Modular Curves and their Arithmetic
Conference Booklet

Date: 6-8 December 2023

Organisers: Elvira Lupoian and Samir Siksek

Supported by:

Contents

Introduction

Location The conference is hosted by the Warwick Mathematics Institute, and will take place in the Zeeman Building. Talks will take place in various rooms in the building, as indicated by the schedule. Participants can navigate their way around campus and the Zeeman building using the interactive campus map:

https://warwick.ac.uk/about/visiting/maps/interactive/

Travel The Warwick University Campus is easily accessible from Coventry city center and Coventry train station via the 12X bus. The campus is also easily accessible from Leamington Spa railway station via bus.

Food Lunch and coffee will be provided in the Maths common room for each day of the conference. Times will vary each day, please refer to the schedule. There are various dinner options on campus, including Varsity, Benugo, Scarman and the Dirty Duck.

Conference Dinner The conference dinner will be held at Scarman conference center at 7pm on Thursday 7th December.

Schedule

Wednesday 6th December

9:00-9:25	Registration	Street
9:25-9:30	Welcome Remarks	IAS
9:30 -10:30	Filip Najman: "Quadratic Points on Modular Curves"	IAS
10:30 -11:30	Coffee Break	
11:30 - 12:30	Contributed Talks	IAS
12:30 -14:00	Lunch	
14:00 - 15:00	Steffen Müller: "p-adic Arakelov theory and quadratic Chabauty"	B3.02
15:00 - 16:00	Coffee Break	
16:00 - 17:00	Samuele Anni	MS.03

Thursday 7th December

10:00 -11:00	David Zywina: "Serre's open image theorem and families of modular curves"	MS.04
11:00 -11:30	Coffee Break	
11:30 - 12:30	Pïerre Parent: "Models for modular curves and some applications"	MS.04
12:30 -13:30	Lunch	
13:30 - 14:30	Hwajong Yoo: "The rational cuspdial subgroup of $J_0(N)$"	MS.04
14:30 - 15:00	Coffee Break	
15:00 - 16:00	Céline Maistret: "Local Arithmetic of Hyperelliptic Curves and Applications"	MS.04
16:00 -17:00	break	
17:00 - 18:00	Open Problem Session	MS.04
19:00	Conference Dinner	

Friday 8th December

9:30-10:30	Contributed Talks	B1.01
10:30-11:00	Coffee Break	
11:00 -12:00	Maarten Derickx: "Conjectures of ranks of modular Jacobians and linear bounds on prime order torsion on elliptic curves"	B1.01
12:00 -13:30	Lunch	
13:30 - 14:30	Jennifer Balakrishnan: "Mock Rational Points in Quadratic Chabauty"	D1.07
14:03 - 15:00	Coffee Break	
15:00 - 16:00	John Cremona: "Computing the isomorphism ring of an elliptic curves over a number field"	D1.07

Abstracts

On smooth plane models of modular curves

Samuele Anni

Aix-Marseille Université

In this talk we will prove that there are only finitely many modular curves that admit a smooth plane model. Moreover, if the degree of the model is greater than or equal to 19, no such curve exists. For modular curves of Shimura type we will show that none can admit a smooth plane model of degree 5, 6 or 7. Further, if a modular curve of Shimura type admits a smooth plane model of degree 8 we will see that it must be a twist of one of four curves.

This is joint work with Eran Assaf and Elisa Lorenzo Garcìa.

Mock rational points in quadratic Chabauty

Jennifer Balakrishnan

Boston University

We discuss the computation of rational points on curves via the quadratic Chabauty method. We also consider the mock rational points (those finitely many points that are not rational) that may arise in the quadratic Chabauty set, in view of a conjecture of Kim. We give conditions under which one can find algebraic mock rational points and present a number of examples, primarily in the case of punctured elliptic curves, produced in ongoing joint work with Betts.

Computing the endomorphism ring of an elliptic curve over a number field

John Cremona

University of Warwick

I will describe joint work with Andrew Sutherland, giving deterministic and probabilistic algorithms to decide whether a given monic irreducible polynomial H in Z[X] is a Hilbert class polynomial, and if so, which one. These algorithms can be used to determine whether a given algebraic integer is the j-invariant of an elliptic curve with complex multiplication (CM), and if so, the associated CM discriminant. Our algorithms admit simple implementations that are asymptotically and in practice faster than existing approaches.

