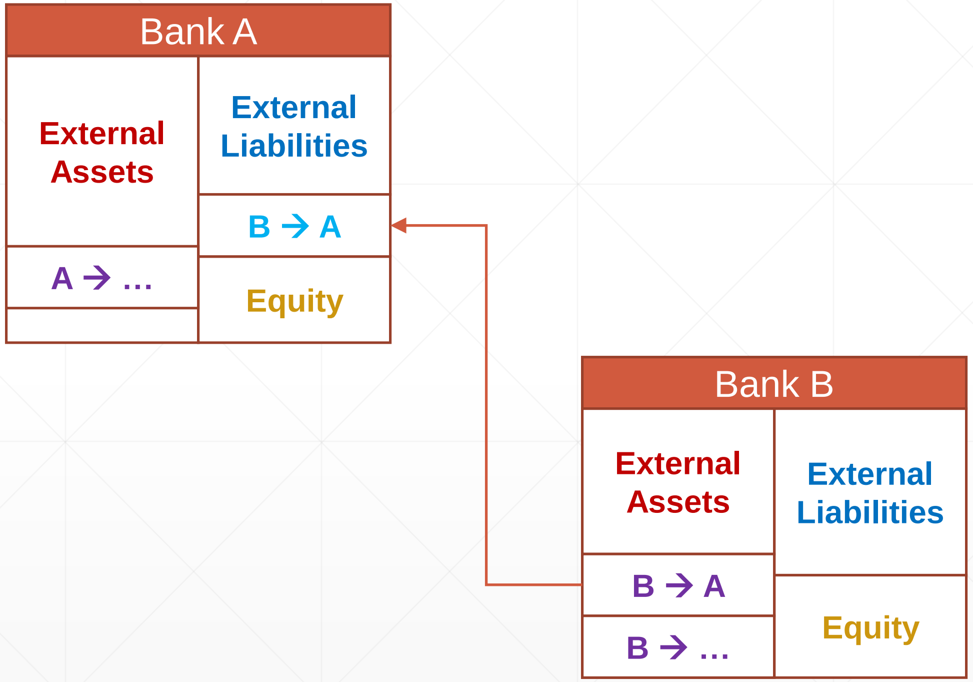
Valuation framework

P Barucca, MB, F Caccioli, M D’Errico, G Visentin, S Battiston, G Caldarelli

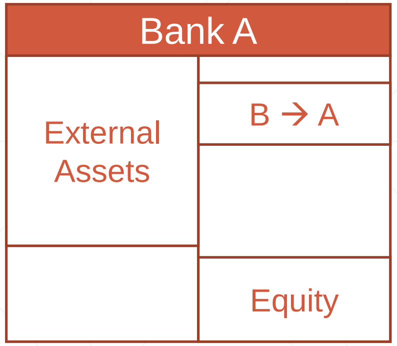
Balance sheet

- Asset side:
  - **External assets** (e.g. loans)
  - **Interbank assets**
- Liability side:
  - **External liabilities** (e.g. deposits)
  - **Interbank liabilities**
  - **Equity**
- Balance sheet identity:

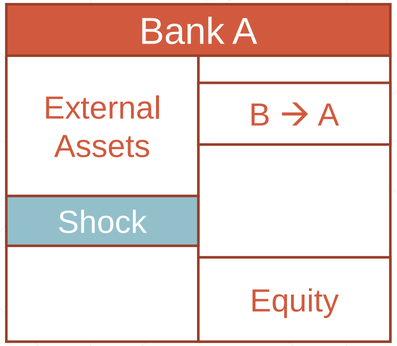
$$E_i(t) = A_i^e(t) + \sum_j A_{ij}(t) - L_i^e(t) - \sum_j L_{ij}(t)$$



