

- Persi Diaconis

**Title :** Using Schur-Weyl duality to diagonalize a Markov chain

**Abstract :** The Burnside process is a simple algorithm for random sampling and counting of a uniform orbit of a finite group  $G$  acting on a finite set  $X$ . It has MANY practical applications –sampling random partitions, conjugacy classes of finite groups(counting conjugacy classes in finite unipotent groups), and unlabelled objects in combinatorial problems (Polya trees). Actually running the Burnside process calls for new group theory. Often, the algorithm seems to work extremely well in practice but careful analysis is challenging. If  $X$  is the space of binary  $n$ -tuples and  $G$  is  $S_n$ , acting coordinatewise, Arun Ram, Andrew Lin and I have carried things out. It turns out that  $SL(2, \mathbb{C})$  also acts and Schur-Weyl duality (and old fashioned hard work) lead to sharp results. Many open problems remain.

- Daniele Dona

**Title :**

**Abstract :**

- Sean Eberhard

**Title :** Diameter bounds for arbitrary finite groups and applications

**Abstract :** I will describe recent joint work with Elena Maini, Luca Sabatini, and Gareth Tracey giving a new general-purpose bound for the diameter of a finite group. The diameter of a group is by definition the maximal diameter of its Cayley graphs. We show that the diameter is bounded in terms of the diameters of the composition factors and the maximal exponent of a normal abelian section.

This leads to a number of consequences. For example, every soluble transitive subgroup of  $\text{Sym}(n)$  has diameter at most  $n^C$  for some constant  $C$ . This extends to transitive subgroups whose nonabelian composition factors are constrained to any given finite set. More generally, assuming Babai’s conjecture, the same conclusion holds for all transitive permutation groups, confirming a folkloric conjecture.

I will also explain how these finite results have applications to infinite groups. In particular, every finitely generated subgroup of the automorphism group of a bounded-degree rooted tree satisfies Grigorchuk’s gap conjecture. Under Babai’s conjecture, this extends to all residually finite groups.

- Valentina Grazian

**Title :** Finite groups with soluble centralisers

**Abstract :** In this talk, we present results on finite groups in which the centralisers of non-central  $\pi$ -elements are soluble, for a given set of primes  $\pi$ . Using a

range of structural and reduction techniques, we classify the possible non-abelian sections of such groups. As particular cases, we consider the situations where  $\pi$  is the set of all prime divisors of the group (so all non-central elements of the group have soluble centralisers) and where  $\pi = \{2\}$ . These results also yield applications to solubility criteria arising from the non-commuting graph of the group. This is joint work with Carmine Monetta and Gareth Tracey.

- Robert Guralnick

**Title :** Fixed points for primitive actions of finite and algebraic groups

**Abstract :** Let  $G$  be a finite primitive permutation group. We discuss the proof of a strong version of Peter Neumann's conjecture on the minimal number of fixed points an element in the point stabilizer in a primitive action can have. We will focus on the affine case and results required for the proof. This is joint work with Garzoni and Liebeck and uses earlier work with Malle.

- Scott Harper

**Title :** Minimal generating sets and the product replacement algorithm

**Abstract :** The set of  $n-1$  transpositions  $\{(1, 2), (2, 3), \dots, (n-1, n)\}$  is a minimal generating set for  $\text{Sym}(n)$ , in the sense that no proper subset of it is a generating set. For a finite group  $G$ , write  $m(G)$  for the maximal size of a minimal generating set in this sense. Whiston (2000) showed that this example is optimal:  $m(G)$  is at most  $n-1$  for any subgroup  $G$  of  $\text{Sym}(n)$  with equality if and only if  $G = \text{Sym}(n)$ . This talk will give best possible bounds on  $m(G)$  for arbitrary subgroups of  $\text{Sym}(n)$  and will highlight some new tools for studying  $m(G)$  in general.

One motivation for this work is the product replacement algorithm, which is the standard way to produce random elements of a group. This algorithm involves a random walk on the product replacement graph, and there are a number of open questions about it. I will give some applications of our work in this direction.

This is joint work with Peter Brooksbank, Maria Elisa Fernandes, Dimitri Leemans and Colva Roney-Dougal.

