The Jacquet-Langlands correspondence on quaternion groups and the transfer operator for the Hecke congruence groups $\Gamma_0(d)$

A special case of the Jacquet-Langlands spectral correspondence shows that the spectrum of the automorphic Laplacian $\Delta_{\mathcal{O}^1}$, $\mathcal{O}^1$ the unit group of a maximal order $\mathcal{O}$ in an indefinite quaternion division algebra $D/\mathbb{Q}$ over the rationals with discriminant d, coincides with the spectrum of the automorphic Laplacian $\Delta_{\Gamma_0(d)}$ when restricted to the space of new Maass cusp forms of the cofinite Hecke congruence group $\Gamma_0(d)$. In the transfer operator approach to the automorphic spectral theory of these Hecke congruence groups $\Gamma_0(n)$ this operator coincides with the transfer operator for the modular group $PSL(2,\mathbb{Z})$ twisted with the permutation representation χ_{ind} induced from the trivial character χ_0 of $\Gamma_0(n)$. Since $\chi_{\text{ind}}(PSL(2,\mathbb{Z}))$ is isomorphic to the finite group $PSL(2,\mathbb{Z})/H(n)$ with $H(n)$ the maximal normal subgroup of $PSL(2,\mathbb{Z})$ contained in $\Gamma_0(n)$, one can decompose χ_{ind} into the irreducible representations χ_i of this finite group. Thereby the spectrum of $\Delta_{\Gamma_0(n)}$ gets decomposed into the spectra of the automorphic Laplacians Δ_{χ_i}. In my talk I will discuss this decomposition for $n = d$ the discriminant of a cocompact quaternion group and show that all the subrepresentations χ_i are singular and hence Δ_{χ_i} has a continuous spectrum and cannot coincide with the purely discrete spectrum of $\Delta_{\mathcal{O}^1}$.