Intuition

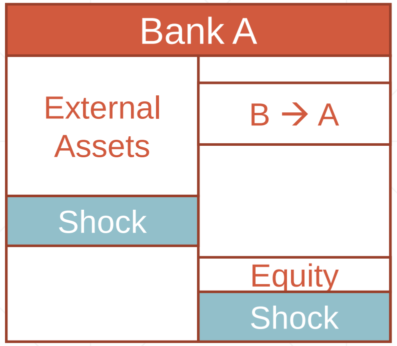


Intuition



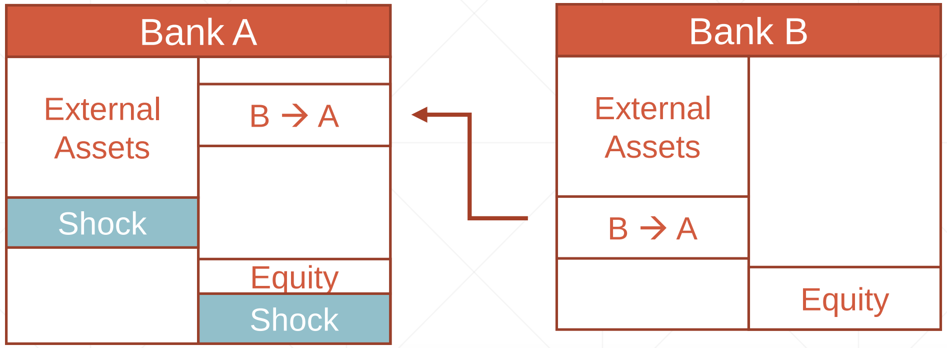
1. Exogenous shock

Intuition



1. Exogenous shock
2. First round: impact on equity

Intuition



1. Exogenous shock
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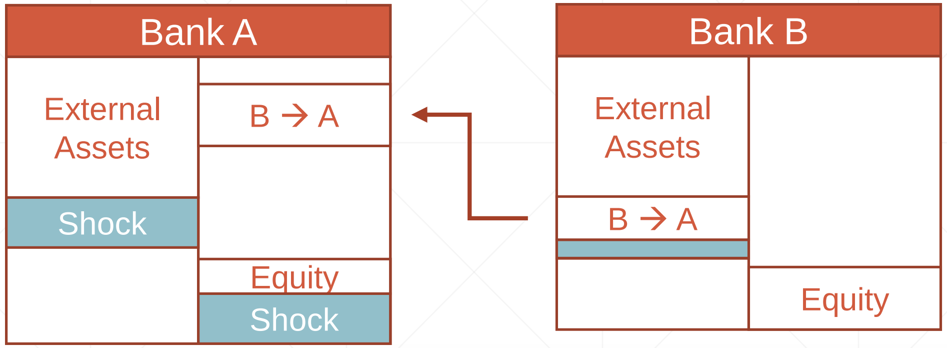
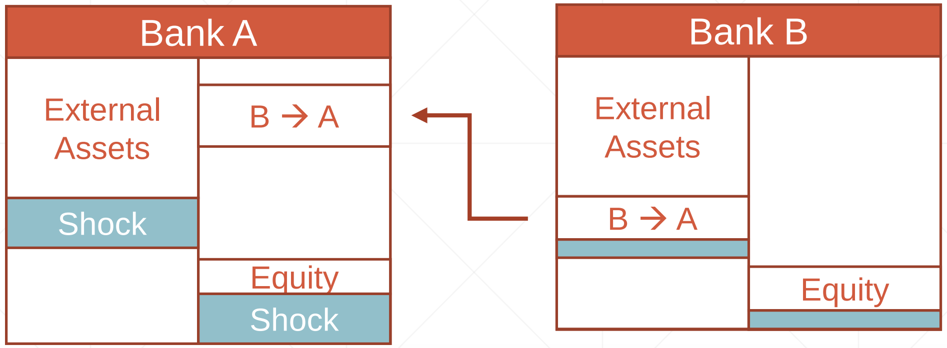


