Workshop Geometric Partial Differential Equations: tentative schedule, titles and abstracts

Location: we are in the Zeeman building on the University of Warwick campus (not the Mathematical Sciences building next door ...)

MONDAY - ROOM: B3.03 (third floor)

9.00 - 9.50 : **Jean Lagacé (KCL)**

10.00 - 10.50 : **Mikhail Karpukhin (UCL)**

11.00 - 11.30 : Coffee/tea in common room

11.30 - 12.20 : **Daniel Stern (Chicago)**

12.30 - 14.00 : Lunch in common room

14.00 - 14.50 : **Marco Guaraco (Imperial)**

15.00 - 15.30 : Coffee/tea in common room

15.30 - 16.20 : **Costante Bellettini (UCL)**

17.30 : wine/beer/food reception in common room (with food from 18.00)

TUESDAY - ROOM: MS.04 (second floor)

9.00 - 9.50 : **Michael Struwe (ETH-Zürich)**

10.00 - 10.50 : **Juan Dávila (Bath)**

11.00 - 11.30 : Coffee/tea in common room

11.30 - 12.20 : **Tobias Lamm (Karlsruhe)**

12.30 - 14.00 : Lunch in common room

14.00 - 14.50 : **Xavier Ros-Oton (Barcelona)**

15.00 - 15.30 : Coffee/tea in common room

15.30 - 16.20 : **Paul Laurain (Paris)**

18.00 - Basic food/beer at Fusion sports bar (on campus, pay yourself) for those interested in the semi-final of the world cup

WEDNESDAY - ROOM: MS.04

9.00 - 9.50 : **Joaquim Serra (ETH-Zürich)**

10.00 - 10.50 : **Connor Mooney (UC Irvine)**

11.00 - 11.30 : Coffee/tea in common room

11.30 - 12.20 : **Manuel del Pino (Bath)**

12.30 - 14.00 : Lunch in common room

14.00 - 14.50 : **Jonas Hirsch (Leipzig)**

17.30 - 19.00 Workshop dinner/buffet at Scarman (on campus) for all participants

19.00 : World Cup semi-final should be available in the bar near the buffet for those interested

THURSDAY - ROOM: B3.03

9.00 - 9.50 : **Tristan Rivière (ETH-Zürich)**

10.00 - 10.50 : **Huy Nguyen (QMUL)**

11.00 - 11.30 : Coffee/tea in common room

11.30 - 12.20 : **Stephen Lynch (Imperial)**

12.30 - 14.00 : Lunch in common room

14.00 - 14.50 : **Roger Moser (Bath)**

15.00 - 15.30 : Coffee/tea in common room

15.30 - 16.20 : **Robert Haslhofer (Toronto)**

19.00 : Dinner at Indian Edge in Kenilworth for speakers only (sorry)

FRIDAY - ROOM: MS.04

9.00 - 9.50 : **Melanie Rupflin (Oxford)**

10.00 - 10.50 : **Mario Schulz (Münster)**

11.00 - 11.30 : Coffee/tea in common room

11.30 - 12.20 **Maxwell Stolarski (Warwick)**

(No lunch provided – available from various outlets on campus)

Titles and Abstracts

Speaker: Costante Bellettini (UCL)

Title: Hypersurfaces with mean curvature prescribed by an ambient function

Abstract: Let N be a compact Riemannian manifold of dimension 3 or higher, and g a Lipschitz non-negative (or non-positive) function on N. In joint works with Neshan Wickramasekera we prove that there exists a two-sided (well-defined global unit normal) closed hypersurface M whose mean curvature attains the values prescribed by g. Except possibly for a small singular set (of codimension 7 or higher), the hypersurface M is C^2 immersed; more precisely, the immersion is a quasi-embedding, namely the only non-embedded points are caused by tangential self-intersections (around such a non-embedded point, the local structure is given by two disks, lying on one side of each other, and intersecting tangentially, as in the case of two spherical caps touching at a point). A special case of PMC (prescribed-mean-curvature) hypersurfaces is obtained when g is a constant, in which case the above result gives a CMC (constant-mean-curvature) hypersurface for any prescribed value of the mean curvature. The proof employs an Allen--Cahn approximation scheme, classical minmax, and gradient flow arguments. A key issue in the construction (and, more generally, in related compactness questions) is the possible formation of "hidden boundaries".

Speaker: Juan Dávila (Bath)

Title: Singularity formation for the two-dimensional harmonic map flow into S^2.

Abstract: We construct finite time blow-up solutions to the harmonic map flow into the sphere S^2, in a smooth bounded domain in 2 dimensions. We find initial and boundary data so that the solution blows-up precisely at prescribed points. The profile around each point is close to an asymptotically singular scaling of a 1-corrotational harmonic map. We analyze stability of this phenomenon if k=1 and discuss then what happens in the direction of instability.
This is joint work with Manuel del Pino (U of Bath) and Juncheng Wei (UBC).

