

Combinatorics Seminar

Friday November 9, 2012 at 2PM

Room MS.03

Matthew Yancey

(University of Illinois at Urbana-Champaign)

Ore's conjecture on color-critical graphs is almost true

A graph G is k -critical if it has chromatic number k , but every proper subgraph of G is $(k - 1)$ -colorable. Let $f_k(n)$ denote the minimum number of edges in an n -vertex k -critical graph. We give a lower bound, $f_k(n) \geq F(k, n)$, that is sharp for every $n = 1 \pmod{k - 1}$. It is also sharp for $k = 4$ and every $n \geq 6$. The result improves the classical bounds by Gallai and Dirac and subsequent bounds by Krivelevich and Kostochka and Stiebitz. It establishes the asymptotics of $f_k(n)$ for every fixed k . It also proves that the conjecture by Ore from 1967 that for every $k \geq 4$ and $n \geq k + 2$, $f_k(n + k - 1) = f_k(n) + \frac{k-1}{2}(k - \frac{2}{k-1})$ holds for each $k \geq 4$ for all but at most $k^3/12$ values of n . This is joint work with Alexandr Kostochka.



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