measures & materials

BOOK OF ABSTRACTS

25–28 MARCH 2024

UNIVERSITY OF WARWICK

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M&Ms in Warwick - Schedule

	Mon 25/3	Tue 26/3	Wed 27/3	Thu 28/3
9:00 - 10:00	Registration	Garroni	Francfort	Babadjian
	Coffee break	Coffee break	Coffee break	Coffee break
10:30 - 11:30	Mielke	Arroyo-Rabasa	Del Nin	Stefanelli
11:30 - 12:30	Maor	Marziani	Benesova	Scardia
	Lunch	Lunch	Lunch	Lunch
14:00 - 15:00	Marchese		Stuvard	
	Coffee Break	Excursion	Coffee break	
15:30 - 16:30	Zwicknagl		Iurlano	
18:30			Social Dinner	

All talks will take place in Zeeman Building, B3.02.

M&Ms in Warwick - Abstracts

Local Poincaré constants and BMO functionals for BV functions

Adolfo Arroyo-Rabasa
University of Bonn

Since the characterization of constant maps without using classical derivatives (Bourgain et al.), there has been a growing interest on the link between derivatives and limits of BMO seminorms. Ambrosio et al. described perimeter using mean oscillations, later extended to describe the total variation of SBV functions as limits of BMO-type seminorms on ε-size cubes (as $\varepsilon \to 0$). However, this uniform ε-size constraint hinders a precise description for general BV functions. Can we relax this model to find a BMO-type description of BV functions?

In this talk, I will give an answer to this question. First, I will introduce the concept of local Poincaré constant (LPC) for a BV function, as a tool to understand the relation between its mean oscillation and its total variation at small scales. Along with this new quantity, I will introduce a geometric relaxation of the functionals by Ambrosio et al., by considering cubes of sidelength smaller than or equal to ε. I will then explain how our functionals converge (as $\varepsilon \to 0$) to a functional defined on BV, represented by integration in terms of the LPC and the total variation.

Additionally, I will discuss a cell-formula representation for the LPC, as the maximum mean oscillation amongst BV blow-ups. As we shall see, this demonstrates our new functional extends the original one from SBV to all BV functions. If time permits, I will also share with you an interesting conjecture concerning a more general representation of our functional, connected with our understanding of the LPC, the fine properties of BV functions, and covering theorems for gradient measures.

This is joint work with Paolo Bonicatto (Trento) and Giacomo Del Nin (MPI).

Uniqueness and characteristic flow for a non strictly convex singular variational problem

Jean-François Babadjian
Université Paris Saclay

This talk addresses the question of uniqueness of the minimizers of a convex but not strictly convex integral functional with linear growth in a two-dimensional setting. The integrand - whose precise form derives directly from the theory of perfect plasticity - behaves quadratically close to the origin and grows linearly once a specific threshold is reached. We make use of spatial hyperbolic conservation laws hidden in the structure of the problem to tackle uniqueness. Our argument strongly relies on the regularity of a vector field - the Cauchy stress in the terminology of perfect plasticity - which allows us to define characteristic lines, and then to employ the method of characteristics. Using the detailed structure of the characteristic landscape, we show that this vector field is actually continuous, save for possibly two points. The different behaviors of the energy density at zero and at infinity imply an inequality constraint on the Cauchy stress. Under a barrier type convexity assumption on the set where the inequality constraint is saturated, we show that uniqueness holds for pure Dirichlet boundary data. This is a joint work with Gilles Francfort.

Non-interpenetration of rods derived by Γ-limits

Barbora Benesova

Charles University Prague

Ensuring non-interpenetration of matter is a fundamental prerequisite when modeling the deformation response of solid materials. In this contribution, we thoroughly examine how this requirement, equivalent to the injectivity of deformations within bulk structures, manifests itself in dimensional-reduction problems. Specifically, we focus on the case of rods embedded in a two-dimensional plane. Our results focus on Γ-limits of energy functionals that enforce an admissible deformation to be a homeomorphism. These Γ-limits are evaluated along a passage from the bulk configuration to the rod arrangement. The proofs rely on the equivalence between the weak and strong closures of the set of homeomorphisms from $\mathbb{R}$ to $\mathbb{R}^2$, a result that is of independent interest and that we establish in this paper, too. This is joint work with D. Campbell, S. Hencl and M. Kružík.

Existence and uniqueness for the transport of currents by Lipschitz vector fields

Giacomo Del Nin

MPI Leipzig

Currents represent a weak notion of surface that has been used, among many things, to model dislocations in materials, i.e., defects in their crystalline structure that appear when they are subject to deformation. With this modeling problem in mind, I will introduce the transport equation for currents, which describes their movement driven by a given vector field. I will then present an existence and uniqueness result when the vector field has Lipschitz regularity. This result was obtained in collaboration with Paolo Bonicatto (Trento) and Filip Rindler (Warwick).

