

## Test 2

- Write your full name and email on the first sheet
- Time: **50 minutes**
- Books, notes and calculators **are not allowed**

**Problem 1** Find all rational solutions  $(x, y) \in \mathbb{Q}^2$  to the equation  $x^2 - x + y^2 = 1$ . [Hint: one solution is  $(0, 1)$ .]

**Problem 2** Let  $M$  be a convex polygon in  $\mathbb{R}^2$  and let  $N = \{-\frac{1}{2} \cdot x : x \in M\}$ . Show that there is a vector  $v \in \mathbb{R}^2$  such that the translated copy  $N + v := \{x + v : x \in N\}$  lies inside  $M$ .