Speaker: Manuel del Pino (Bath)

Title: Dynamics of concentrated vortices in 2D and 3D Euler flows

Abstract: A classical problem that traces back to Helmholtz and Kirchhoff is the understanding of the dynamics of solutions to the Euler equations of an inviscid incompressible fluid when the vorticity of the solution is initially concentrated near isolated points in 2d or vortex lines in 3d. We discuss some recent results on these solutions' existence and asymptotic behavior. We describe, with precise asymptotics, interacting vortices, and traveling helices, and extension of these results for the 2d generalized SQG. This is research in collaboration with J. Dávila, A. Fernández, M. Musso, and J. Wei.

Speaker: Marco Guaraco (Imperial)

Title: Solving Plateau's problem via semilinear elliptic equations: boundary behavior

Abstract: Plateau's problem is a central problem in Geometric Analysis and PDEs. Given an arbitrary closed curve in R^3, it asks for the existence of an area minimizing embedded surface with boundary equal to the given curve. In 1960, Federer and Fleming conceived the theory of currents as a framework for solving this problem. A different approach was proposed in 1990 by Fröhlich and Struwe, through the study of level sets of semilinar elliptic equations. They showed the existence of a minimal surface which was smooth away from the curve. I will talk about joint work with Stephen Lynch, in which we show that the surface is also smooth up to the boundary, thus completing a new solution to the (smooth) Plateau's problem.

Speaker: Robert Haslhofer (Toronto)

Title: Classification of compact ancient noncollapsed flows in R^4

Abstract: To capture singularities under mean curvature flow one wants to understand all ancient solutions. In addition to shrinkers and translators one also encounters ancient ovals, namely compact ancient noncollapsed flows that are not self-similar. In this talk, I will explain that any bubble-sheet oval for the mean curvature flow in R^4, up to scaling and rigid motion, either is the O(2)×O(2)-symmetric ancient oval constructed by White, or belongs to the one-parameter family of Z_2^2×O(2)-symmetric ancient ovals constructed by Du and myself. In particular, this seems to be the first instance of a classification result for geometric flows that are neither cohomogeneity-one nor selfsimilar. This is joint work with Beomjun Choi, Toti Daskalopoulos, Wenkui Du and Natasa Sesum. I will also briefly mention the noncompact case, which is joint work with Kyeongsu Choi and Or Hershkovits.

Speaker: Jonas Hirsch (Leipzig)

Title: Interior regularity for two-dimensional stationary Q-valued maps

Abstract: We consider 2-dimensional Q-valued maps that are stationary with respect to outer and inner variations of the Dirichlet energy.

In this talk I would like to present some idea's that can be used to show that these maps are actually Hölder continuous and that the dimension of their singular set is at most one.

I would like to highlight how one can adapt ideas of M. Grüter to ``reverse'' the Douglas and Rado approach. We will use properties of minimal surfaces to show continuity of stationary ``harmonic'' functions. This idea helps us to localize in the target space \mathcal{A}_Q and overcome the problem of the absence of an ``honest'' PDE.

Joint work with L. Spolaor.

Speaker: Mikhail Karphukin (UCL)

Title: Optimization of eigenvalues and minimal surfaces

Abstract: The study of optimal upper bounds for Laplace eigenvalues on closed surfaces is a classical problem of spectral geometry going back to J. Hersch, P. Li and S.-T. Yau. Its most fascinating feature is the connection to the theory of minimal surfaces in spheres. Optimization of Steklov eigenvalues is an analogous problem on surfaces with boundary. It was popularized by A. Fraser and R. Schoen, who discovered its connection to the theory of free boundary surfaces in Euclidean balls. Despite many widely known empiric parallels, an explicit link between the two problems was discovered only in the last two years. In the present talk, we will show how Laplace eigenvalues can be recovered as certain limits of Steklov eigenvalues and discuss the applications of this construction to the geometry of minimal surfaces. The talk is based on joint works with D. Stern.

Speaker: Jean Lagacé (Kings)

Title: Measure eigenvalues: theory and applications

Abstract: In recent years, eigenvalues defined variationally with respect to a measure have provided a good abstract framework to study both the Laplace and Steklov eigenvalue problem. In this talk, I will explain this framework, provide a few useful continuity results and describe some applications

Based on joint work with A. Girouard (Laval) and M. Karpukhin (UCL)

Speaker: Tobias Lamm (KIT Karlsruhe)

Title: Index estimates for sequences of harmonic maps

Abstract: We study the limiting behavior of the index and the nullity of sequences of harmonic maps from a two-dimensional Riemann surface into a general target manifold. We show upper and lower bounds for the index of the sequence in terms of the index of the so-called bubble limit.