Homogenization in linearized elastoplasticity

Gilles Francfort

Flatiron Institute New York City

In this talk I will present old joint work with Alessandro Giacomini. The goal is to perform periodic homogenization in the class of, say, two-phase elastoplastic mixtures of Von Mises type. As I will explain, the resulting model has a far more intricate rheology than that of each constituent casting (additional) doubt on the consistency of those models. I (we) have no clue as to what should be done when departing from this two (or multi)-phase periodic framework.

Variational modelling of grain boundaries in dimension 2

Adriana Garroni

University of Rome Sapienza

I will describe a variational model introduced by Lauteri and Luckhaus which accounts for the presence of material defects in polycrystals. In this model incompatibilities at the boundary of two grains may be resolved by the presence of dislocations. The asymptotic analysis in terms of Gamma-convergence yields an anisotropic energy for grain boundaries which scales for small angles as the so called Read and Shockley energy. A phase field approximation of such a limiting energy which can be used to detect grains in crystal imagines will be also presented.

Existence of solutions for the 2d contact problem with friction

Flaviana Iurlano

University of Genoa

In this talk, we focus on the incremental quasistatic contact problem with Coulomb friction in linearised elasticity (Signorini-Coulomb problem) and present optimal existence results for the most general 2D problem with arbitrary geometry and elasticity modulus tensor. The problem is reduced to a variational inequality involving a nonlinear operator which handles both elasticity and friction. This operator is proved to fall into the class of the so-called Leray-Lions operators, so that a result of Brézis can be invoked to solve the variational inequality. It turns out that one property in the definition of Leray-Lions operators is difficult to check and requires proving a new fine property of the 2D linear elastic Neumann-to-Dirichlet operator. In the case of isotropic elasticity, either homogeneous or heterogeneous, we prove the existence of solutions to the Signorini-Coulomb problem for arbitrarily large friction coefficient. In the case of anisotropic elasticity, we exhibit an example of nonexistence of a stationary solution for large friction coefficient; then, we provide a condition on the friction coefficient under which anisotropic solutions are proved to exist. This is a joint work with P. Ballard.

Gamma-convergence for plane to wrinkles transition problem

Roberta Marziani

University of L'Aquila

We consider a variational problem modelling transition between flat and wrinkled region in a thin elastic sheet, and identify the Γ-limit as the sheet thickness goes to 0. The limiting problem is scalar and convex, but constrained and posed for measures. For the Γ-liminf inequality we first pass to quadratic variables so that the constraint becomes linear, and then obtain the lower bound by Reshetnyak's theorem. The construction of the recovery sequence for the Γ-limsup inequality relies on mollification of quadratic variables , and careful choice of multiple construction parameters. Eventually for the limiting problem we show existence of a minimizer and equipartition of the energy for each frequency. This is a joint work with Peter Bella.

Strain-gradient plasticity from a multiplicative decomposition approach

Cy Maor

Hebrew University of Jerusalem

Dislocations can be viewed as stress-free elastic bodies that undergo some "cut and weld" procedures (corresponding to a glide mechanism in a crystal), and are closely related to plastic deformations in crystalline materials. In the last 15 years there have been significant advancements in rigorously deriving plasticity models from elemental models of finitely many dislocations. In most of these works, the elemental model is that of an additive decomposition of the strain: the variable is a strain field with a prescribed curl (concentrated at the core of the dislocations). This model is, in a sense, a pre-assumed linearization of the material internal geometry, which is more accurately represented by a multiplicative strain decomposition. In this talk I will describe this multiplicative decomposition model, and show how strain gradient plasticity in two-dimensions can be directly obtained from it. No previous knowledge in dislocation theory or plasticity will be assumed. Joint work with Raz Kupferman.

The singularities of mass minimizing currents modulo p

Andrea Marchese
University of Trento

Mass minimizing integral currents modulo p are minimal surfaces exhibiting a geometric complexity which is observed in soap films but it is not present in classical solutions of the Plateau's problem. For instance, the constraint allows p minimal surfaces to meet together with the same orientation across a common interface, without creating a boundary. In a series of joint works with De Lellis, Hirsch, Spolaor, and Stuvard we described the fine structure of their singularities.