1. Exogenous shock
2. First round: impact on equity
3. Counterparties revaluate interbank assets

Intuition



1. Exogenous shock
2. First round: impact on equity
3. Counterparties revaluate interbank assets
4. Second round: impact on counterparties' equities
5. Contagion spreads to Bank B's counterparties

Valuation functions

$$E_i(t) = A_i^e(t) + \sum_j A_{ij}(t)\boxed{\mathbb{V}_{ij}(\mathbf{E}(t)|\ldots)} - L_i^e(t) - \sum_j L_{ij}(t)$$

- Between 0 and 1
  - When equal to one, interbank assets are worth the book value
  - When equal to zero, interbank assets are worth nothing
- Non decreasing in the equity vector
- Continuous from above
- Greatest and least solutions computed iteratively

Valuation functions: Zoology

By using specific valuation functions we can recover several contagion algorithms:

- Eisenberg and Noe (2001)
- Furfine (2003)
- Rogers and Veraart (2013)
- DebtRank (2015)

Forward-looking solvency contagion

MB, P Barucca, A Brinley Codd, J Hill

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Valuation functions: At maturity

- Valuation that accounts for uncertainty on solvency of counterparties:

$$\tilde{E}_i(T) = A_i^e(T) + \sum_{j=1}^{n} A_{ij}(T)\mathbb{V}_j(\tilde{E}_j(T);\ldots) - L_i^e(T) - \sum_{j=1}^{n} L_{ij}(T)$$

- The valuation of interbank assets is performed via a discount factor:

$$\mathbb{V}_j(\tilde{E}_j(T);\ldots) = \begin{cases} 1 & \text{for } \tilde{E}_j(T) > 0 \\ r_j(\tilde{E}_j(T);\ldots) & \text{for } \tilde{E}_j(T) \le 0 \end{cases}$$

If the borrower has not defaulted, then the discount factor is equal to one and the interbank asset is worth its book value; otherwise it will be worth less.

Valuation functions: Forward-looking

- We now perform a forward-looking valuation at time $t < T$: Average over the risk-neutral measure:

$$\tilde{E}_i(t) = A_i^e(t) + \sum_{j=1}^{n} A_{ij}(t)\mathbb{E}^{\mathbb{Q}}\left[\mathbb{V}_j(\tilde{E}_j(T);\ldots)|\mathbf{A}^e(t)\right] - L_i^e(t) - \sum_{j=1}^{n} L_{ij}(t)$$

- We also account for the possibility that banks can default at any point in time

$$\mathbb{V}_j(\tilde{E}_j(T);\ldots) = \begin{cases} 1 & \text{for } \tilde{E}_j(s) > 0, \forall s < T \\ \rho_j & \text{otherwise} \end{cases}$$

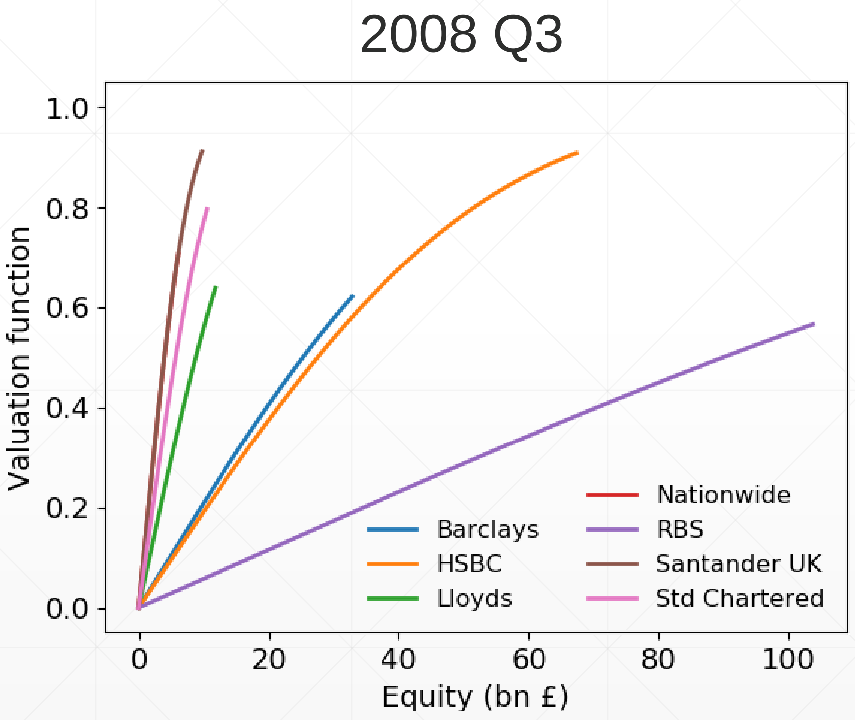
- It turns out that:

