Dynamical properties of biparametric skew tent maps (joint work with Zoltan Buczolich)

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We consider skew tent maps $T_{\alpha,\beta}(x)$ with $T_{\alpha,\beta}(x) = \frac{\beta}{\alpha}x$ for $0 \leq x \leq \alpha$ and $T_{\alpha,\beta}(x) = \frac{\beta}{1-\alpha}(1-x)$ for $\alpha < x \leq 1$. With this choice of parameters $T_{\alpha,\beta}$ maps $[0,1]$ into $[0,1]$ and (α, β) is the vertex of $T_{\alpha,\beta}$. The dynamics of $T_{\alpha,\beta}$ for $(\alpha,\beta) \in [0,1]^2$ is interesting when $(\alpha,\beta) \in U = \{(\alpha,\beta) : 0.5 < \beta \leq 1,\ 1-\beta < \alpha < \beta\}$. Denote by $h(\alpha,\beta)$ the topological entropy of $T_{\alpha,\beta}$. It is well-known that $h(\alpha,\beta)$ is strictly monotone increasing along vertical line segments in U. It is natural to ask what happens if we move in the horizontal direction, that is for fixed β we consider $h(\alpha) = h(\alpha,\beta)$, for $(\alpha,\beta) \in U$. Turned out that $h(\alpha)$ is strictly monotone increasing on $(1-\beta,\beta)$. To deal with this question one needs to consider equi-topological entropy curves in the square. We denote by $\underline{M} = K(\alpha,\beta)$ the kneading sequence of $T_{\alpha,\beta}$ and by $\Lambda = \Lambda_{\alpha,\beta}$ its Lyapunov exponent. For a given kneading squence $\underline{M}$ we consider isentropes (or equi-topological entropy, or equi-kneading curves), $(\alpha, \Psi_{\underline{M}}(\alpha))$ such that $K(\alpha, \Psi_{\underline{M}}(\alpha)) = \underline{M}$. On these curves the topological entropy $h(\alpha, \Psi_{\underline{M}}(\alpha))$ is constant. We show that $\Psi'_{\underline{M}}(\alpha)$ exists and the Lyapunov exponent $\Lambda_{\alpha,\beta}$ can be expressed by using the slope of the tangent to the isentrope. Since this latter can be computed by considering partial derivatives of an auxiliary function $\Theta_{\underline{M}}$ a series depending on the kneading sequence which converges at an exponential rate, this provides an efficient new method of finding the value of the Lyapunov exponent of these maps.