This is joint work with Jonas Hirsch (Leipzig).

Speaker: Paul Laurain (Paris Cité)

Title: Fine multibubble analysis for the Brezis-Nirenberg problem

Abstract: I am going to expose some recent results on the behavior, on a domain, of the Yamabe type equation
\Delta u + a u =u^{2^*-1}
where 2^* is the critical Sobolev exponent. After remembering how this equation plays a central role in the prescription of the scalar curvature and the seminal result of Brezis and Nirenberg. We will see that there exists a critical potential for which there is no more positive solutions. Hence, I will describe our main result which fully describes the behavior of solutions when the potential approaches the critical one. In particular, we will see that the Robin function of the operator \Delta +a plays a crucial role in this analysis. This work is a collaboration with Tobias König.

Speaker: Stephen Lynch (Imperial)

Title: Ancient solutions of Ricci flow with Type I curvature growth

Abstract: A solution of Ricci flow is called ancient if it extends infinitely far back in time. Such solutions arise naturally as blow-up limits at singularities of the flow. Classifying the possible geometries of ancient solutions is a fascinating problem which has received much attention. I will describe work with A. Royo Abrego, where we consider nonnegatively curved Type I ancient solutions. The main result is a dimension-reduction statement, asserting that a simply connected, noncollapsed Type I ancient solution with nonnegative sectional curvature is the product of a Euclidean factor with a compact solution. If the solution is additionally weakly PIC2, then it is a locally symmetric space, and the compact factor splits as a product of shrinking solitons.

Speaker: Connor Mooney (UC Irvine)

Title: The anisotropic Bernstein problem

Abstract: The Bernstein problem asks whether entire minimal graphs in R^{n+1} are necessarily hyperplanes. It is known through spectacular work of Bernstein, Fleming, De Giorgi, Almgren, Simons, and Bombieri-De Giorgi-Giusti that the answer is positive if and only if n < 8. The anisotropic Bernstein problem asks the same question about minimizers of parametric elliptic functionals, which are natural generalizations of the area functional that both arise in many applications and offer important technical challenges. We will discuss the recent solution of this problem (the answer is positive if and only if n < 4). This is joint work with Y. Yang.

Speaker: Roger Moser (Bath)

Title: Infinity-harmonic functions and the inverse mean curvature flow

Abstract: Infinity-harmonic functions are solutions of the Aronsson equation and solutions of the optimal Lipschitz extension problem. For sufficiently regular solutions, the norm of the gradient is constant along their streamlines. For a two-dimensional domain, it can be seen that these streamlines coincide with the time slices of a solution of the inverse mean curvature flow (the evolution equation such that the velocity is the reciprocal of the mean curvature).

In general, however, solutions are not smooth enough to draw these conclusions. Nevertheless, there is a deeper connection between the two problems, which can still be proved in the weak sense under certain assumptions.

Speaker: Huy The Nguyen (QMUL)

Title: An \varepsilon-regularity theorem for the Allen-Cahn flow

Abstract: We will consider the relationship between the Allen-Cahn flow, a semilinear parabolic PDE and the mean curvature flow. In particular, we will show an analogue of the Brakke's local regularity theorem for the Allen-Cahn flow that is we show uniform C^{2,\alpha}-regularity for the transition layers converging to smooth mean curvature flows as \varepsilon tends to 0 under an almost unit-density assumption and show how to use this theorem to answer a question of Ilmanen on no cancellation in the limit. We will also discuss some more recent relations between these two equations. This talk is based on joint work with Shengwen Wang.

Speaker: Tristan Rivière (ETH Zürich)

Title: Morse Index Stability in Bubble Tree Analysis

Abstract: The lower semi-continuity of the Morse index for weakly converging sequences of critical points to coercive variational problems is a rather robust property.

In this talk we will be interested with the reverse inequality which is much more delicate. We will be presenting a general strategy for proving the upper semi continuity of the Morse index plus nullity for critical points to general conformally invariant variational problems including Harmonic Maps and Prescribed Mean Curvature surfaces into arbitrary manifolds, Willmore surfaces, Yang-Mills Fields...etc. If time permits, we will mention possible extensions to approximated functional to which Palais-Smale theory applies with the perspective of implementing these bounds to limits of solutions to concrete minmax problems.

Speaker: Xavier Ros-Oton (Barcelona)

Title: The singular set in the Stefan problem

Abstract: The Stefan problem, dating back to the XIXth century, is probably the most classical and important free boundary problem. The regularity of free boundaries in the Stefan problem was developed in the groundbreaking paper (Caffarelli, Acta Math. 1977). The main result therein establishes that the free boundary is C^\infty in space and time, outside a certain set of singular points.
The fine understanding of singularities is of central importance in a number of areas related to nonlinear PDEs and Geometric Analysis. In particular, a major question in such a context is to establish estimates for the size of the singular set. The goal of this talk is to present some new results in this direction for the Stefan problem. This is a joint work with A. Figalli and J. Serra.

