THERMODYNAMIC FORMALISM FOR GEODESIC FLOW ON CAT(-1) SPACES

DAN THOMPSON (OHIO STATE)

In the 1980s, Gromov popularized the study of CAT(-1) spaces, spaces which satisfy a metric notion of negative curvature. These spaces are not necessarily manifolds, covering examples such as graphs equipped with an interior metric, yet they still have a geodesic flow defined on them. While there are analogies with the dynamics of the geodesic flow on a negative curvature manifold, the full power of uniformly hyperbolic dynamics is not currently available in this setting: there is still a coding of the geodesic flow by a suspension flow over a shift of finite type, but rather than being a conjugacy, it is via an orbit semi-equivalence.

In general, orbit equivalence is too weak a relationship to preserve any refined dynamical information. However, in this setting, we are able to use a geometric argument to show that the geodesic flow inherits the weak specification property from the suspension flow. We then use the specification property directly on the geodesic flow for CAT(-1) spaces, obtaining thermodynamic results analogous to the negative curvature Riemannian setting. In particular, we prove uniqueness of equilibrium states for Holder potential functions, the large deviations principle, and the equidistribution of weighted periodic orbits. This is joint work with Dave Constantine (Wesleyan) and Jean-Francois Lafont (Ohio State).