- Harald Helfgott

**Title :**

**Abstract :**

- Ellen Henke

**Title :** Fusion systems, localities and the classification of finite simple groups

**Abstract** : Saturated fusion systems are categories modelling the  $p$ -local structure of finite groups (i.e. the structure of the normalizers of non-trivial  $p$ -subgroups of finite groups). They play a role in Aschbacher’s program to revisit the proof of the classification of finite simple groups, in the modular representation theory of finite groups and in certain parts of homotopy theory. Linking systems associated to fusion systems were introduced by Broto, Levi and Oliver to study “ $p$ -completed classifying spaces of fusion systems”, but recent results suggest that they are also important from a purely algebraic point of view. Chermak showed that linking systems exist and that every linking system corresponds to a group-like structure called a locality. After an introduction to the subject, I will report in this talk on an attempt at simplifying and extending parts of Aschbacher’s program using the theory of localities.

- Derek Holt

**Title** : Computation in large finite groups

**Abstract** : We survey some of the significant recent developments in Computational Group Theory. These include:

- Completion of the classification of the maximal subgroups of the Monster.
- Computing in large finite matrix groups by constructing composition trees of the groups, and using constructive recognition algorithms for the finite almost simple groups.
- Using “hybrid group” data structures to compute in groups with a large normal solvable subgroup. This enabled Alexander Hulpke to compute the character table of the maximal subgroup of the Monster with structure  $2^{10+16}O^+(10, 2)$ .

- Martin Kassabov

**Title** : Monotone parameters on Cayley graphs of finitely generated groups

**Abstract** : We construct a new large family of finitely generated groups with uncountably many values of the following monotone parameters: spectral radius, critical probabilities, and asymptotic entropy. We also present several open problems on other monotone parameters. The construction is quite flexible and is based on limits of marked groups (joint with I. Pak).

- Martin Liebeck

**Title** : Congruence classes in general linear groups

**Abstract** : There is a substantial literature on classifying congruence classes of matrices in  $GL(n, K)$  and the isometry groups of the corresponding bilinear forms. However, even for finite fields  $K$  these questions are not completely resolved. I will

survey some of the literature and present some recent work with Eamonn O'Brien and Scott Harper which finishes the problem in the finite case.

- Richard Lyons

**Title :** The Classification Grid

**Abstract :** This will be a report on the progress of the CGLSS (or GLS) revision project for the classification of the finite simple groups. The Classification Grid, in particular, is very near completion.

- Gunter Malle

**Title :** Overgroups of Sylow normalisers and character correspondences

**Abstract :** We report about the determination of subnormalisers of elements in finite groups of Lie type. These are certain, often maximal, overgroups of normalisers of Sylow subgroups. Their investigation is motivated by conjectured character correspondences proposed by Moreto and Rizo, widely generalising the McKay conjecture.

- Chris Parker

**Title :**

**Abstract :**

- Kevin Piterman

**Title :** Quillen's conjecture for  $p = 2$  and Levi subgroups

**Abstract :** Quillen's conjecture states that if a finite group  $G$  has no non-trivial normal  $p$ -subgroup, then its poset of non-trivial  $p$ -subgroups is not contractible. The conjecture has been approached through a combination of group-theoretic and topological methods. In 1992, Michael Aschbacher and Stephen D. Smith showed that the conjecture holds for  $p > 5$ , provided that certain almost simple groups with unitary socle satisfy the Quillen dimension property, a homological condition on the  $p$ -subgroup poset. A few years ago, in joint work with S.D. Smith, we extended their result to every odd prime and obtained further reductions towards the case  $p = 2$ . More recent work of Antonio Daz Ramos establishes the required homological condition for unitary groups, completing the proof of the conjecture for odd primes  $p$ .

In this talk, I will discuss the topological perspective in the context of the Aschbacher–Smith approach to the conjecture, with emphasis on the prime  $p = 2$ . More specifically, any minimal counterexample at  $p = 2$  must involve simple components of classical Lie type in odd characteristic, and for every such component

$L$ , an almost simple group with socle  $L$  must fail the Quillen dimension property. The problem then reduces to a question about geometries associated with classical groups; in particular, it leads naturally to the study of the homology of Levi subgroup posets and direct-sum decomposition posets.

- Cheryl Praeger

**Title** : Nicely embedded simple subgroups of finite simple groups

**Abstract** : Simple groups often contain families of smaller simple subgroups, and usually there is a “most natural” or “nicest” kind. Our interest is in constructing these nice embeddings efficiently. In the case of finite classical groups our objective is a new fast constructive recognition algorithm for these groups (the vision of our former colleague Akos Seress): the talk addresses one crucial blockage which we have just overcome after more than a decade of work - to show how these nicely embedded subgroups could be constructed with high probability by two very special random elements called stingray elements. This is joint work with Alice Niemeyer and Stephen Glasby.

