Problem 1. Find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x) - f(y) \in \mathbb{Q} \quad \text{whenever} \quad x - y \in \mathbb{Q}.$$

Problem 2. For each nonnegative integer k, let $d(k)$ denote the number of 1's in the binary expansion of k (for example $d(0) = 0$ and $d(5) = 2$). Let m be a positive integer. Express

$$\sum_{k=0}^{2^m-1} (-1)^{d(k)} k^m$$

in the form $(-1)^m a^{f(m)}(g(m))!$, where a is an integer and f and g are polynomials.

Problem 3. Let $n \geq 4$ be given,and suppose that the points $P_1, P_2, ..., P_n$ are randomly chosen on a circle. Consider the convex n-gon whose vertices are these points. What is the probability that at least one of the vertex angles of this polygon is acute?

Problem 4. Let p be a prime number. Call a positive integer n *interesting* if

$$x^n - 1 = (x^p - x + 1)f(x) + pg(x)$$

for some polynomials $f, g \in \mathbb{Z}[x]$.

(a) Prove that $p^p - 1$ is interesting.

(b) For which primes p is $p^p - 1$ the smallest interesting number?