Title: On $l^2$-Betti numbers and their analogues in positive characteristic

Let $G$ be a group, $K$ a field and $A$ a $n$ by $m$ matrix over the group ring $K[G]$. Let $G = G_1 > G_2 > G_3 \cdots$ be a chain of normal subgroups of $G$ of finite index with trivial intersection. The multiplication on the right side by $A$ induces linear maps

$$\begin{array}{rccc} \phi_i : & K[G/G_i]^n & \to & K[G/G_i]^m \\ & & & \\ & (v_1, \ldots, v_n) & \mapsto & (v_1, \ldots, v_n)A. \end{array}$$

We are interested in properties of the sequence $\{\frac{\dim_K \ker \phi_i}{|G:G_i|}\}$. In particular, we would like to answer the following questions.

(1) Is there the limit $\lim_{i\to\infty} \frac{\dim_K \ker \phi_i}{|G:G_i|}$?
(2) If the limit exists, how does it depend on the chain $\{G_i\}$?
(3) What is the range of possible values for $\lim_{i\to\infty} \frac{\dim_K \ker \phi_i}{|G:G_i|}$ for a given group $G$?

It turns out that the answers on these questions are known for many groups $G$ if $K$ is a number field, less known if $K$ is an arbitrary field of characteristic 0 and almost unknown if $K$ is a field of positive characteristic.

In my talk I will give several motivations to consider these questions, describe the known results and present recent advances in the case where $K$ has characteristic 0.