

IMC SEMINAR
Selection Test 1

Instructions.

- Time: **1 hour**.
- Books, notes, and calculators **are not allowed**.
- Good luck!

Problem 1. Let $a > 0$ and let $f(x)$ be a continuous function on $[0, a]$ such that $f(x) > 0$ and $f(x)f(a - x) = 1$ for every $x \in [0, a]$. Evaluate

$$\int_0^a \frac{dx}{1 + f(x)}.$$

Problem 2. Let H be an $n \times n$ matrix all of whose entries are ± 1 and whose rows are mutually orthogonal. Suppose H has an $a \times b$ submatrix whose entries are all 1. Show that $ab \leq n$.

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