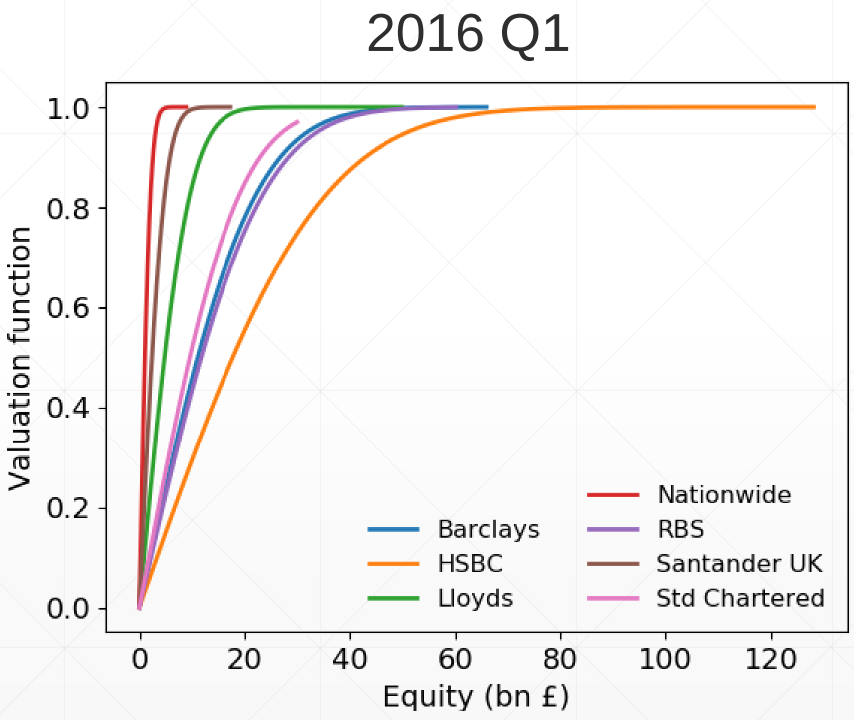
$$\mathbb{E}^{\mathbb{Q}}\left[\mathbb{V}_j(\tilde{E}_j(T);\ldots)|\mathbf{A}^e(t)\right] = 1 - p_j^D(\tilde{E}_j(t)) + \rho_j\, p_j^D(\tilde{E}_j(t))$$

Data

- We use real interbank exposures between banks part of the Bank of England's annual concurrent stress test:
  - 7 banks, which account for 80% of the regulated UK lending
  - 2008 – 2013: exposures larger than 10% of capital
  - 2014 – 2015: no threshold, more granular data (subordinated and unsecured lending)
- When possible (2014 – 2016) we interpret the equity of our model as the CET1 buffer, otherwise we use shareholders' equity for consistency.
- Valuation functions:
  - Volatilities estimated from share prices
  - Forward-looking horizon: 1 year

