An elementary proof for the dimension of the graph of the classical Weierstrass function

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Let $W_{\lambda,b}(x)=\sum_{n=0}^{\infty}\lambda^n g(b^n x)$ where $b\geqslant 2$ is an integer and $g(u)=\cos(2\pi u)$ (classical Weierstrass function) or $b=2$ and $g(u)=\mathrm{dist}(u,\mathbb{Z})$. Building on recent work by Baránsky, Bárány and Romanowska and on a 2001 paper by Tsujii, we provide elementary proofs that the Hausdorff dimension of $W_{\lambda,b}$ equals $2+\frac{\log\lambda}{\log b}$ for all $\lambda\in(\lambda_b,1)$ with a suitable $\lambda_b<1$. This reproduces results by Ledrappier and Baránsky, Bárány and Romanowska without using the dimension theory for hyperbolic measures of Ledrappier and Young, which is replaced by a simple telescoping argument together with a recursive multi-scale estimate.