

## THE BETA TRANSFORMATION WITH A HOLE AT 0

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Motivated by the open dynamical systems and Diophantine approximation in dynamical systems, we consider the beta transformation with a hole at 0. Given  $\beta \in (1, 2)$  the beta transformation  $T_\beta$  on the circle  $[0, 1]$  is defined by  $T_\beta(x) = \beta x \pmod{1}$ . For  $t \in [0, 1]$  let  $K_\beta(t)$  be the survivor set of  $T_\beta$  with hole  $(0, t)$  given by

$$K_\beta(t) := \{x \in [0, 1] : T_\beta^n(x) \notin (0, t) \text{ for all } n \geq 0\}.$$

In this talk I will characterize the bifurcation set  $E_\beta$  of all parameters  $t \in [0, 1]$  for which the set-valued function  $t \mapsto K_\beta(t)$  is not locally constant. It turns out to be rather different from that for doubling map. This is based on using the Lyndon words and Farey words from the symbolic interpretation of  $E_\beta$ . Furthermore, I will describe for each  $\beta \in (1, 2)$  the critical value  $\tau_\beta$ , such that the Hausdorff dimension of  $K_\beta(t)$  is positive if and only if  $t < \tau_\beta$ . This is a joint work with Charlene Kalle, Niels Langeveld and Wenxia Li.