STRONG CONVERGENCE OF RANDOM PERMUTATIONAL REPRESENTATIONS

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Warwick, 08 . 07 . 2026

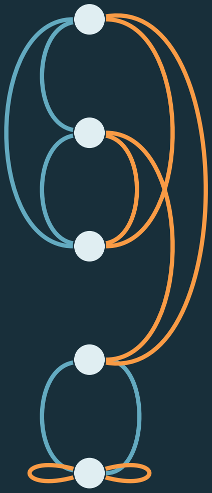
- **Expander family:** Sequence (X_n) of d-regular connected graphs such that

$$d - \|A(X_n)\downarrow_{\mathbf{1}^\perp}\| \geq \epsilon > 0\,.$$

- **Friedman (2003):** Random graphs are **optimal expanders** with high probability.
- In this model,

$$A(X_n)\downarrow_{\mathbf{1}^\perp} = \sum P_\sigma \downarrow_{\mathbf{1}^\perp} + P_\sigma^* \downarrow_{\mathbf{1}^\perp}\ .$$

- **Bordenave and Collins (2019):** Permutational matrices restricted to $\mathbf{1}^\perp$ **strongly converge** to the left regular representation of the free group.



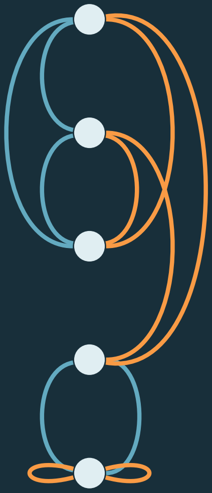
- **Random permutational representation:**

$$\Gamma \xrightarrow{\varphi_n} G_n \xrightarrow{\pi_n} \mathrm{Sym}(n) \xrightarrow{\mathrm{std}} U_{n-1}(\mathbf{C}).$$

- **Strong convergence:** For every $x \in \mathbf{C}[\Gamma]$,

$$\lim_n \|\mathrm{std} \circ \pi_n \circ \varphi_n(x)\| = \|\lambda(x)\| .$$

- **Chen, Garza-Vargas, Tropp and van Handel (2026):** Proofs of strong convergence can be accessed via the **polynomial method**.
  - **Effective asymptotic expansion.**
  - **Temperedness of leading coefficients.**



- **B. and Jezernik (2026+).**
  - **Γ free products of finite groups.**
  - $G_n = \mathrm{Sym}(n)$.
  - $\pi_n = \mathrm{id}$.
- **Cassidy (2025+).**
  - $\Gamma = F_k$.
  - $G_n = \mathrm{Sym}(n)$.
  - **π_n action on k-subsets, $k \leq n^{1/20}$.**
- **Which other permutational representations might we consider?**