Speaker: Melanie Rupflin (Oxford)

Title: Quantitative estimates for almost harmonic maps

Abstract: For geometric variational problems one often only has weak, rather than strong, compactness results and hence has to deal with the problem that sequences of (almost) critical points u_j can converge to a limiting object with different topology.

A major challenge posed by such singular behavior is that the seminal results of Simon on Lojasiewicz inequalities, which are one of the most powerful tools in the analysis of the energy spectrum of analytic energies and the corresponding gradient flows, are not applicable.

In this talk we present a method that allows us to prove Lojasiewicz inequalities for almost harmonic maps that form simple singularities and explain how these results allow us to draw new conclusions about the energy spectrum of harmonic maps and the convergence of harmonic map flow for low energy maps from surfaces into analytic manifolds.

Speaker: Mario Schulz (Münster)

Title: Non-uniqueness of free boundary minimal surfaces

Abstract: Free boundary minimal surfaces arise naturally in partitioning problems for convex bodies and capillarity problems for fluids. The case of the Euclidean unit ball as ambient manifold is of particular interest for the study of extremal metrics for Steklov eigenvalues on surfaces with boundary. The theory has been developed in various interesting directions, yet many fundamental questions remain open: Can a surface of any given topology be realized as an embedded free boundary minimal surface in the 3-dimensional Euclidean unit ball? And, if so, in which class are such embeddings unique up to ambient isometries? The topology alone does do not determine a free boundary minimal surface uniquely in general. I will present an even stronger non-uniqueness result by providing pairs of non-isometric free boundary minimal surfaces having not only the same topology but also the same symmetry group. Joint work with Alessandro Carlotto and David Wiygul.

Speaker: Joaquim Serra (ETH Zürich)

Title: Nonlocal approximation of minimal surfaces: optimal estimates from stability

Abstract: Minimal surfaces in closed 3-manifolds are classically constructed via the Almgren-Pitts approach. The Allen-Cahn approximation has proved to be a powerful alternative, and Chodosh and Mantoulidis (in Ann. Math. 2020) used it to give a new proof of Yau's conjecture for generic metrics and establish the multiplicity one conjecture.

In a recent paper with Chan, Dipierro, and Valdinoci we set the ground for a new approximation based on nonlocal minimal surfaces. More precisely, we prove that stable s-minimal surfaces in the unit ball of R^3 satisfy curvature estimates that are robust as s approaches 1 (i.e. as the energy approaches that of classical minimal surfaces).

Moreover, we obtain optimal sheet separation estimates and show that critical interactions are encoded by nontrivial solutions to a (local) "Toda type" system.

As a nontrivial application, we establish that hyperplanes are the only stable s-minimal hypersurfaces in R^4, for s sufficiently close to 1.

Speaker: Daniel Stern (Chicago)

Title: Existence of harmonic maps and applications in spectral geometry

Abstract: I'll discuss recent progress on the existence theory for harmonic maps, in particular the existence of harmonic maps of optimal regularity from manifolds of dimension n>2 to every non-aspherical closed manifold containing no stable minimal two-spheres. As an application, we'll see that every manifold carries a canonical family of sphere-valued harmonic maps, which (in dimension<6) stabilize at a solution of a spectral isoperimetric problem generalizing the conformal maximization of Laplace eigenvalues on surfaces. Based on joint work with Mikhail Karpukhin.

Speaker: Maxwell Stolarski (Warwick)

Title: Mean Curvature Flow Singularities with Bounded Mean Curvature

Abstract: In 1984, Huisken showed that the second fundamental form always blows up at a finite-time singularity for the mean curvature flow. Naturally, one might then ask if the mean curvature must also blow up at a finite-time singularity. We'll discuss work that shows the answer is "no" in general, that is, there exist mean curvature flow solutions that become singular with uniformly bounded mean curvature.

Speaker: Michael Struwe (ETH Zürich)

Title: Plateau flow, alias the heat flow for half-harmonic maps

Abstract: Using the Millot-Sire/Moser interpretation of the half-Laplacian on S^1 as the Dirichlet-to-Neumann operator for the Laplace equation on the ball B, we devise a classical approach to the heat flow for half-harmonic maps from S^1 to a closed target manifold N\subset R^n, recently studied by Wettstein, and for arbitrary finite-energy data we obtain a result fully analogous to classical results for the harmonic map heat flow of surfaces and in similar generality.

When N is a smoothly embedded, oriented closed curve \Gamma\subset R^n the half-harmonic map heat flow may be viewed as an alternative gradient flow for the Plateau problem of disc-type minimal surfaces.