WEAK INVARIANCE PRINCIPLE FOR THE LOCAL TIMES OF GIBBS-MARKOV PROCESSES

ABSTRACT. A classical invariance principle (central limit theorem) for random walks states that if $X_1, X_2, \ldots$ is a sequence of independent, identically distributed random variables with mean 0 and finite variance, then $\frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i$ converges in law to the Gaussian distribution. The functional invariance principle states that if we define $S_n := \sum_{k=1}^{n} X_i$ and $W_n(t) := \sum_{k=1}^{[nt]} S_k$ then the random functions $\frac{1}{\sqrt{n}} W_n$ converge in law to the Brownian motion. An enhanced invariance principle for the local time process (which does not seem to follow from the functional invariance principle) states that the suitably normalized occupation times of the processes S_n, namely the random functions $l_n(x) := \frac{1}{n} \# \left\{ 1 \leq k \leq n : S_k = \left[\sqrt{n} x \right] \right\}$ converge in law to the local time of the Brownian motion (the definition above is suitable for integer valued processes and there exists a corresponding definition for $\mathbb{R}$ valued processes).

While there exist generalizations of the CLT and the functional invariance principle for weakly dependent processes, in particular processes of the form $X_n = \varphi \circ T^n$ where (X, T) is a topological Markov shift endowed with a Gibbs measure , there were no such generalizations (at least known to me) for the invariance principle for local times. We discuss the main elements of the proof of the invariance principle for local times of topological Markov shifts together with some recent applications to the measure theoretical isomorphism problem of dynamical systems.