Statistical aspects of geodesic flows in nonpositive curvature

Titles and abstracts

Viviane Baladi. There are no deviations to the ergodic integrals of Giulietti-Liverani flows on the two-dimensional torus.

Abstract. Giulietti and Liverani introduced a class of uniquely ergodic flows associated with Anosov diffeomorphisms F of the torus $\mathbb{T}^2$, and obtained expansions for their ergodic integrals in terms of the spectral data of the transfer operator of F. Using the Artin-Mazur zeta function and suitable anisotropic spaces, we show that this transfer operator has no non-trivial eigenvalues, and therefore there are no deviations to the ergodic integrals. Our proof also gives information on the speed of mixing of the measure of maximum entropy of Anosov diffeomorphisms of the torus $\mathbb{T}^2$.

Péter Bálint. Soft dispersing billiards.

Abstract. Soft dispersing billiards are obtained if in a Sinai billiard configuration with circular scatterers the usual hard core collisions are replaced by a centrally symmetric potential, allowing this way the particle to enter the scatterers. In joint work with Imre Péter Tóth, we proved that, for a class of potentials, the resulting discrete time dynamical system has good statistical properties. Although these results are not new, there remain several open problems.

Carl Dettmann. Infinite horizon Lorentz gases with static or pulsating scatterers.

Abstract. The Lorentz gas consists of a point particle colliding with an extended array of hard obstacles. It has been studied widely in statistical mechanics and ergodic theory as a deterministic diffusion process. Recent interest has focused on the infinite horizon case, where the scatterers are periodic and it is possible (but of zero probability) for the particle to avoid any collisions. In the two-dimensional static case this leads to logarithmic superdiffusion, and anomalous convergence of the second moment to twice the variance of the limiting normal distribution. Higher dimensions or flat points lead to a number of interesting variations. When the scatterers have a time-periodic radius, collisions typically increase the average velocity of the particle. In this case heuristic arguments and numerical simulations show that the infinite horizon enhances the acceleration, despite longer periods without collisions.

Sébastien Gouëzel. An approximate Livsic theorem.

Abstract. Livsic theorem in hyperbolic dynamics ensures that a Hölder function whose average vanishes along periodic orbits has to be a coboundary. I will discuss the situation where one only knows that the average is small, along periodic orbits of large but finite length: can one deduce that the function is close to being a coboundary, in a quantitative sense? Motivations and applications will also be discussed. Joint work with T. Lefeuvre.

Gerhard Knieper. Geodesic stretch and the rigidity of the marked length spectrum for closed manifolds of negative curvature.

Abstract. Recently Colin Guillarmou and Thibault Lefeuvre proved the local rigidity of the marked length spectrum for closed manifolds of negative curvature. More precisely, they showed that two Riemannian manifolds of negative curvature which are sufficiently close in the C^k topology, for a suitable k, are isometric provided they have the same marked length spectrum. Using the geodesic stretch and methods from thermodynamical formalism we will give a different proof for the local rigidity. At the same time we obtain new stability estimates for the length spectrum. Based on the geodesic stretch we are also able to define a metric on the space of negatively curved isometry classes. For Teichmüller spaces this metric has been introduced by Thurston. This is joint work with Colin Guillarmou and Thibault Lefeuvre.

Carlos Matheus. Counting special Lagrangian fibrations on generic twistor families of K3 surfaces.

Abstract. Partly motivated by a sort of "analogy" between translation and K3 surfaces, Simion Filip extended the classical results of Howard Masur and William Veech for the problem of counting cylinders in translation surfaces by showing that the number $N(V)$ of special Lagrangian fibrations with volume $< V$ on generic twistor families of K3 surfaces is $N(V) = cV^{20} + O(V^{20-a})$ for some constants $c > 0, a > 0$. In this talk, we discuss a joint work with Nicolas Bergeron proving that Filip's theorem is valid for any $0 < a < 4/697633$.

Françoise Pène. Mixing estimates for periodic Lorentz gases with finite or infinite horizon.

Abstract. Periodic Lorentz gases are particular cases of a general framework of $\mathbb{Z}^d$–cover of hyperbolic dynamical system. In this context, the rate of mixing is directly related to the local limit theorem of the step function. This enables us, when the horizon is finite, to obtain a mixing expansion of every order for the collision map, but also for the flow. Contrarily to previous expansions obtained in other contexts of dynamical systems preserving an infinite measure, the coefficients appearing in our expansion are linearly independent. This provides in particular mixing rate for null integral observables. The result for the flow (in the finite horizon case) is a recent joint work with Dmitry Dolgopyat and Péter Nandori. In the more complicated case of infinite horizon, error terms in mixing estimates for the collision map, including results for some null integral observables, have been obtained very recently in a joint work with Dalia Terhesiu.

Mark Pollicott. Two questions on going from $\kappa < 0$ to $\kappa \leq 0$.

Abstract. We want to consider two possible properties of smooth Riemannian metrics on compact surfaces. Firstly, the differentiability of topological entropy. Secondly, the density of closed geodesics which are null in homology. Both properties are well-known and easy to establish in the case of metrics of strictly negative curvature ($\kappa < 0$). We want to consider the more general case of rank one nonpositive curvature ($\kappa \leq 0$).

Barbara Schapira. Strong positive recurrence in negative curvature and applications.

Abstract. I will present a geometric notion of strong positive recurrence on noncompact negatively curved manifolds, which has many applications, in particular to the existence of a measure of maximal entropy, the regularity of entropy along smooth variations of the metric, and many other things. This is a joint work with Samuel Tapie.

Richard Sharp. Periodic orbit growth on covers of Anosov flows.

Abstract. It is well-known that the topological entropy of an Anosov flow on a compact manifold describes the exponential growth rate of its periodic orbits. If we pass to a regular cover of the manifold then we can consider a corresponding growth rate for periodic orbits of the lifted flow. This growth rate is bounded above by the original topological entropy but if the cover is infinite then the growth rate may be strictly smaller. In the important special case of a geodesic flow over a compact manifold with negative sectional curvatures, we have equality if and only if the cover is amenable (Dougall & Sharp, 2016). This statement fails for general Anosov flows but we will discuss a recent result that gives a natural generalisation. (Joint work with Rhiannon Dougall.)

Dalia Terhesiu. Sharp mixing for Lorentz gases (maps) with infinite horizon.

Abstract. In joint work with Françoise Pené, we obtain sharp error terms in local limit theorems and mixing for Lorentz maps with infinite horizon. First order terms in both local limit theorems and mixing have been previously covered, but higher order expansion pose new challenges. I will present part of our main results, focusing on the new ideas.

Dan Thompson. The K-property and a Lindeberg CLT for equilibrium states in non-positive curvature.

Abstract. Equilibrium states for geodesic flows over compact rank 1 manifolds and sufficiently regular potential functions were studied by Burns, Climenhaga, Fisher and myself. We showed that if the higher rank set does not carry full topological pressure then the equilibrium state is unique. In this talk, I will describe new joint work with Ben Call, which shows that these equilibrium states have the Kolmogorov property. If time permits, I will also describe a new result with Tianyu Wang, which shows that certain sequences of measures supported on collections of regular closed geodesics asymptotically satisfy a type of Central Limit Theorem. This extends ideas of Denker-Senti-Zhang who recently obtained analogous results in a uniform discrete-time setting.