Linear algebra

1. Do there exist two $n \times n$ matrices A and B such that $AB - BA$ is the unit matrix?

2. Let A, B, C and D be $n \times n$ matrices. Suppose that the matrices AB^T and CD^T are symmetric and $AD^T - BC^T$ is the unit matrix. Show that $A^T D - C^T B$ is also the unit matrix.

3. Do there exist polynomials $a(x)$, $b(x)$, $c(y)$ and $d(y)$ such that $a(x)c(y) + b(x)d(y) = 1 + xy + x^2y^2$?

4. Let A be an $n \times n$ matrix such that $|a_{ii}| > \sum_{j \neq i} |a_{ij}|$ for every $i = 1, \ldots, n$. Show that A is regular.

5. Let A and B be two $n \times n$ matrices such that the rank of $AB - BA$ is one. Show that $(AB - BA)^2 = 0$.

6. Let A be an $n \times n$ matrix such that $A^3 - A$ is the unit matrix. Show that the determinant of A is positive.

7. Let A be an $n \times n$ skew-symmetric matrix, i.e., $A = -A^T$. Show that the determinant of A is non-negative.