- Colva Roney-Dougal

**Title** : Cameron’s greedy conjecture

**Abstract** : A base for a subgroup  $G$  of  $\text{Sym}(\Omega)$  is a sequence of points of  $\Omega$  whose pointwise stabiliser in  $G$  is trivial. For  $G$  finite, the base size  $b(G)$  of  $G$  is the size of a smallest possible base. Blaha showed in 1992 that computing  $b(G)$  is  $NP$ -hard, but since many permutation group algorithms have running time depending on the size of a known base, we are interested in efficiently finding small bases. There is a natural greedy algorithm to construct a base, which repeatedly stabilises any point in any longest remaining orbit until the resulting group is trivial. In 1999, Peter Cameron conjectured that there exists a constant  $c$  such that if  $G$  is primitive then every base constructed by this algorithm has size at most  $cb(G)$ . I will report on progress towards proving this conjecture, including some recent joint work with Hong Yi Huang on the case where  $G$  is of diagonal type.

- Ronald Solomon

**Title** : On strongly closed  $p$ -subgroups and the CFSG

**Abstract** : Representation theory played a fundamental role in the early phase of work on the classification of the finite simple groups. Beginning in the 1970s, this was largely superseded by the methods of local group theory, except for the repeated application of Glauberman’s  $Z^*$ -Theorem concerning finite groups with a strongly closed subgroup of order 2. In the recent work of Lyons and myself, we encountered serious difficulties involving strongly closed subgroups of order 3.

Fortunately for us, using representation theorem, Glauberman and Geoff Robinson were able to provide a very useful theorem in this setting. I will state a stronger conjecture, discuss the Glauberman-Robinson Theorem, and give an example of how Lyons and I apply it.

- Britta Späth

**Title** : The Alperin-McKay conjecture and blocks of maximal defect

**Abstract** : For blocks of maximal defect the Alperin-McKay conjecture can be seen as a blockwise refinement of the McKay conjecture. Some of the techniques and results used in proving the McKay conjecture help in the context, other need some refinement. I will explain what additional block-theoretic problems arise in this situation and how they can be solved using Dade's ramification groups. This is joint work with Lucas Ruhstorfer.

- Gernot Stroth

**Title** : The Uniqueness Case: The unique uniqueness group

**Abstract** : The purpose of this talk is to sketch a proof of the following: If  $G$  is a  $\mathcal{K}$ -proper simple group, then all uniqueness subgroups are conjugate. This is a milestone when dealing with groups in the uniqueness case. The methods used in this proof in a more sophisticated form eventually are used for a proof that there is no  $\mathcal{K}$ -proper simple group in the uniqueness case.

- Jay Taylor

**Title** : The Challenges of Navarro's Galois-McKay Conjecture

**Abstract** : Recently the long-standing McKay Conjecture has been resolved by Cabanes' Späth using the classification of finite simple groups so, naturally, attention has turned to the stronger forms of the McKay conjecture. In this talk I'll discuss the challenges presented by one of these, Navarro's Galois-McKay Conjecture (which incorporates fields of values), as well as describe some of the tools that can be used to attack the conjecture for groups of Lie type.

- Gareth Tracey

**Title** : The Goldschmidt-Sims conjecture

**Abstract** : In 1967, Charles Sims conjectured that point stabilisers in primitive permutation groups have order bounded above by a function of their smallest non-trivial orbit size. The conjecture was proved by Cameron, Praeger, Saxl and Seitz in 1983. In this talk, I will speak about the Goldschmidt-Sims conjecture: a

generalisation of the Sims conjecture which has applications in a variety of different areas of mathematics. Joint work with Laci Pyber.

- Martin Van Beek

**Title** : Fusion systems and simple groups with class two Sylow  $p$ -subgroups

**Abstract** : In this talk, I will describe how to determine all reduced saturated fusion systems supported on finite  $p$ -groups of nilpotency class two. I will then describe how to use this result at the prime 2 to obtain a new proof of Gilman and Gorenstein's classification of finite simple groups which have a Sylow 2-subgroup of nilpotency class at most two.