

## Test 3

- Write your full name and email on the first sheet
- Time: **50 minutes**
- Books, notes and calculators **are not allowed**

**Problem 1** Let an integer  $n \geq 2$  be given. Two players alternatively name distinct positive divisors of  $n$  with the restriction that no multiple of a previously named divisor can be named. The player who is forced to name 1 loses the game. Prove that there is a winning strategy for the player who starts.

**Problem 2** Is there a continuous function  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x)) = -x$$

for all  $x \in \mathbb{R}$ ?