Balanced-Viscosity Solutions for generalized gradient systems and a delamination problem

Alexander Mielke
WIAS Berlin

In generalized gradient systems the kinetic relation is nonlinear and may contain a rate-independent part. Under slow loading conditions the solutions will follow a rate-independent evolution except when instabilities arising from non-convexity of the energy lead to the formation of jumps on a short time scale. The understanding of the behavior in this vanishing-viscosity limit can be attacted by different methods, like the direct vanishing-viscosity approach introduced in [DD*08], the two-speed solutions in [RSV21], or the visco-energetic solutions in [DR*21].

We will show how the theory of Balanced-Viscosity solutions, which was developed in [MRS09, MRS16, MiR23], can be used to define a notion of solutions that has desirable upper-semicontinuity properties (also called stability). Finally, we will provide an application to a model where an elastic body can delaminate along a given crack path.

References

[DD*08] G. Dal Maso, A. DeSimone, M. G. Mora, and M. Morini: *A vanishing viscosity approach to quasistatic evolution in plasticity with softening.* Arch. Rational Mech. Anal. **189**:3 (2008) 469–544.

[DR*21] G. Dal Maso, R. Rossi, G. Savaré, and R. Toader: *Visco-energetic solutions for a model of crack growth in brittle materials.* arXiv:2105.00046 (2021).

[MiR23] A. Mielke and R. Rossi: *Balanced-Viscosity solutions to infinite-dimensional multi-rate systems.* Arch. Rational Mech. Anal. **247**:53 (2023) 1–100.

[MRS09] A. Mielke, R. Rossi, and G. Savaré: *Modeling solutions with jumps for rate-independent systems on metric spaces.* Discr. Cont. Dynam. Systems Ser. A **25**:2 (2009) 585–615.

[MRS16] ______: *Balanced Viscosity (BV) solutions to infinite-dimensional rate-independent systems.* J. Europ. Math. Soc. **18** (2016) 2107–2165.

[RSV21] F. Rindler, S. Schwarzacher, and J. L. Velázquez: *Two-speed solutions to non-convex rate-independent systems.* Arch. Rational Mech. Anal. **239** (2021) 1667–1731.

Shape Optimisation for nonlocal anisotropic energies

Lucia Scardia
Heriot-Watt University

We consider shape optimisation problems for sets of prescribed mass, where the driving energy functional is nonlocal and anisotropic. More precisely, we deal with the case of attractive/repulsive interactions in two and three dimensions, where the attraction is quadratic and the repulsion is given by an anisotropic variant of the Coulomb potential. Under the sole assumption of strict positivity of the Fourier transform of the interaction potential, we show the existence of a threshold value for the mass above which the minimiser is an ellipsoid, and below which the minimiser does not exist. If, instead, the Fourier transform of the interaction potential is only nonnegative, we show the emergence of a dichotomy: either there exists a threshold value for the mass as in the case above, or the minimiser is an ellipsoid for any positive value of the mass. This is joint work with Riccardo Cristoferi and Maria Giovanna Mora.

Nonassociative plasticity at finite strains: modeling and analysis

Ulisse Stefanelli
University of Vienna

I will present a variational model for nonassociative elastoplastic evolution at finite strains. The constitutive material model features the occurrence of two distinct plastic potentials for activation and normality, respectively. The flow rule is variationally formulated by means of a specific state-dependent dissipation potential. I will introduce a suitably weak notion of quasistatic evolution in terms of measure-valued energetic solutions and prove existence. This is joint work with Andreas Vikelis (Vienna).

Surface tension revisited: from minimal surfaces to the soap films capillarity model

Salvatore Stuvard
University of Milan

Minimal surfaces have represented for decades the classical mathematical model for thin material structures in equilibrium subject to surface tension (e.g. soap films). By neglecting the intrinsic three-dimensional nature of such structures, this model fails at capturing some of the physical properties observed in experiments.

In this talk, I will take this point of view and show how a suitable version of the classical Plateau's problem can be recast as the singular limit of a family of minimisation problems defined in terms of Gauss' free energy in capillarity theory

under an additional spanning constraint. I will discuss existence of minimisers, their regularity, and their geometry.

Based on joint works of the speaker with King and Maggi (UT Austin), and with Bevilacqua and Velichkov (U Pisa).

On variational models for hetero-epitaxial growth with dislocations

Barbara Zwicknagl

Humboldt University Berlin

We study a variational model for the hetero-epitaxial deposition of a crystalline film on a rigid substrate. The energy functional accounts for the surface energy of the film's free surface, the elastic energy due to the crystallographic misfit between the film and the substrate, and the nucleation energy of dislocations. We discuss the scaling law for the infimal energy. The upper bound constructions suggest that in some parameter regimes isolated island structures or the presence of dislocations is energetically favorable. The lower bound builds on a new variant of a ball construction. This talk is based on joint work with Lukas Abel and Janusz Ginster (both HU Berlin).
