

Practice problems on convergence and limits

These problems give extra practice for the ideas/concepts discuss in the class. They are not part of selection for Warwick's IMC team but you are strongly encouraged to work on them (and then check your solutions against those published on the IMC seminar webpage).

Problem 1 Let $s_0 := 1$ and, inductively on $n = 1, 2, 3, \dots$, define $s_n := (\frac{6n+1}{n+1} + s_{n-1})^{1/3}$. Does the limit $\lim_{n \rightarrow \infty} s_n$ exist? If yes, determine its value.

Problem 2 Is the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^{(n+1)/n}}$$

convergent?

Problem 3 Let a_0, a_1, \dots be a sequence of positive reals such that a_n is either $a_{n-1}/2$ or $\sqrt{a_{n-1}}$ for each $n \geq 1$. Can this sequence converge to a limit α with $0 < \alpha < 1$? (Fully justify your answer.)

Problem 4 What is the limit

$$\lim_{m \rightarrow \infty} \frac{\sum_{n=1}^m n^{-1/2}}{m^{1/2}} ?$$

Problem 5 Is the sum

$$\sum_{n=10^7}^{\infty} \frac{1}{n \times \ln n \times \ln(\ln(\ln n))}$$

convergent or not? (Justify your answer.)