Valuation functions: Calibration

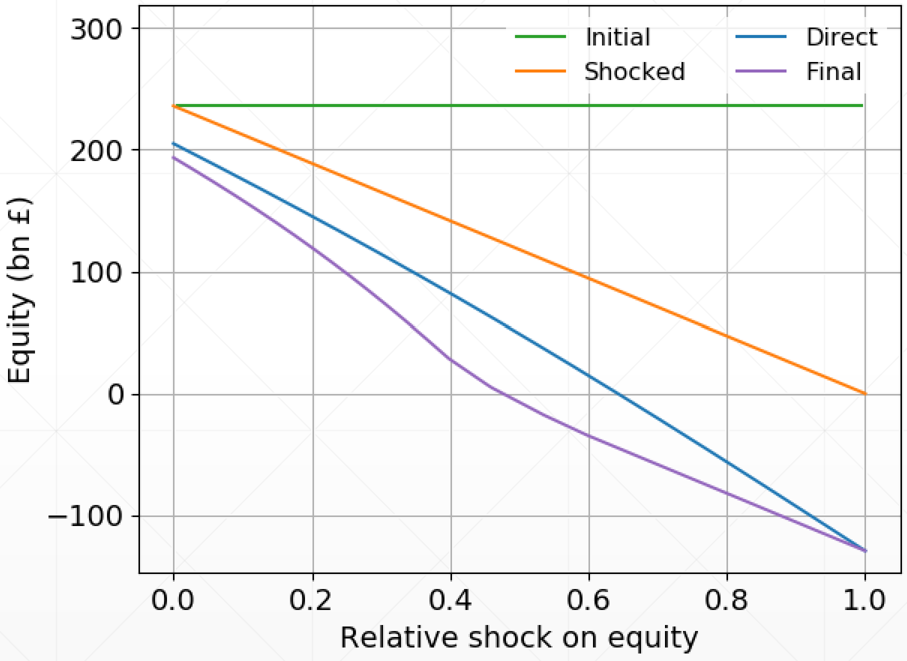




Recovery rate = 0

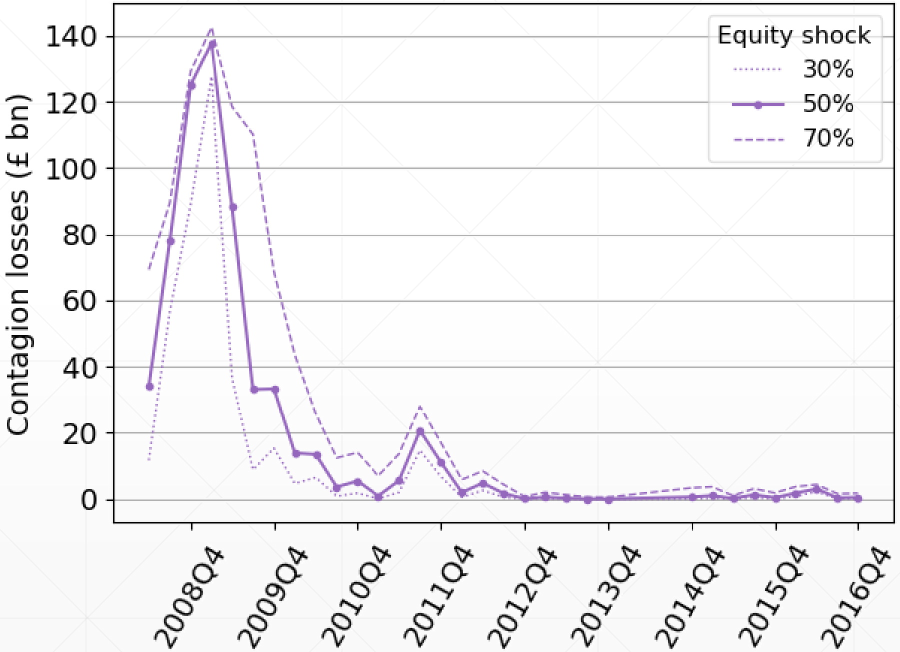
Simplified stress tests

- We run simplified stress tests. In the first “scenario” all banks suffer a homogeneous (relative) shock to their equity.
- Losses due to contagion (orange to purple) can be as large as the exogenous shock.
- Losses caused by direct exposures (orange to blue) can be a large as those caused by indirect exposures (blue to purple).

