Abstract: When $L_\infty(X, \mathcal{L}, \lambda)$ is a σ-finite measure space, a necessary and sufficient test for a bounded sequence in $L_\infty(X, \mathcal{L}, \lambda)$ that is pointwise convergent λ-almost-everywhere to be weakly convergent follows from a crash course on the representation of $L_\infty(X, \mathcal{L}, \lambda)^*$ as a class of finitely additive measures [Yosida-Hewitt 1952].

When, in addition, $(X, \mathcal{L}, \lambda)$ is the Borel measure space of a locally compact Hausdorff topological space X, and X_∞ is its one-point-compactification, the resulting criterion can be localized to points of X_∞ .

The purpose is mainly pedagogical and the target audience includes students, or anyone who is nervous about representing the dual of $L_\infty(X, \mathcal{L}, \lambda)$ by finitely additive measures, knowing that

- on $\mathbb{N}$ with the power set σ-algebra there are uncountably many distinct finitely additive measures $\nu \geq 0$ with $\nu(\mathbb{N}) = 1$, but $\nu(S) = 0$ for any finite set $S \subset \mathbb{N}$.

- *there exist uncountably many, linearly independent, finitely additive measures* $\nu \geq 0$ *defined on the Lebesgue* σ*-algebra of* $(0, 1)$ *with the property that*

$$\int_0^{1-\frac{1}{k}} u \, d\nu = 0 \text{ for all } u \in L_\infty(0,1) \text{ and } k \in \mathbb{N}, \text{ but } \int_0^1 1 \, d\nu = 1.$$

A goal will be to come to terms with observations such as these, that violate intuition based on classical measure theory, and answer questions such as

- does $\left\{\sin\left(\frac{1}{kx}\right)\right\}_{k \in \mathbb{N}} \subset L_\infty(0, \pi)$ have a weakly convergent subsequence?

- what is meant by weak convergence to zero in ℓ_∞?