Conjectures on ranks of modular jacobians and linear bounds on prime order torsion on elliptic curves

Maarten Derickx

Let S(d) denote the set of primes p such that there exists an elliptic curve over a number field of degree d with a point of order p. Oesterlé showed that the largest prime in S(d) is smaller than $(3^{d/2}+1)^2$, giving an exponential upperbound. However, explicit calculations for small values of d have shown that this bound is far from sharp. In this talk we will present some conjectures on the Mordell-Weil ranks of modular jacobians that imply a linear bound on the largest prime in S(d). We provide numerical evidence for these conjectures by verifying them for p < 100,000. This is joint work with Michael Stoll

Local arithmetic of hyperelliptic curves and applications

Céline Maistret

University of Bristol

In this talk, I will consider hyperelliptic curves over local fields of odd characteristics. I will first survey the "Cluster Picture" approach to understanding various arithmetic invariants of the curves and their Jacobians. Then, I will present two recent applications: computing Euler factors of curves (joint with A. Sutherland) and computing conductor exponents of family of Frey hyperelliptic curves (joint with M. Azon, M. Curco Iranzo, M. Khawaja and D. Mocanu).

p-adic Arakelov theory and quadratic Chabauty

Steffen Müller

University of Groningen

The quadratic Chabauty method for the computation of rational points on certain curves, due to Balakrishnan and Dogra, is the simplest nontrivial instance of Kim's nonabelian Chabauty. Based on p-adic Hodge theory, it produces a p-adic function F and a finite set T such that F maps the rational points into T. While T can be shown to be trivial for several interesting modular curves, its computation is in general a difficult problem.
I will present an alternative approach to quadratic Chabauty that is based on p-adic Arakelov theory. Time permitting, I will also discuss work in progress that uses this to obtain an algorithm to compute the set T. This is joint work with Amnon Besser and Padmavathi Srinivasan.

Quadratic points on modular curves

Filip Najman

University of Zagreb

In this talk I will survey results, old and new, on quadratic points on modular curves, as well as the arithmetic consequences of these results. I will also describe the techniques used to determine all the quadratic points on modular curves. The talk is based partially on joint work with Adžaga, Bruin, Keller, Michaud-Jacobs, Ozman and Vukorepa.

Models for modular curves and some applications

Pïerre Parent

Université de Bordeaux

If one wants to study rational points of an algebraic curve over Q, say, it may help a lot to be able to "reduce the situation modulo prime numbers". But that implies to define and compute good models for those objects over the integers. In that talk, I will present recent results about semistable or regular models for modular curves associated with maximal subgroups of GL_2 in prime level. Then I will try to give diophantine applications, such as effective versions of the Bogomolov conjecture in some new cases.

The rational cuspidal subgroup of $J_0(N)$

Hwajong Yoo

Seoul National University

We introduce several questions related to generalized Ogg's conjecture. One important object to study is the rational cuspidal subgroup of $J_0(N)$, which we still do not know in general. We discuss a certain strategy to compute this group, and prove that it works when the prime divisors of N are at most two odd primes. This is joint with M. Yu.

Serre's open image theorem and families of modular curves

David Zywina

Cornell University

Consider a non-CM elliptic curve E/Q. The Galois action on the torsion points of $E(\overline{Q})$ can be expressed in terms of a Galois representation $\rho_E : Gal(\overline{Q}/Q) \to GL_2(\widehat{Z})$. A famous theorem of Serre say that the image of ρ_E is an open, and hence finite index, subgroup of $GL_2(\widehat{Z})$. We shall describe ideas that allows us to explicitly compute the image in Serre's theorem for any non-CM E/Q. When viewed in terms of modular curves, this seems very difficult at first since there are infinitely many curves that need to be considered. However, it becomes tractable when we put the relevant modular curves in suitable families of twists.

Contributed Talks

Wednesday Session

Petar Orlić (University of Zagreb) : "Gonality of modular curves and their quotients"

Kenji Terao (University of Warwick): "Maps between isolated points on modular curves"

Sho Yoshikawa (Tokyo University of Science) : "Modularity of Elliptic Curves over some totally real fields"

Friday Session

Mar Curcó-Iranzo (Utrecht University): "Q-torsion of generalised modular Jacobians"

Diana Mocanu (University of Warwick): "Conductor computations for the modular method via cluster pictures"

Elie Studnia (Université Paris Cité): "(Anti-)symplectic congruences of elliptic curves mod 7"

List of Participants

David Kurniadi Angdinata	University College London
Samuele Anni	Aix-Marseille Université
Jennifer Balakrishnan	Boston University
Lee Berry	King's College London
Bijay Raj Bhatta	University of Manchester
Pedro-José Cazorla Garcia	University of Manchester
John Cremona	University of Warwick
Mar Curcó-Iranzo	Utrecht University
Mike Daas	University of Leiden
Maarten Derickx	
Xenia Dimitrakopoulo	University of Warwick
Tommaso Faustini	University of Warwick
Stevan Gajović	Charles University
Alexandros Groutides	University of Warwick
Joseph Harrison	University of Warwick
Maleeha Khawaja	University of Sheffield
Alexandros Leivaditis	University of Leiden
Elvira Lupoian	University of Warwick
Steffen Müller	University of Groningen
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