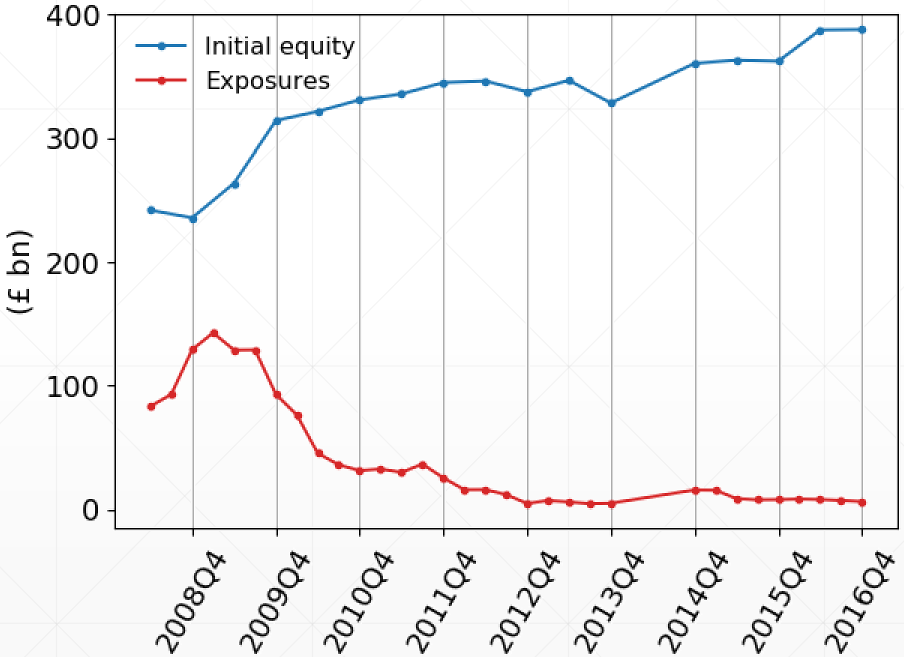


2008, recovery rate = 0

Contagion losses decline

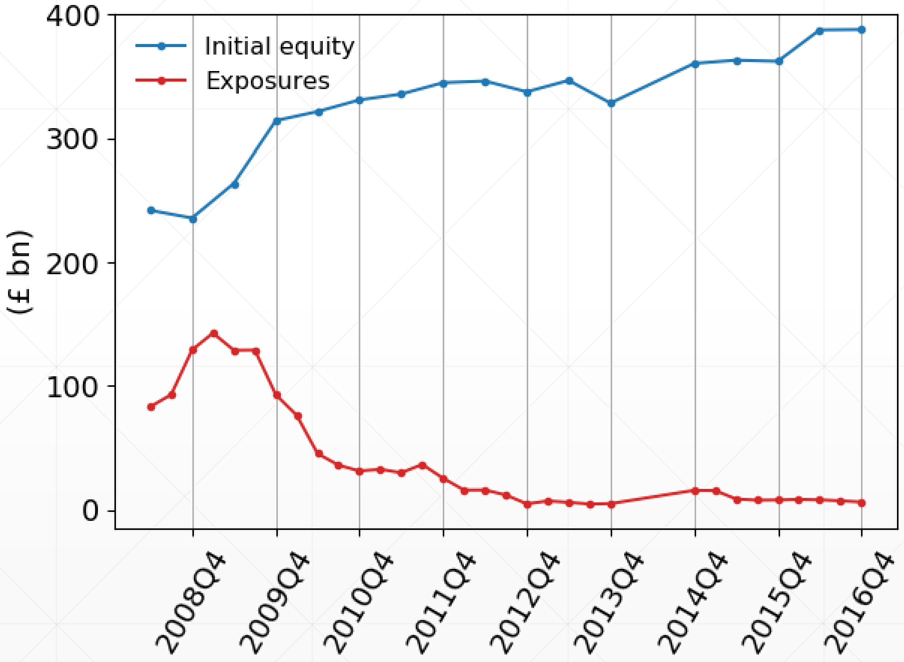


Recovery rate = 0

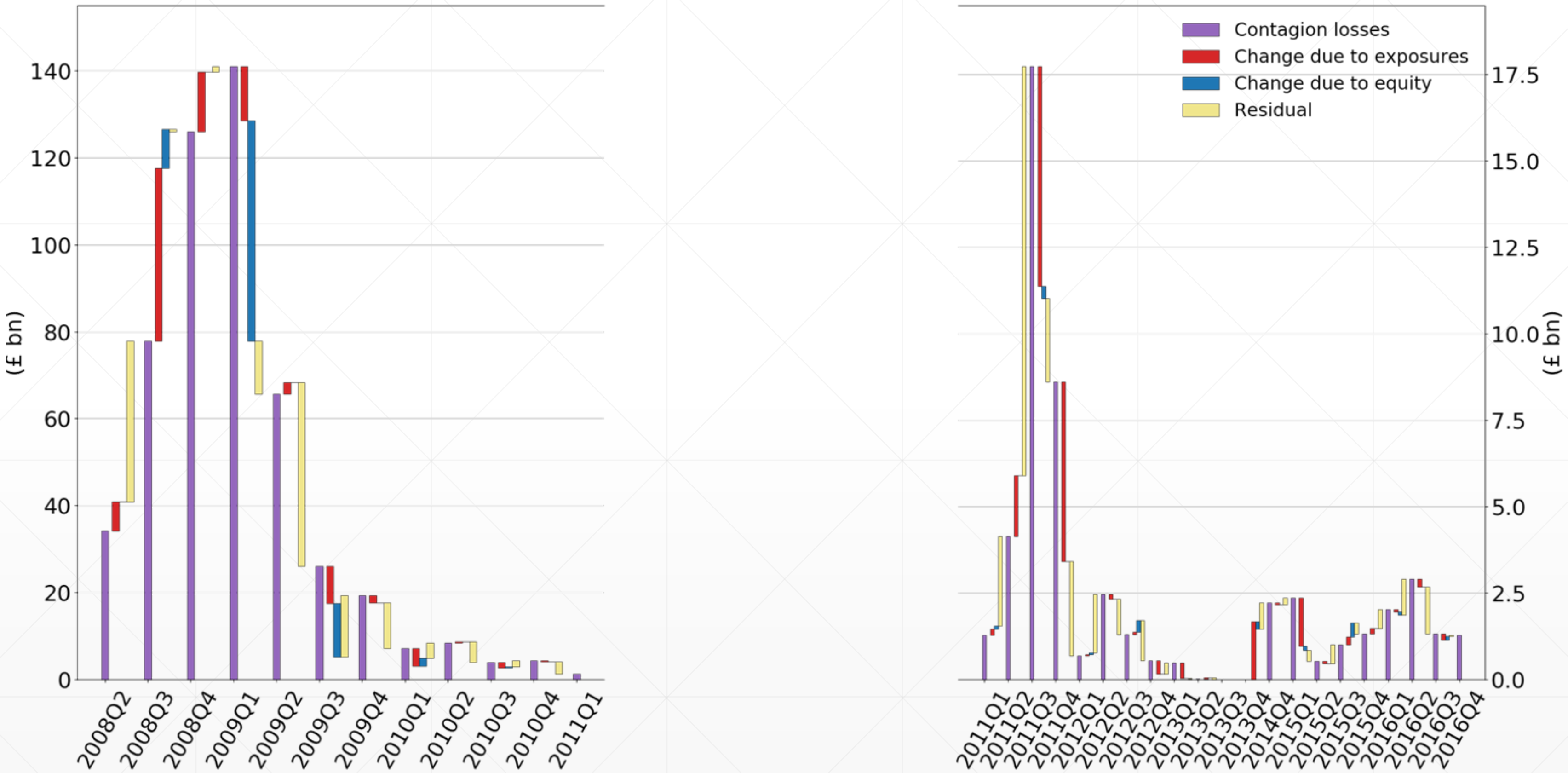


Contagion losses decline

- In order to isolate the effect in changes of equity and exposures we build synthetic balance sheets:
    1. Balance sheets of period 1 with exposures from period 2,
    2. Balance sheets of point 1 with initial equity from period 2
- As a robustness check we also do vice versa.



