

Test 2

- Write your full name and email on the first sheet
- Time: **50 minutes**
- Books, notes and calculators **are not allowed**

Problem 1 Compute the determinant of the skew-symmetric $2n \times 2n$ matrix A_n with all entries above the diagonal being 1:

$$A_n = \begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ -1 & 0 & 1 & \dots & 1 \\ -1 & -1 & 0 & \dots & 1 \\ & \vdots & & \ddots & \vdots \\ -1 & -1 & -1 & \dots & 0 \end{pmatrix}.$$

Problem 2 Suppose that the real polynomial

$$P(x) = x^n + a_{n-1}x^{n-1} + \dots + a_1x + 1$$

with non-negative coefficients has n real roots. Prove that

$$P(2) \geq 3^n.$$