IMC Selection Test 1

Problem 1. Determine all polynomials $P(x)$ such that $P(x^2+1) = (P(x))^2 + 1$ and $P(0) = 0$.

Problem 2. Let A, B be two square complex matrices satisfying $AB - BA = A$. Prove that $\det(A) = 0$.

Problem 3. Let $x_1, \cdots, x_n \in \mathbb{R}^2$ satisfy $\|x_i\|_2 \leq 1$ for all $1 \leq i \leq n$ and $\|x_i - x_j\|_2 > \frac{1}{2}$ for all $1 \leq i < j \leq n$, where $\|(x,y)\|_2 = \sqrt{x^2+y^2}$ is the Euclidean norm. Show that $n \leq 25$.

Problem 4. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(|f(x) - f(y)|) = f(f(x)) - 2x^2 f(y) + f(y^2) \tag{1}$$

for all $x, y \in \mathbb{R}$.