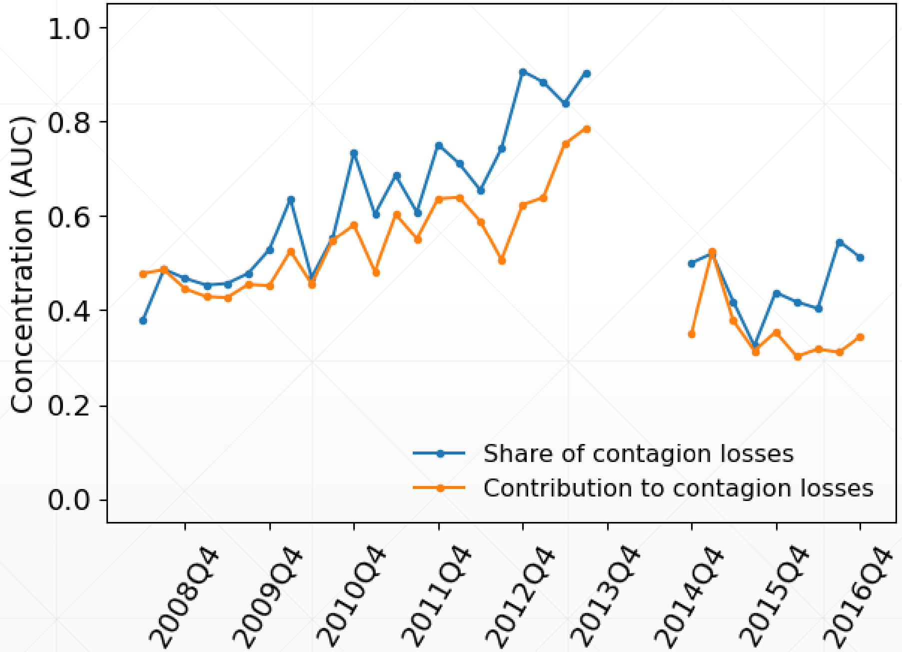
Decomposing the fall



Shock on equity = 50%, recovery rate = 0

Measures of concentration

- Relative vulnerability: fraction of contagion losses experienced by a bank
- Systemic importance: fraction of contagion losses caused by a bank
  - Selective bail-outs
  - Shapley decomposition of aggregate contagion losses



Shock on equity = 50%, recovery rate = 0

Conclusions

- When clearing payments money is locally conserved.
- Bankruptcy costs or ex-ante valuation can break conservation.
- There is a valuation framework that includes all those cases.
- We apply the framework to stress testing: The risk related to solvency contagion has shapely decreased from the peak of the crisis to today.

Info

- Neva Working Paper:
  https://ssrn.com/abstract=2795583
- Bank of England Staff Working Paper:
  https://ssrn.com/abstract=2996689
- Run solvency contagion on your own data!
  https://github.com/marcobardoscia/neva