How sticky is the chaos/order boundary?

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Hamiltonian dynamical systems with mixed phase space are ubiquitous, but notoriously poorly understood, partly due to complicated structure of the boundary between chaotic and ordered regions. Mushroom billiards were introduced by Bunimovich in 2001 as an example of sharply divided phase space. A billiard comprises a point particle moving uniformly in a specified region except for mirror-like reflections from the boundary, here shaped like a mushroom with a semicircular cap and polygonal stem. Later, Altmann and others pointed out that almost all mushroom billiards have parabolic orbits embedded in the chaotic region leading to "stickiness", algebraic slowing of the chaotic expansion and mixing properties. A zero measure set of mushroom parameters for which these orbits are absent, and the remaining stickiness, originating from the boundary of the chaotic region itself, can be characterised using Diophantine approximation methods. The results may shed light on the parameter dependence of stickiness in more general Hamiltonian systems.