

Test 1

- Write your full name and email on the first sheet
- Time: **50 minutes**
- Books, notes and calculators **are not allowed**

Problem 1 Let $A = (a_{ij})_{i,j=1}^n$ be an $n \times n$ -matrix with real entries such that, for every $i \in \{1, \dots, n\}$, we have $|a_{ii}| > \sum_{j \in \{1, \dots, n\} \setminus \{i\}} |a_{ij}|$ (that is, the absolute value of each diagonal entry is strictly larger than the sum of the absolute values of the other entries from the same row). Prove that the matrix A is invertible.

Problem 2 Compute the limit of $n^2 f(n)$ as $n \rightarrow \infty$, where

$$f(n) = \left(1 + \frac{1}{n+1}\right)^{n+1} - \left(1 + \frac{1}{n}\right)^n, \quad n \in \mathbb{N}.$$